Image preprocessing in a local neighborhood

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Outline of the lecture:

- Noise filtration (and edge detection in other lecture.
- Noise and its statistical nature.
- Space invariant filters.
- Discrete 2D convolution.
- Separable filters.
- Nonlinear noise filtration.

Image preprocessing, the intro

The input is an image, the output is an image too.

The image is not interpreted.

The aim

- To suppress the **distortion** (e.g., correction of the geometric distortion caused by spherical shape of the Earth taken from a satellite).
- **Contrast** enhancement (which is useful only if the human looks at the image).
- Noise suppression.
- **Enhancement of some image features** needed for further image processing, e.g., edge finding.

Local image preprocessing operations

The specific knowledge about the image and distortions (i.e., semantics) in not used in preprocessing.

Taxonomy from the usage point of view:

1. Smoothing.
2. Edge detection (gradient operators, image sharpening).

Taxonomy according character of the mathematical description:

1. Linear.
2. Nonlinear.

Statistical principal of noise filtration

Assume that each pixel is contaminated by additive noise:

- which is statistically independent of the image function,
- has a zero mean μ,
- and has a zero standard deviation σ.

Let have i realization of the image, $i = 1, \ldots n$. The estimate of the correct value is

$$\frac{g_1 + \ldots + g_n}{n} + \frac{\nu_1 + \ldots + \nu_n}{n}\,.$$

The outcome is a random variable with $\mu' = 0$ and $\sigma' = \sigma/\sqrt{n}$.

Smoothing from several images without blurring

Assumptions: n images of the same unchanged scene, in which it can be assumed that noise is independent to the imaage.

The correct intensity value: $f(i,j)$ is estimated from a random population given by all pixels at the same position in all input images $g_k(i,j)$, e.g., by ordinary averaging,

$$f(i,j) = \frac{1}{n} \sum_{k=1}^{n} g_k(i,j) \, .$$

Example: Suppression of thermal noise for cameras utilized for precise measurements. The correct value is usually estimated from 50 images at least.

The trouble: need to filter noise from a single image

- No other choice, but to resort to the data redundancy in the image.
- Neighboring pixels have the same or similar intensity value.
- The intensity value can be corrected based on the analysis of intensities in the neighborhood. A single typical sample or the combination of several intensity values in the neighborhood is taken.
- The trouble with blurring on the steep intensity transitions occurs..

General local filtration

- The correct (new) intensity value is estimated in the small neighborhood of the current pixel.
- Imagine that the image is systematically traversed line by line, e.g. from top left. The small neighborhood $\mathcal{O}$ around the representative pixel is analyzed. This neighborhood is often a small rectangle, called also a window..
- The result of the analysis is written to the output image at the same position as the representative pixel has in the input image..
- In general, the filter properties can vary with the position of the representative point.

Operators independent to the shift also space invariant filters

- It is a special case of the local filtration.
- The properties of the filter remain the same in all positions of the representative point in the image.
- These filters have a counterpart in filtration in the frequency domain, e.g. in Fourier spectrum.

Local linear preprocessing

- Nová hodnota je vypočítána jako lineární kombinace hodnot obrazové funkce v okolí.
- Připomínka: Linearita, tj. 2 vlastnosti: aditivita a homogenita.
- U skutečných obrazů je předpoklad linearity narušen
  - Hodnotu pixelu nelze vynásobit libovolným skalárem nebo výsledek součtu nemůže být jakýkoliv. Problém saturace díky omezenému rozsahu hodnot obrazové funkce, typicky $\langle 0, 255 \rangle$.
  - Problémy na okraji obrazu. Maska přesahuje.

A discrete 2D convolution

Příspěvek jednotlivých pixelů v okolí $\mathcal{O}$ je vážen v lineární kombinaci koeficienty h podle

$$g(x,y) = \sum_{(m,n)} \sum_{\in \mathcal{O}} h(x-m,\, y-n)\; f(m,n)\,.$$

Konvoluční jádro h, též konvoluční maska.

Často se používá obdélníkové okolí $\mathcal{O}$ s lichým počtem řádků a sloupců, a tak může reprezentativní bod ležet uprostřed konvoluční masky.

Ordinary averaging

Averaging in the 3×3 neighborhood

$$h = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} .$$

Modifications, which stress importance of pixels close to the center of the mask

$$h = \frac{1}{10} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix} , \quad h = \frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix} .$$

Ordinary averaging, example 1

Original 256×256

Added artif. noise

Averaging 3×3

Ordinary averaging, example 2

Original 256×256

Added artif. noise

Averaging 7×7

Separable filters

Example: a binomical 2D filter of the size 5×5. The filter element is constituted as the edition of two preceding elements in Pascal triangle.

$$\begin{bmatrix} 1 & 4 & 6 & 4 & 1 \\ 4 & 16 & 24 & 16 & 4 \\ 6 & 24 & 36 & 24 & 6 \\ 4 & 16 & 24 & 16 & 4 \\ 1 & 4 & 6 & 4 & 1 \end{bmatrix} = \begin{bmatrix} \\ \\ h_1 \\ \\ \\ \end{bmatrix} \begin{bmatrix} & & h_2 & & \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 6 \\ 4 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & 6 & 4 & 1 \end{bmatrix}.$$

Separability $\Rightarrow$ savings in calculations

The size of the convolution mask is $2N + 1$.

$$
\begin{aligned}
g(x, y) &= \sum_{m=-N}^{N} \sum_{n=-N}^{N} h(x - m,\, y - n)\, f(m, n) \\
&= \sum_{m=-N}^{N} \sum_{n=-N}^{N} h(m, n)\, f(x + m, y + n) \\
&= \sum_{m=-N}^{N} h_1(m) \sum_{n=-N}^{N} h_2(n)\, f(x + m, y + n)
\end{aligned}
$$

Separability $\Rightarrow$ savings in calculations 2

- Our filter of the size 5×5 needs 25 multiplications and 24 additions in each pixel.
- If a separable filter is used, only 10 multiplications and 8 additions.

- The saving would be more dramatic in the case of the convolution-based filtration in the 3D image, e.g. from a tomograph. For the convolution mask of the size $9 \times 9 \times 9$, there is a need for 729 multiplications and 728 additions.
- 27 multiplications and 24 suffice per voxel. voxel.

Separability check and related decomposition

Each filter with the rank 1 is separable.

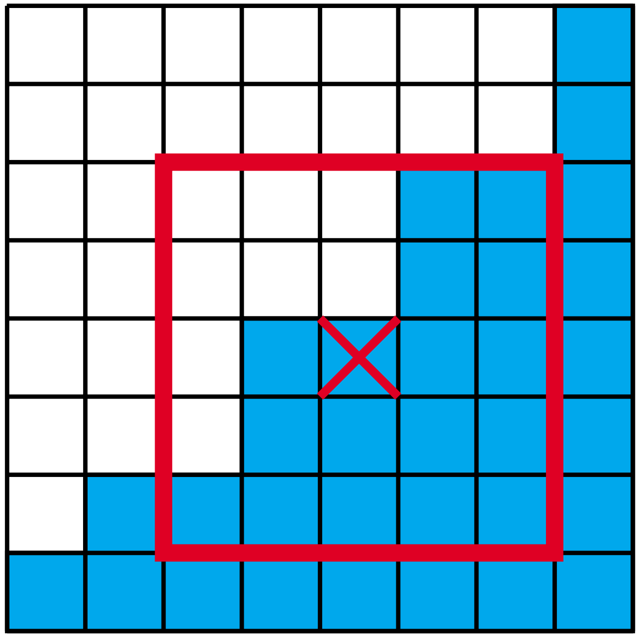
The singular decomposition (SVD).

```
[u,s,v] = svd(A);
s = diag(s);
tol = length(A) * max(s) * eps;
rank = sum(s > tol);
if (rank == 1)
  hcol = u(:,1) * sqrt(s(1));
  hrow = conj(v(:,1)) * sqrt(s(1));
  y = conv2(hcol, hrow, x, shape);
else
  %Nonseparable stencil
end
```

Nonlinear smoothing

The aim: to reduce the the blur of edges while smoothing.

1. principle: find such a subset of the current pixel neighborhood, in which the intensity does not change much.

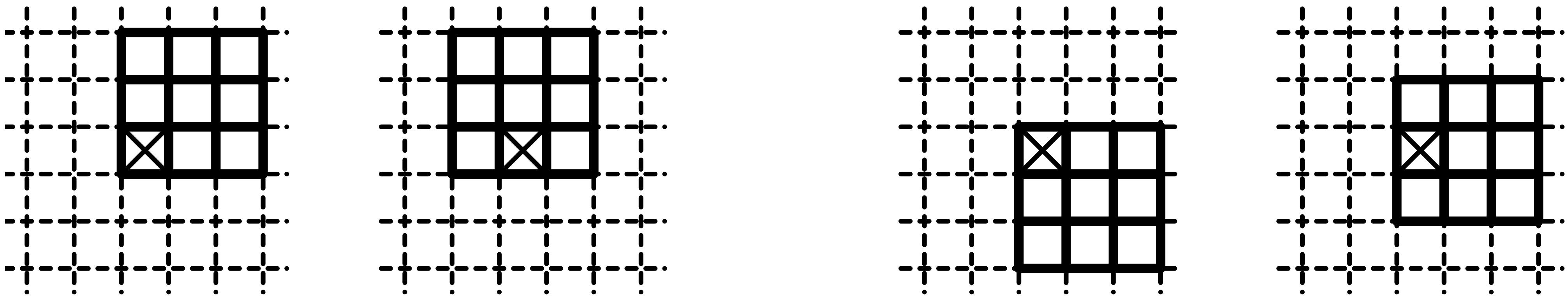


2. principle – robust statistics: The mean value is a bad estimate if outliers are present.

Rotating mask method

V okolí 5×5 vyhledává homogenní část rotující maska 3×3.

Celkem 9 poloh, 1 uprostřed + 8 na obrázku.



1 2 $\cdots$ 7 8

Z masek se vybere ta, kde má jas nejmenší rozptyl.

Median filtration

- Medián $=$ výběrový kvantil.
- Nechť je x náhodnou veličinou. Medián M je hodnota, pro kterou je pravděpodobnost jevu $x < M$ rovna jedné polovině.
- Výpočet mediánu je pro diskrétní obrazovou funkci jednoduchý. Stačí uspořádat vzestupně hodnoty jasu v lokálním okolí a medián určit jako prvek, který je uprostřed této posloupnosti.
- Aby se snadno určil prostřední prvek, používají se posloupnosti s lichým počtem prvků, typicky okolí 3×3, 5×5, atd.
- Výpočet ještě urychlí skutečnost, že k nalezení mediánu stačí částečné uspořádání posloupnosti.

Median, the example

100	98	102
99	105	101
95	100	255

Aritmetický průměr = 117,2

Medián: 95 98 99 100 100 101 102 105 255

Robustní, protože snese méně než 50 % vychýlených hodnot.

Median example, Prague castle

Originál 256×256

Přidán umělý šum

Medián 3×3

- Filtraci mediánem lze použít iterativně.
- Hlavní nevýhodou filtrace mediánem v obdélníkovém okolí je to, že porušuje tenké čáry a ostré rohy v obraze.