Digital Image Processing

Course organization

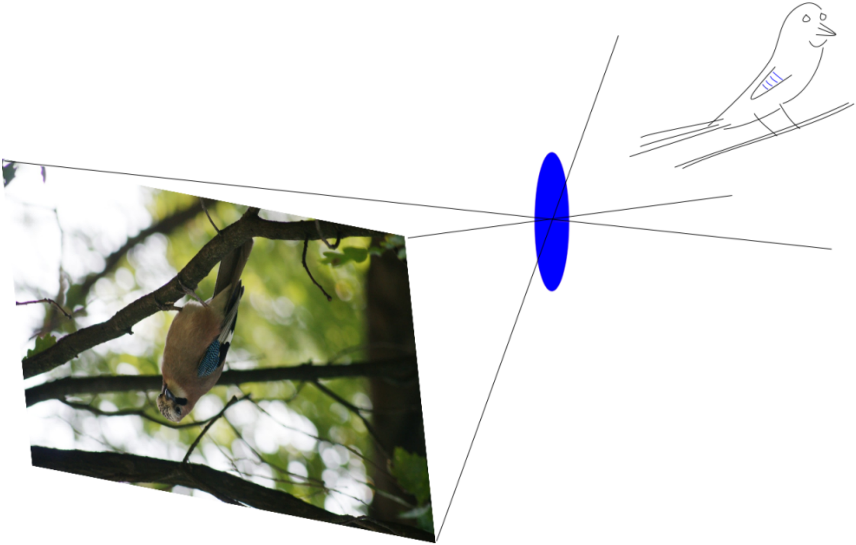
Teachers 2011:

- Lecturer & lab tutor: Ondřej Drbohlav

Courseware:

- `http://cw.felk.cvut.cz`
- ⇒ online discussion of conditions and rules

Digital image - Origin



Digital image - Origin

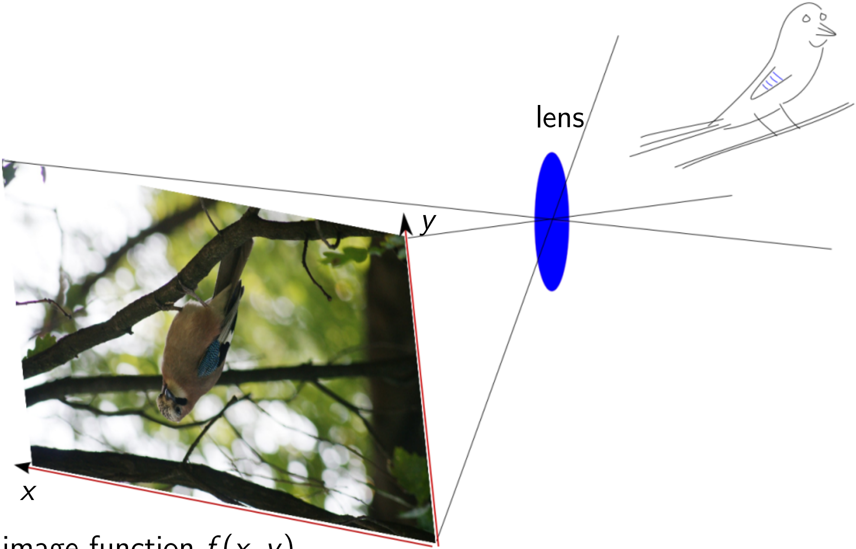


image function $f(x, y)$

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Image function is a mapping:

$$f : Q \mapsto R$$

	domain Q	range R
lives in	$Q \subset \mathbb{R}^2$	various: color $R \subset \mathbb{R}^3$
unit	x, y each: [mm]	each channel $[Wm^{-2}]$

Image function $f(x, y)$

Image function is a mapping:

$$f : Q \mapsto R$$

	domain Q	range R
lives in	$Q \subset \mathbb{R}^2$	various: grayvalue $R \subset \mathbb{R}$
unit	x, y each: [mm]	each channel $[\mathrm{Wm}^{-2}]$

Image function $f(x, y)$ (2)

Image function is a mapping:

$$f : Q \mapsto R$$

This can be regarded as a **set** of ordered pairs $([x, y], value)$.

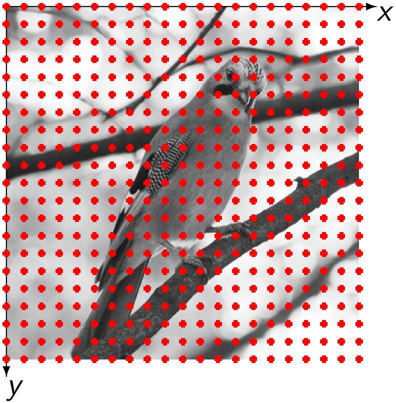
Both Q and R are continuous!

The major part of this lecture will be concerned with how to **represent** the image function in a **digital** form.

Representing image function

This requires use of finite memory space.

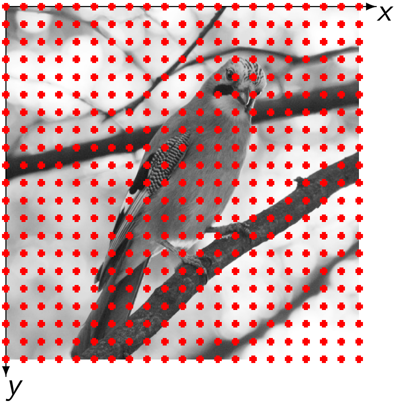
Representing image function



This requires use of finite memory space.

- representing f by finite number of numbers $\Rightarrow$ **sampling**

Representing image function



This requires use of finite memory space.

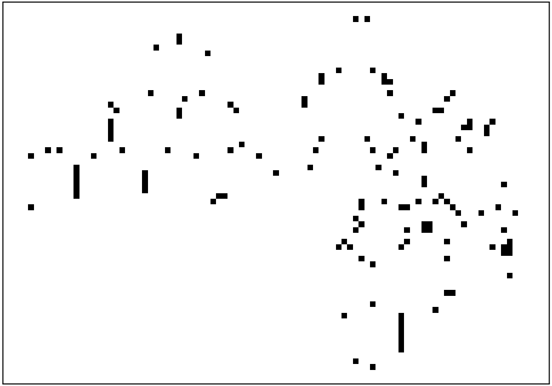
- representing f by finite number of numbers $\Rightarrow$ **sampling**
- at each such point, store the *value* in finite precision $\Rightarrow$ **quantization**.

Sampling (1)

- Representing f using values sampled on a regular grid is by far the most common choice.
- There can be other representations (functional forms, etc.)
- There can be other sampling schemes (hexagonal, irregular, etc.)

Sampling (2)

How to sample properly? Intuitively, the function should not change much between two sampling points. Compare these 60x90 images . . .



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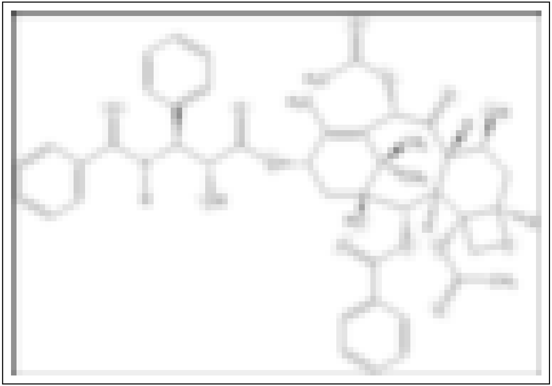
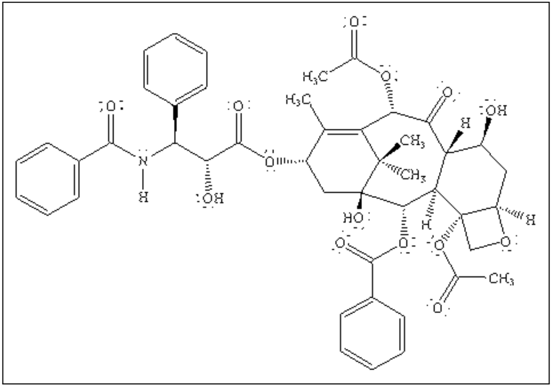


Sampling (2)

How to sample properly? Intuitively, the function should not change much between two sampling points. Compare these 60x90 images . . . and the source image function!



Sampling (3)

- necessary to ensure that there are no high-frequency oscilations in the image function before sampling
- if necessary, filter the function before sampling
- this has relation to aliasing and Nyquist theory — we will be talking about it later.

link: some blackboard scribble

Quantization

256 levels

Quantization

64 levels

Quantization

32 levels

Quantization

16 levels

Quantization

8 levels

Quantization

4 levels

Quantization

2 levels

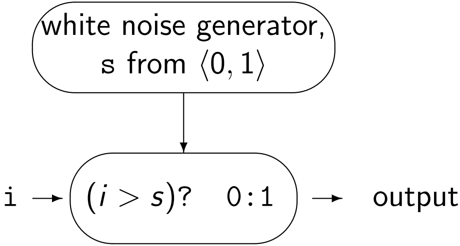
Quantization & sampling — interplay

- Would it be possible to trade quantization for resolution? E.g. using only 2 levels but increasing sampling rate
- . . . not attractive from coding/compression point of view
- . . . but necessary for **creating** the image function at some output devices which use limited number of levels
- E.g. black & white printers
- Displays (Amazon Kindle)
- ⇒ **Dithering**

Dithering (random) (1)

Simple but effective: random dithering

- Idea: represent a number $i \in \langle 0, 1 \rangle$ by an ensamble of 0's and 1's such that their expected value is i.
- How:



Dithering (random) (1)

Simple but effective: random dithering

- Idea: represent a number $i \in \langle 0, 1 \rangle$ by an ensamble of 0's and 1's such that their expected value is i.
- How: (matlab code)

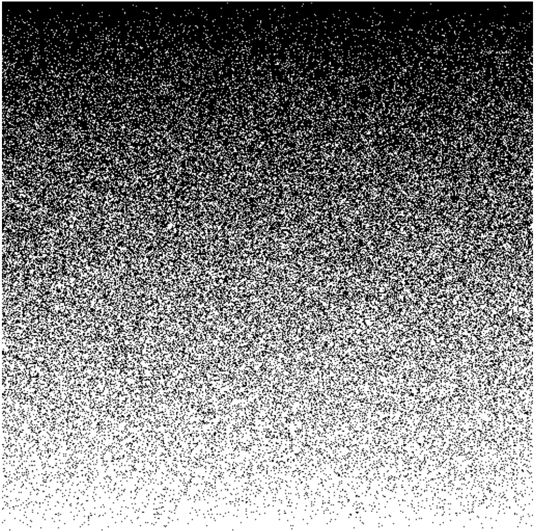
```
function o = dither_randomly(im);
% function o = dither_randomly(im);
% dithers a uint8 image using random
% sampling.
t = 255*rand(size(im));
o = t < im;
```

Dithering (random) (2, Examples)

ramp, 0-255

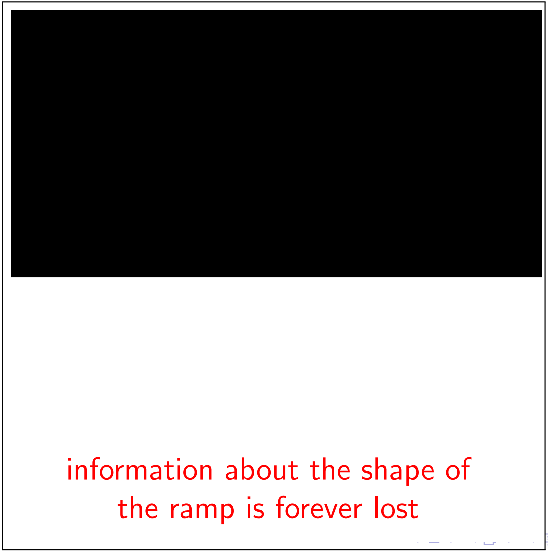
Dithering (random) (2, Examples)

dithered, 0/1



Dithering (random) (2, Examples)

thresholded, 0/1



Dithering (random) (2, Examples)

original, 256 levels

Dithering (random) (2, Examples)

dithered, 0/1

Dithering (3)

Can we do better?

- with the previous approach, the advantage is simplicity
- . . . but the problem is that the output image neighboring pixels are generated completely independently
- leading to sub-optimal result

Dithering (3)

Can we do better?

- with the previous approach, the advantage is simplicity
- ...but the problem is that the output image neighboring pixels are generated completely independently
- leading to sub-optimal result
- another easy way: code and distribute the residuum to neighboring pixels
- $\Rightarrow$ Floyd-Steinberg dithering

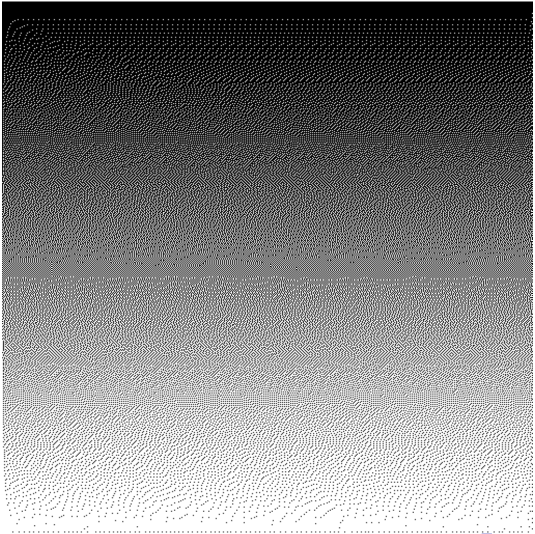
link: blackboard explanation

Dithering (4, Floyd-Steinberg)

ramp, 0-255

Dithering (4, Floyd-Steinberg)

dithered, 0/1



Dithering (4, Floyd-Steinberg)

original, 256 levels

Dithering (4, Floyd-Steinberg)

dithered, $0/1$

Dithering (5, Comparison)

random

Floyd-Steinberg

Dithering (6, Comparison II)

filtered by a Gaussian, $\sigma = 3$

random

Floyd-Steinberg

Dithering (6, Comparison II)

original

Information

- So far, we have seen that with different options of sampling/quantization, different amount of information is lost
- Connected to this is information-theoretic view of an image contents

Histogram, entropy

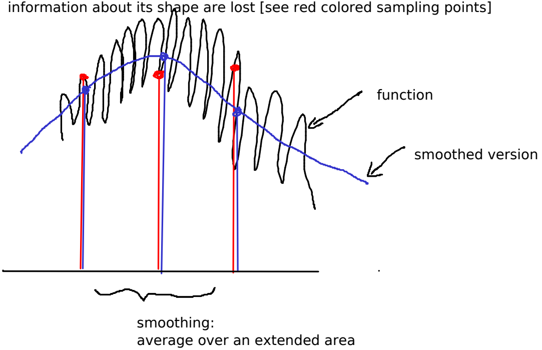
- Histogram: stores frequencies $q(i)$ for all values i in an image
- for a gray-scale, 8 bit image: 256 bins
- probability of a given intensity value is

$$p(i) = q(i)/N\,,$$

  N is the number of pixels in an image
- entropy:

$$H = -\sum_{i=0}^{255} p(i)\log_2 p(i)\,.$$

Problems with sampling an image function. When the function oscilates to a large extent in between the sampling locations, information about its shape are lost [see red colored sampling points]



To retain at least some information about the shape of the function, the function has to be made smooth prior to sampling [see blue colored sampling points].

Floyd - Steinberg dithering algorithm - example. The goal is to represent an image by values which are either 0 or 255.

100 closer to 0 than to 255. Replace this value by 0. Error is 100-0=100. Distribute this error to the 4 surrounding pixels = increment them all by 25.

	100	100	100	100
100	100	100	100	100

→

	0	125	100	100
125	125	125	100	100

Then, continue with the next pixel in a left-to-right, top-to-bottom manner.

Note that this is the demonstration of the principle. In the actual Floyd-Steinberg algorithm, the error is not distributed to the four neighbors evenly, but by the following weights:

$$\frac{1}{16}\begin{bmatrix} & X & 7 \\ 3 & 5 & 1 \end{bmatrix}$$