Digital image, basic concepts

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Outline of the lecture:

- Image, image function $f(x, y)$.
- Image digitization: sampling $+$ quantization.
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- Distance in the image, to be 'contiguous', region.
- Convex set, brightness histogram.
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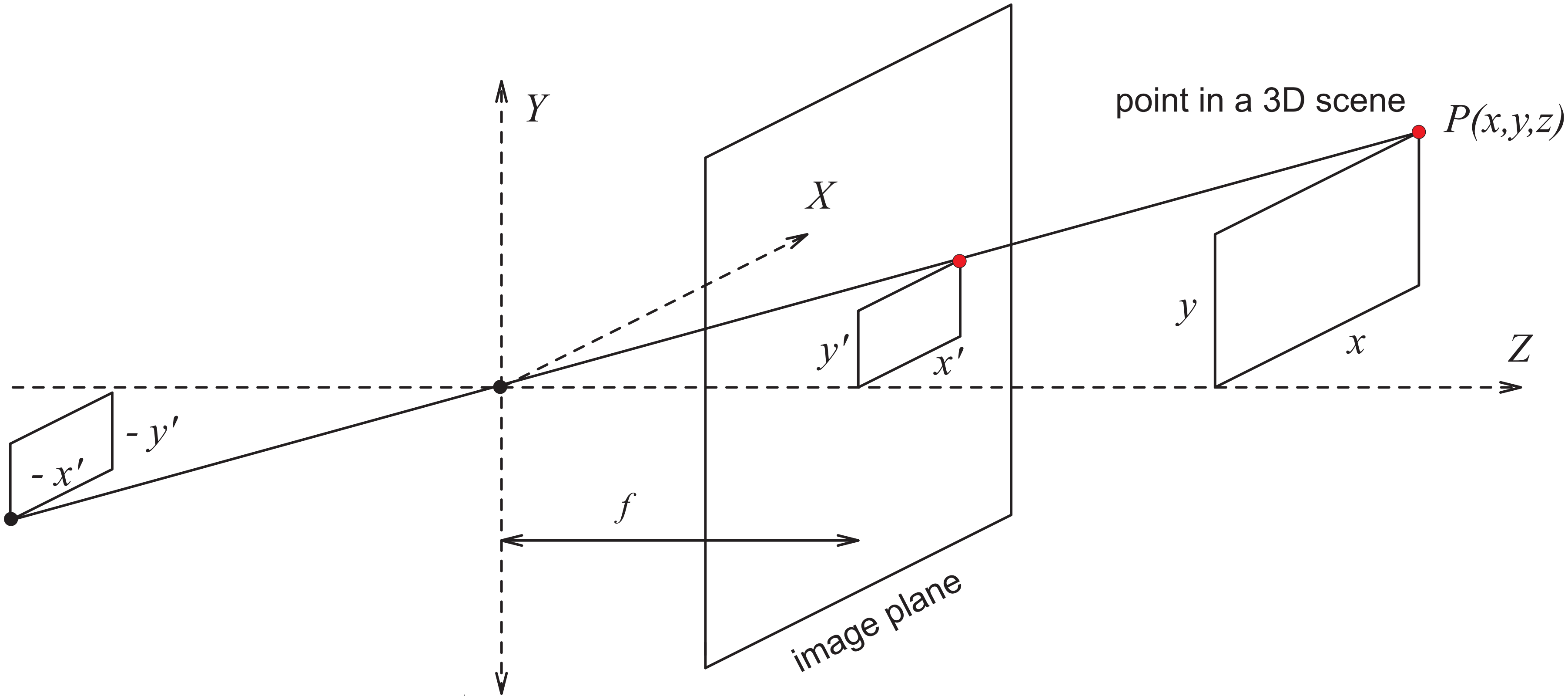
Image

Image is understood intuitively as the visual response on the retina or light sensitive chip in a camera, TV camera, ...

Image function $f(x, y)$, $f(x, y, t)$ is the result of the perspective projection.



$$x' = \frac{x\,f}{z}\,, \qquad y' = \frac{y\,f}{z}\,.$$

Image, digital image, pixel

- (Continuous) image = the input (understood intuitively), e.g., on the retina or captured by a TV camera.
- Let us assume a gray level image for simplicity.
- The continuous image function $f(x,y)$ or a matrix of picture elements, pixels (after digitization).
- (x,y) are spatial coordinates of a pixel.
- Value $f(x,y)$ corresponds usually to brightness.
- $f(x,y,t)$ in the case of an image sequence, t corresponds to time.

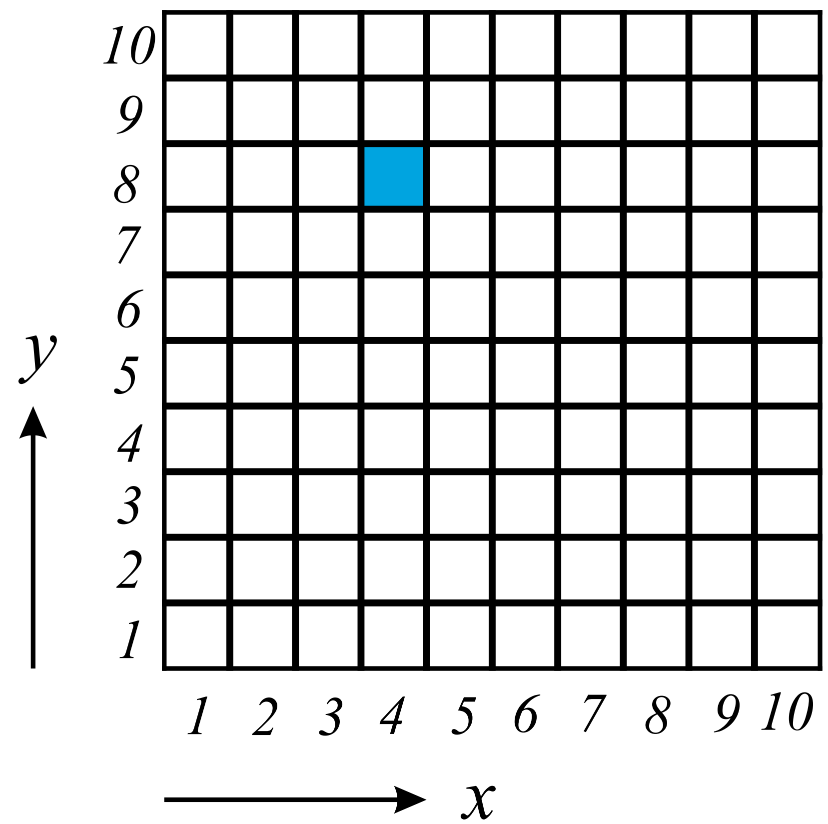


Image function = 2D signal

Monochromatic static image $f(x, y)$, where

(x, y) are coordinates in a plane with the range

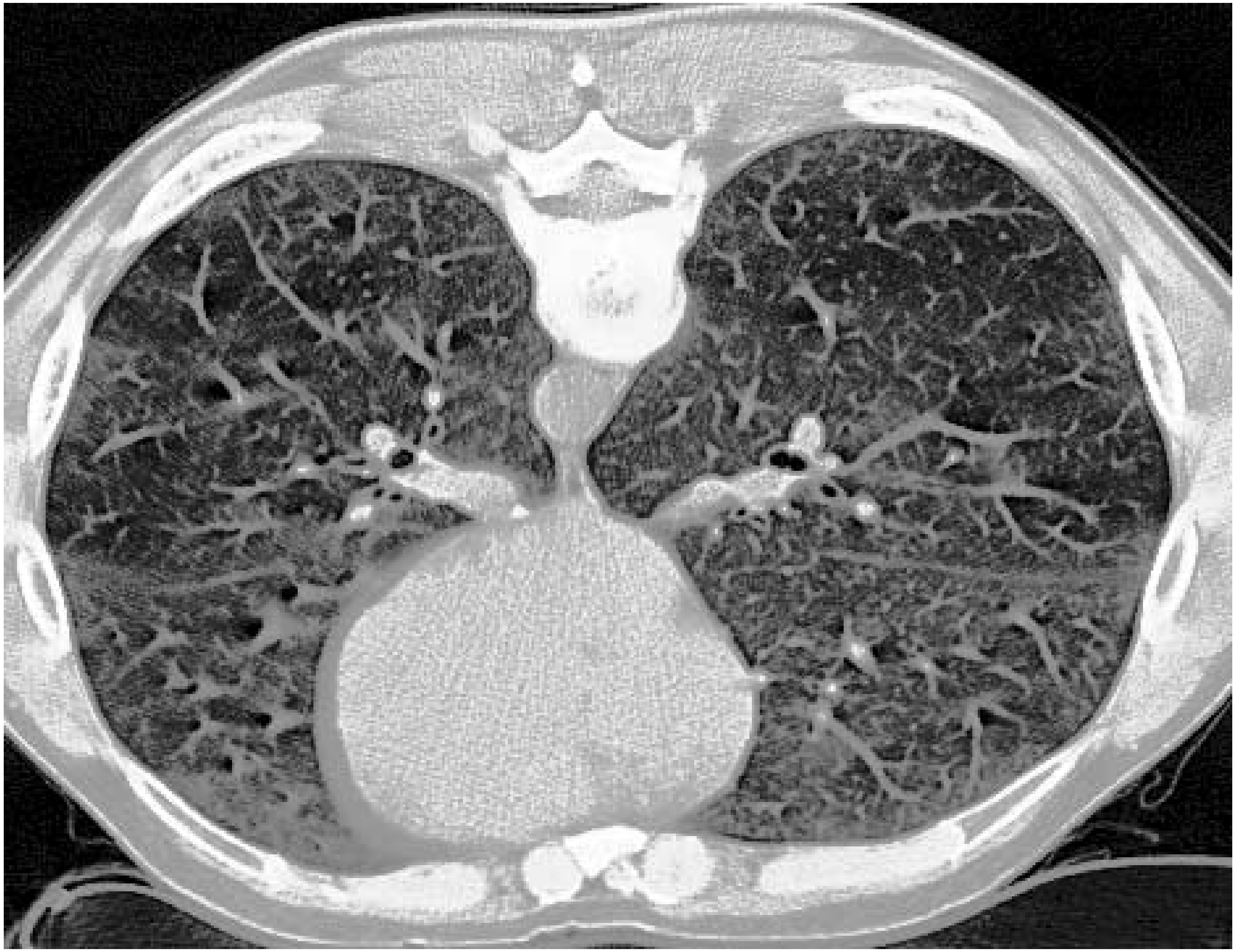
$$R = \{(x, y),\ 1 \le x \le x_m,\ 1 \le y \le y_n\}\ ;$$

f is a value of the image function ($\approx$ brightness, optical density with transparent original, distance to the observer, temperature in termovision, etc.)

(Natural) 2D images:

A thin sample in the optical microscope, image of a letter (character) on a piece of paper, fingerprint, one slice in the tomograph, etc.

Example of a digital image
a single slice from a X-ray tomograph



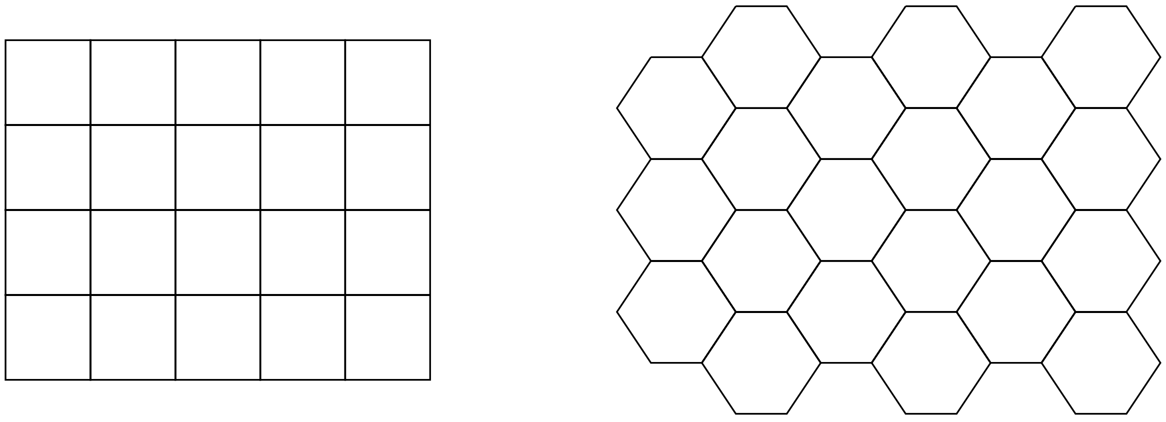
Digitization

- Sampling & quantization of the image function value (also called intensity).
- Digital image is often represented as a matrix.
- Pixel $=$ the acronym from picture element.

Image sampling

Image sampling consists of two tasks:

1. Arrangement of sampling points into a raster.



(a) (b)

2. Distance between samples (Nyquist-Shannon sampling theorem).
   - The sampling frequency must be >2 higher than the maximal frequency; in the sense: which would be possible to reconstruct from the sampled signal. We will be able to derive the theorem after we explain Fourier transformation.
   - Informally: In images the samples size (pixel size) has to be twice smaller than the smallest detail of interest.

First image scanner, 1956

The SEAC Scanner
with control console in background

Image sampling, example 1

Original 256×256

128×128

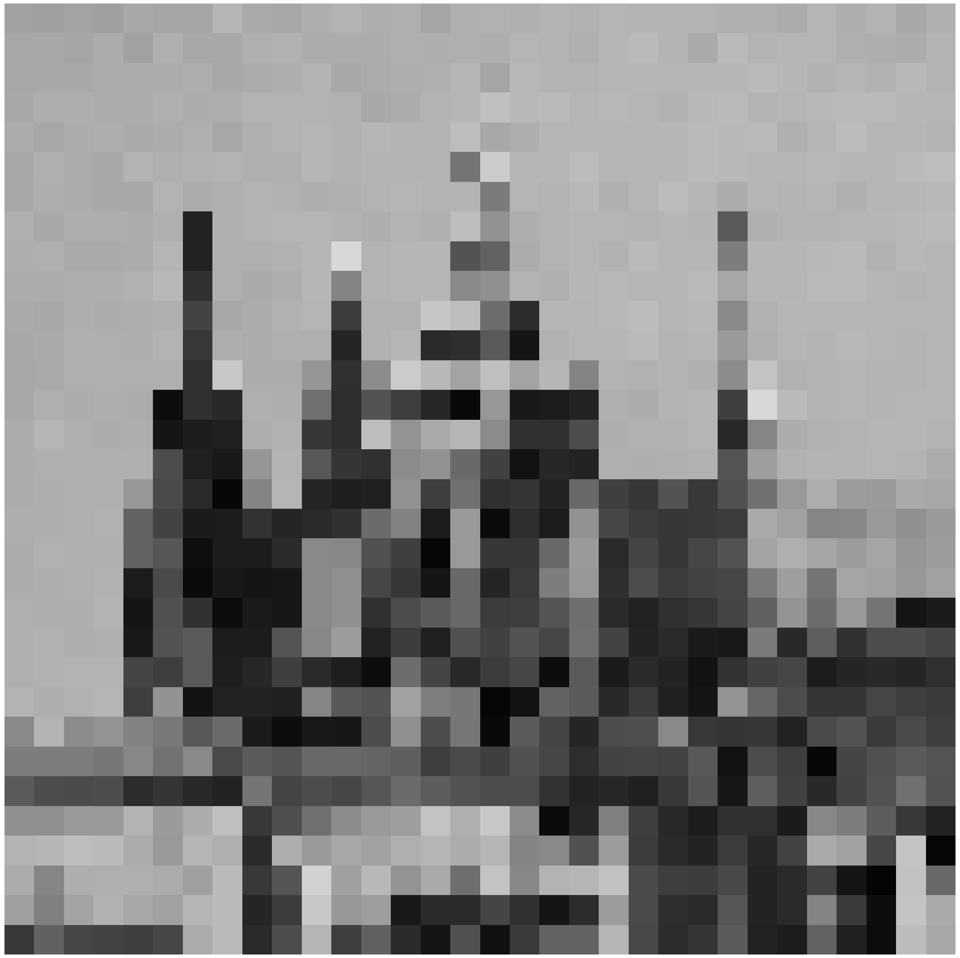
Image sampling, example 2

Original 256×256

64×64

Image sampling, example 3

Original 256×256



32×32

Image quantization, example 1

Original 256 gray levels

64 gray levels

Image quantization, example 2

Original 256 gray levels

16 gray levels

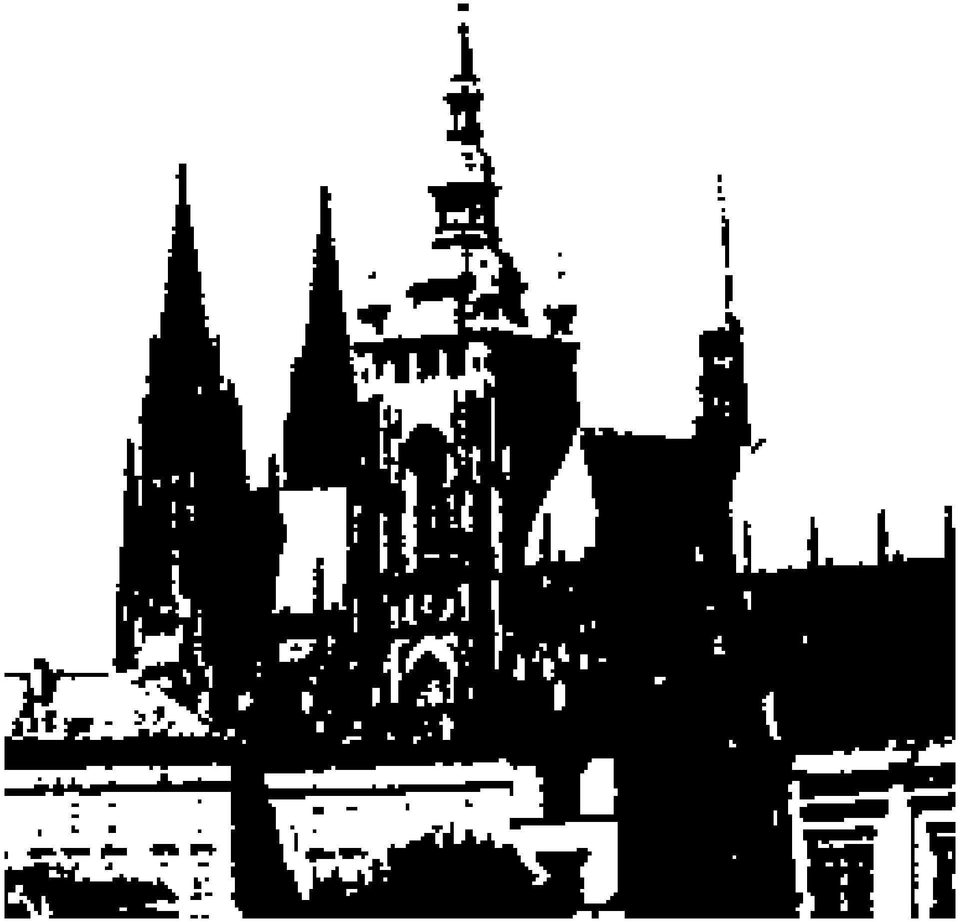
Image quantization, example 3

Original 256 gray levels

4 gray levels

Image quantization, example 4 (binary image)

Original 256 gray levels



2 gray levels

The distance, mathematically

Function D is called the **distance**, if and only if

$$D(p,q) \geq 0\,, \qquad \text{specially } D(p,p) = 0 \text{ (identity).}$$

$$D(p,q) = D(q,p)\,, \qquad \text{(symmetry).}$$

$$D(p,r) \leq D(p,q) + D(q,r)\,, \qquad \text{(triangular inequality).}$$

Several distance definitions in the square grid

Euclidean distance

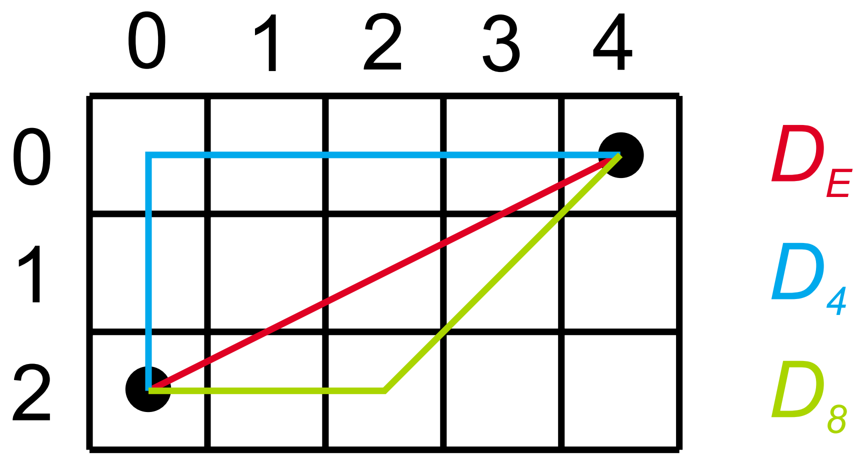
$$D_E((x,y),(h,k)) = \sqrt{(x-h)^2 + (y-k)^2}\,.$$

Manhattan distance (distance in a city with the rectangular street layout)

$$D_4((x,y),(h,k)) = \mid x-h \mid + \mid y-k \mid \;.$$

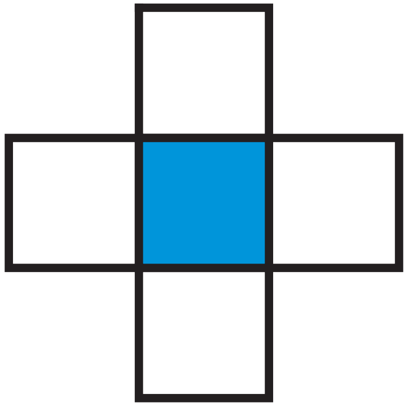
Chessboard distance (from the king point of view in chess)

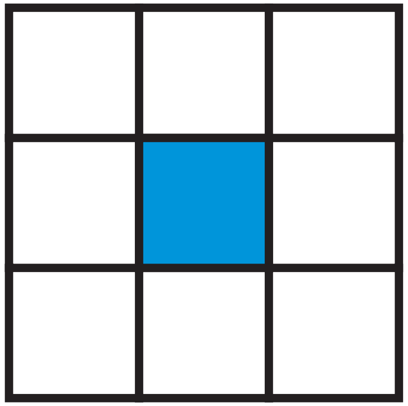
$$D_8((x,y),(h,k)) = max\{\mid x-h \mid, \mid y-k \mid\}\,.$$



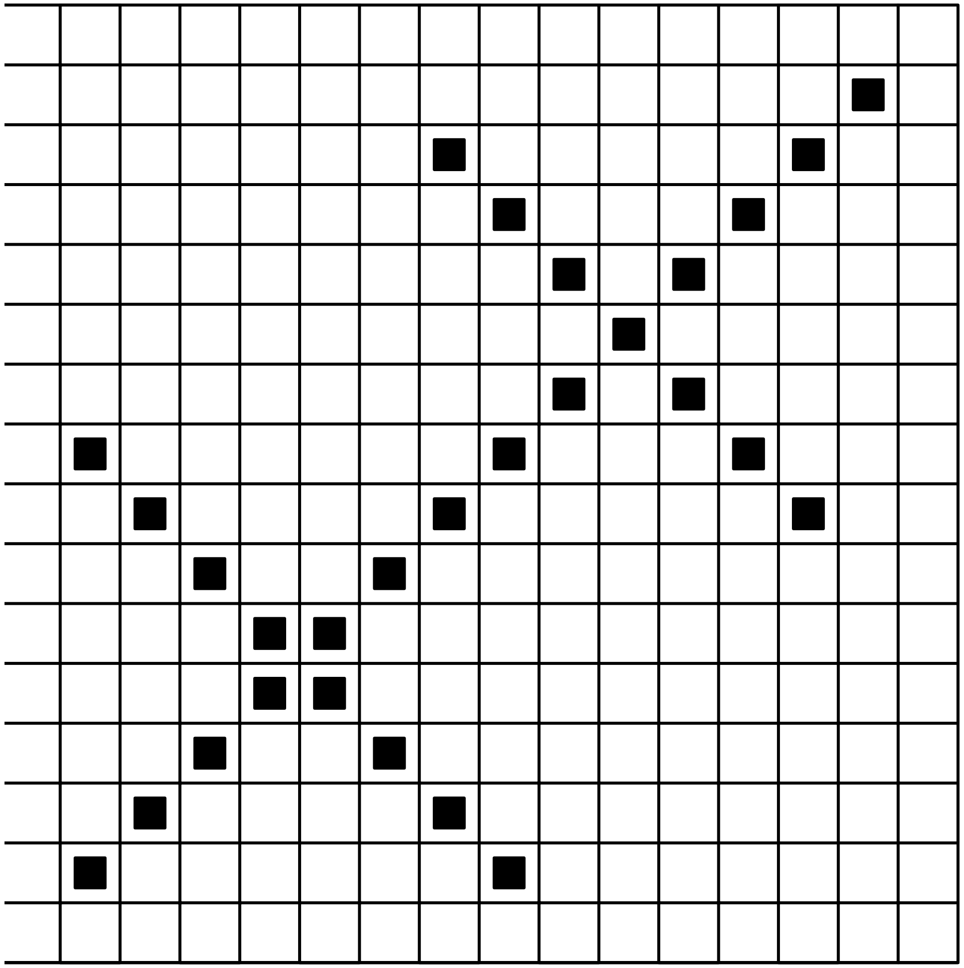
4-neighborhood and 8-neighborhood

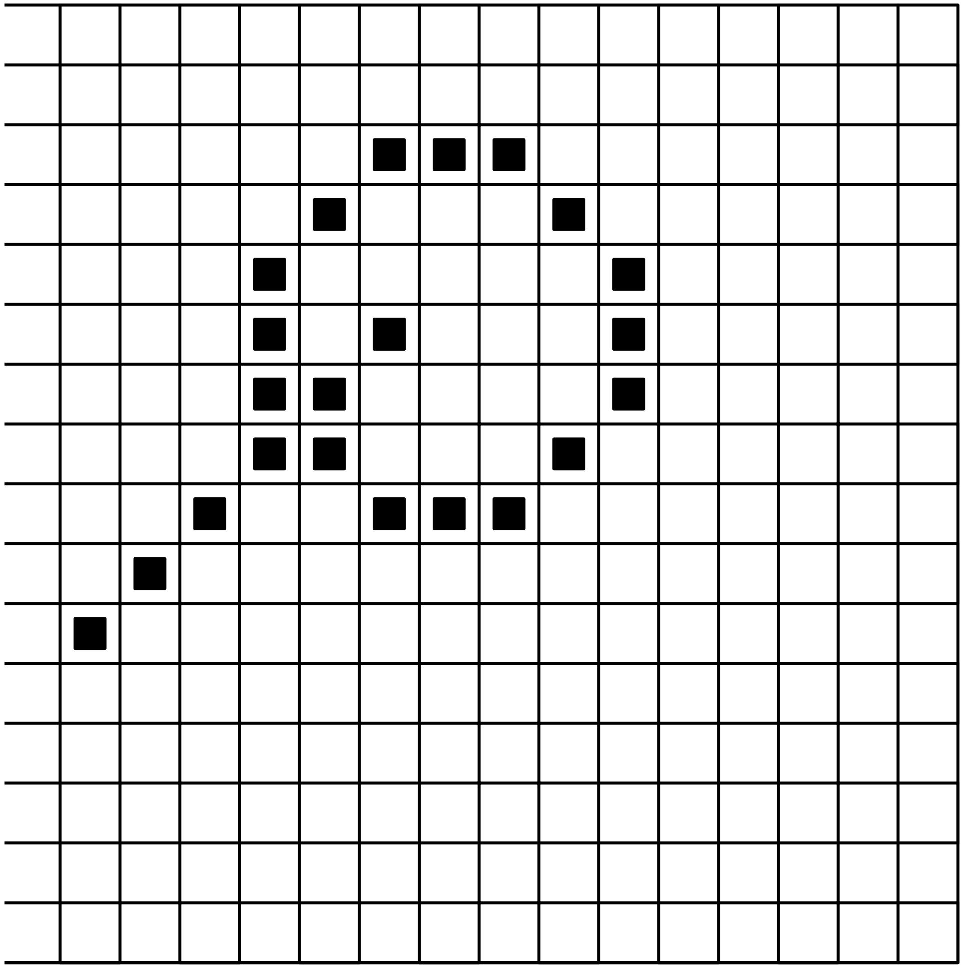
A set consisting of the pixel (called, e.g., a representative pixel or point) and its neighbors of distance 1.





Paradox of crossing line segments





Binary image & the relation ‘be contiguous’

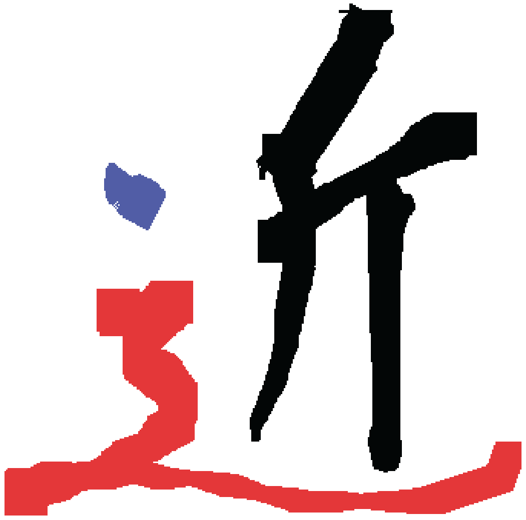
black $\sim$ objects
white $\sim$ background

- *A note for curious. Japanees kanji symbol means ’near to here’.*
- Introduction of the concept ‘object’ allows to select those pixels on a grid which have some particular meaning (recall discussion about interpretation). Here, black pixels belong to the object – a character.
- Neighboring pixels are contiguous.
- Two pixels are contiguous if and only if there is a path consisting of contiguous pixels.

Region = contiguous set

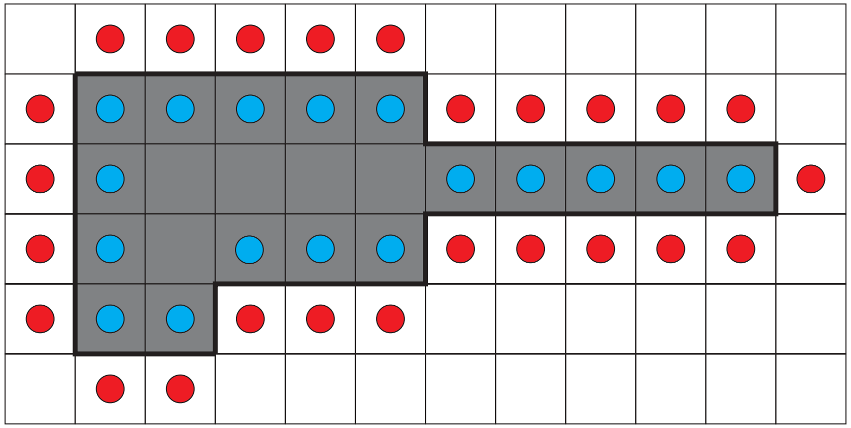
- The relation 'x is contiguous to y' is
  - reflexive, $x \sim x$,
  - symmetric $x \sim y \implies y \sim x$ and
  - transitive $(x \sim y)$ & $(y \sim z)$

  $\implies x \sim z$. Thus it is an equivalence relation.
- Any equivalence relation decomposes a set into subsets called classes of equivalence. These are regions in our particular case of relation "to be contiguous".
- In the image below, different regions are labeled by different colors.



Region boundary

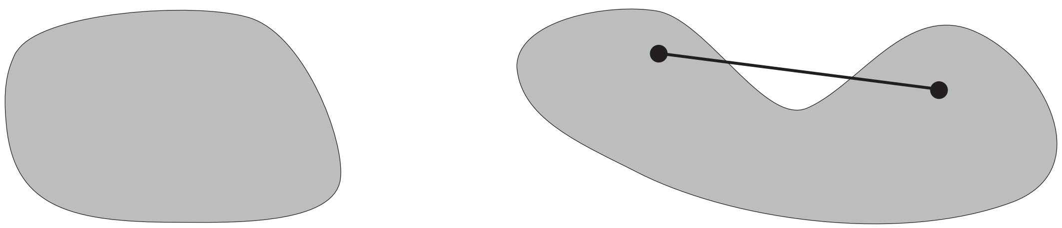
- The Region boundary (also border) R is the set of pixels is the set of pixels within the region that have one or more neighbors outside R.
- Theoretically, the the continuous image function $\Rightarrow$ infinitesimally thin boundary.
- In a digital image, the boundary has always a finite width. Consequently, it is necessary to distinguish inner and outer boundary.



- Boundary (border) of a region $\times$ edge in the image $\times$ edge element (edgel).

Convex set, convex hul

Convex set = any two points of it can be connected by a straight line which lies inside the set.



convex non-convex

Convex hull, lake, bay.

Distance transform, DT

- Called also: distance function, chamfering algorithm (analogy of woodcarving).
- DT provides in each pixel the distance from some image subset (perhaps describing objects).
- The resulting DT 'image' has pixel values of 0 for elements of the relevant subset, low values for close pixels, and then high values for pixels remote from it.
- For a binary image, DT provides the distance from each pixel to the nearest non-zero pixel (object).

input image

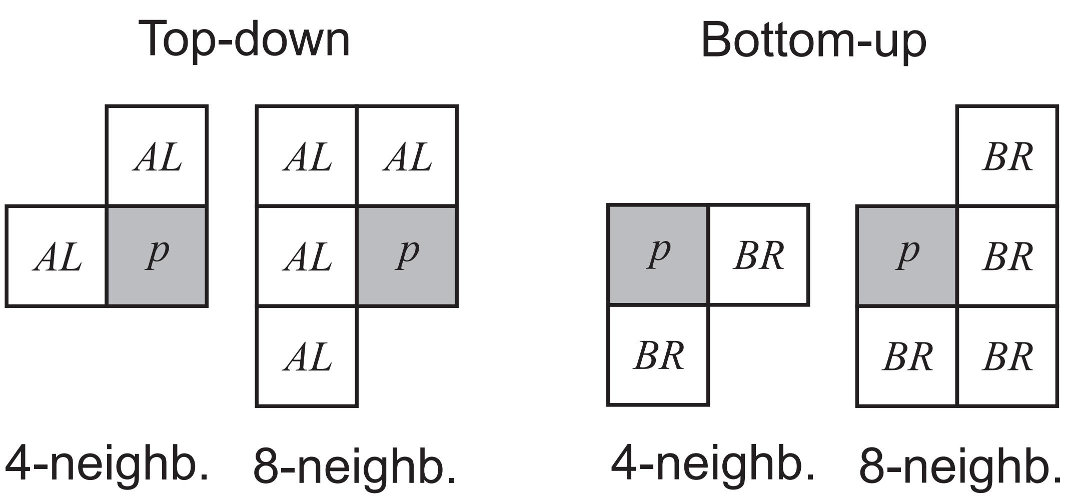
0	0	0	0	0	0	1	0
0	0	0	0	0	1	0	0
0	0	0	0	0	1	0	0
0	0	0	0	0	1	0	0
0	1	1	0	0	0	1	0
0	1	0	0	0	0	0	1
0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0

DT result

5	4	4	3	2	1	0	1
4	3	3	2	1	0	1	2
3	2	2	2	1	0	1	2
2	1	1	2	1	0	1	2
1	0	0	1	2	1	0	1
1	0	1	2	3	2	1	0
1	0	1	2	3	3	2	1
1	0	1	2	3	4	3	2

Distance transform algorithm informally

- The famous two-pass algorithm calculating DT by Rosenfeld, Pfaltz (1966) for distances D_4 and D_8.
- The idea is to traverse the image by a small local mask.
- The first pass starts from top-left corner of the image and moves row-wise horizontally left to right. The second pass goes from the bottom-right corner in the opposite bottom-up manner, right to left.



- The effectiveness of the algorithm comes from propagating the values of the previous image investigation in a 'wave-like' manner.

Distance transform algorithm

1. To calculate the distance transform for a subset S of an image of dimension $M \times N$ with respect to a distance metric D, where D is one of D_4 or D_8, construct an $M \times N$ array F with elements corresponding to the set S set to 0, and all other elements set to infinity.
2. Pass through the image row by row, from top to bottom and left to right. For each neighboring pixel above and to the left (illustrated in Figure **??** by the set AL) set

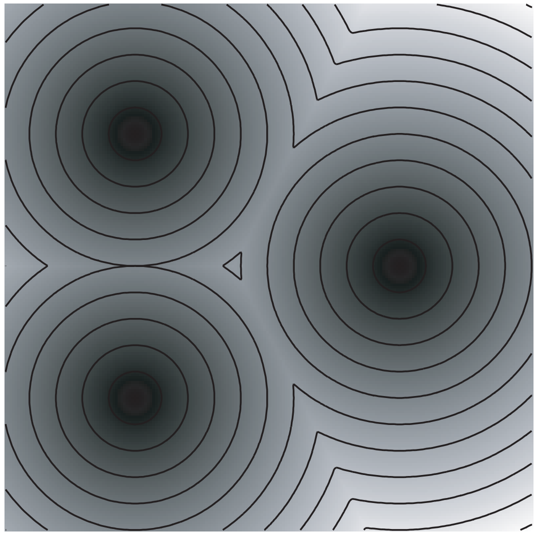
$$F(p) = \min_{q \epsilon AL} \big(F(p), D(p,q) + F(q)\big) \, .$$

3. Pass through the image row by row, from bottom to top and right to left. For each neighboring pixel below and to the right (the set BR in Figure **??**), set

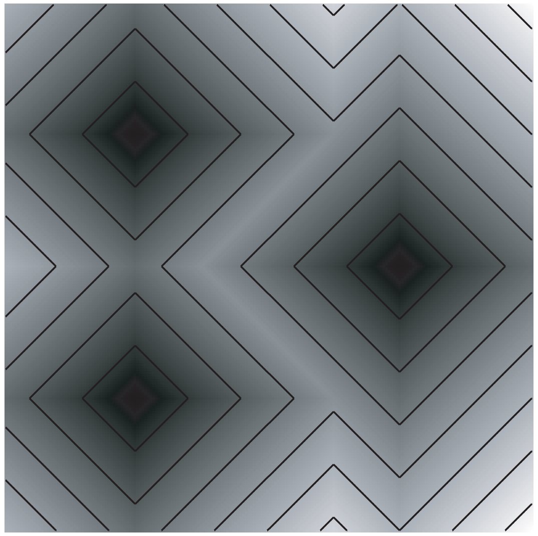
$$F(p) = \min_{q \epsilon BR} \big(F(p), D(p,q) + F(q)\big) \, .$$

4. The array F now holds a chamfer of the subset S.

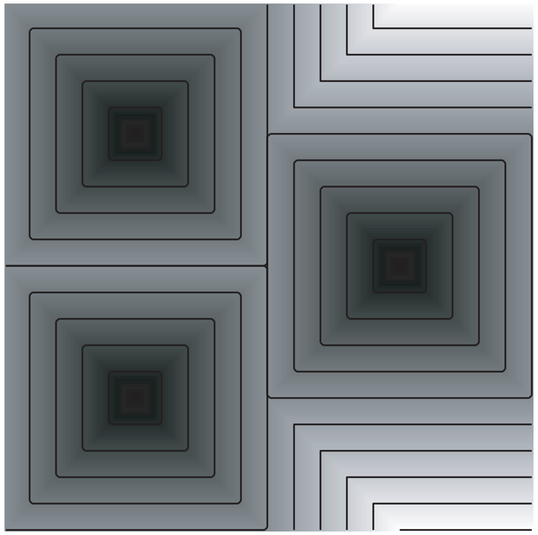
DT illustration for three distance definitions



Euclidean



D_4

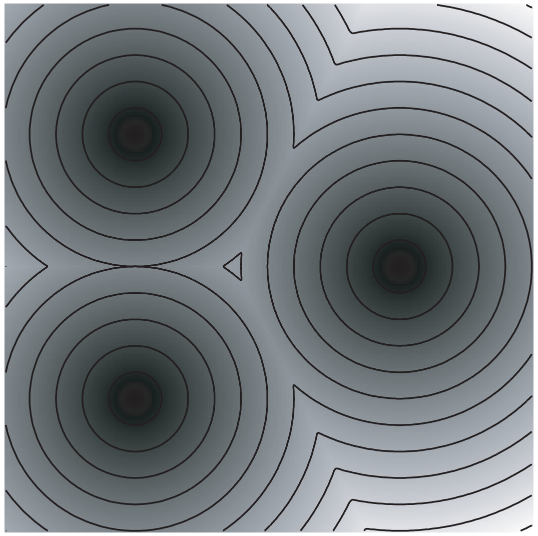


D_8

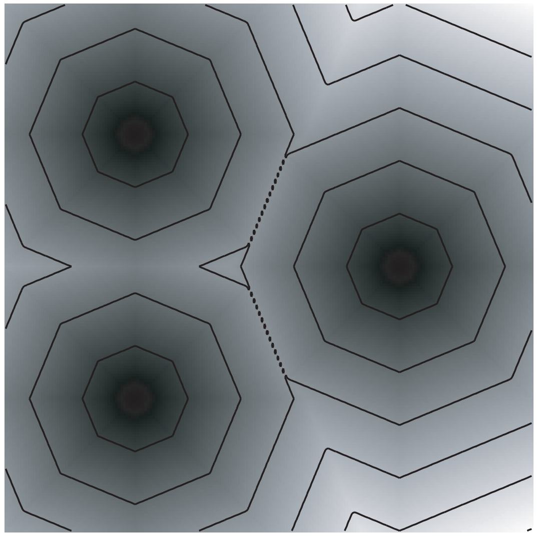
Quasieucledean distance

Eucledean DT cannot be easily computed in two passes only. The quasieucledean distance approximation is often used which can be obtained in two passes.

$$D_{\mathrm{QE}}\big((i,j),(h,k)\big) = \begin{cases} |i-h| + (\sqrt{2}-1)\,|j-k| & \text{for } |i-h| > |j-k|\,, \\ (\sqrt{2}-1)\,|i-h| + |j-k| & \text{otherwise.} \end{cases}$$

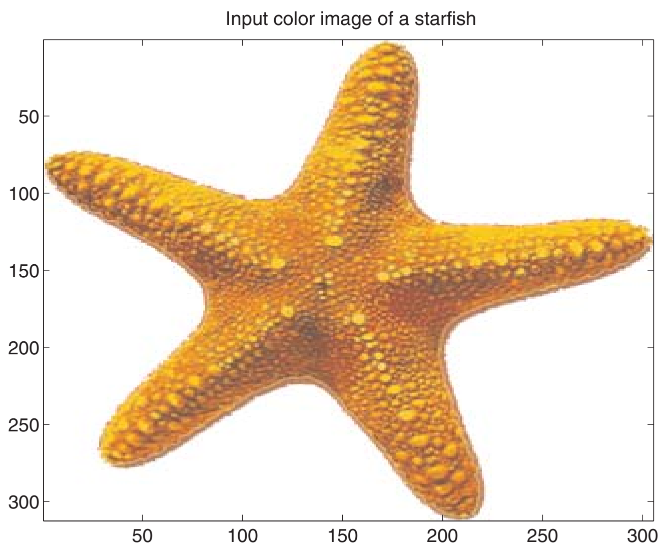


Euclidean

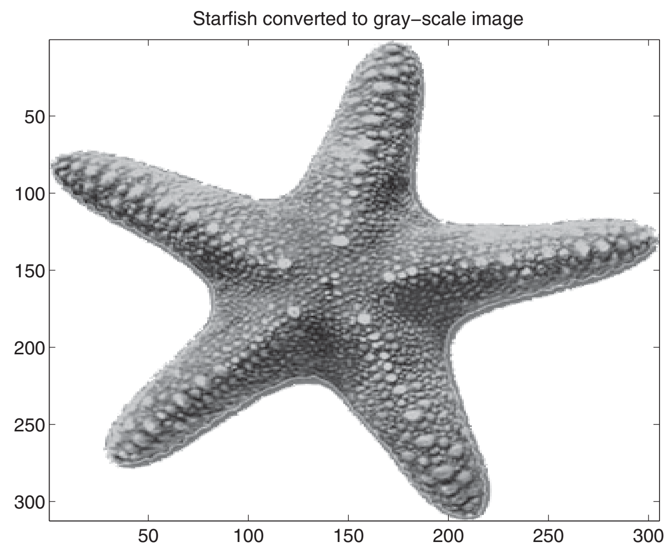


quasieuclidean

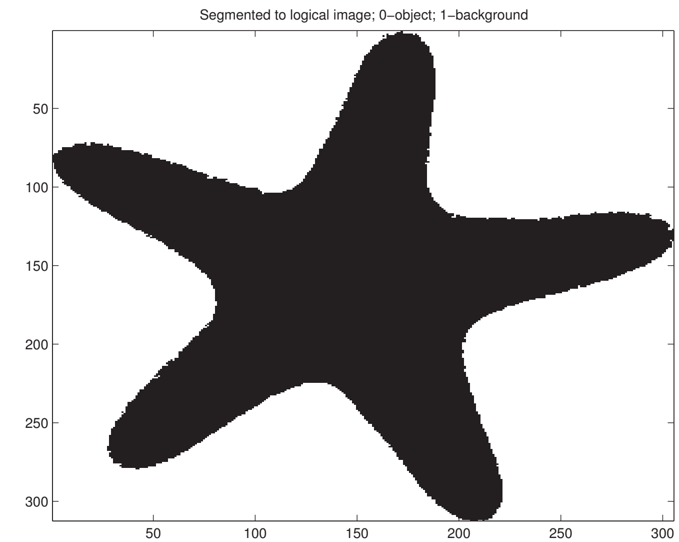
DT, starfish example, input image



color

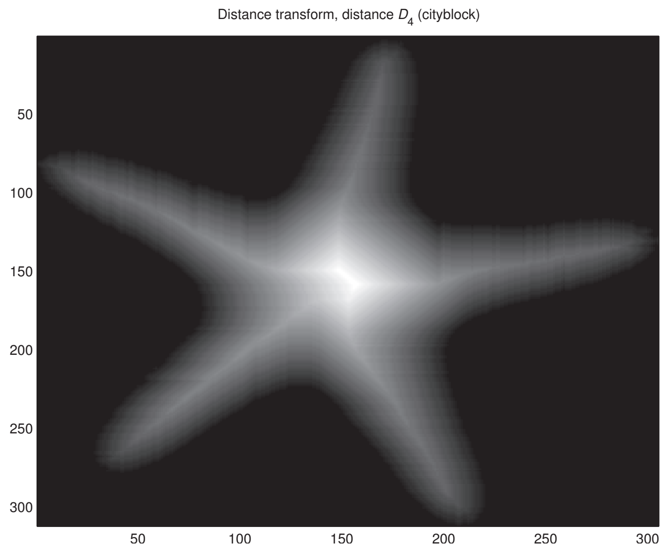


grayscale

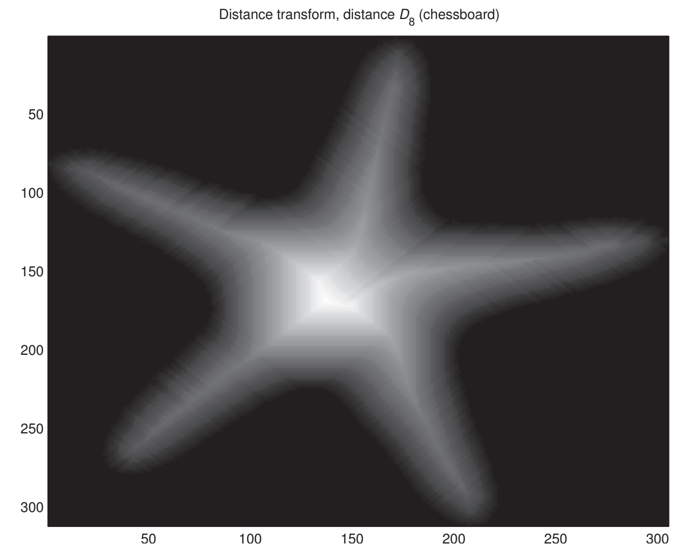


binary

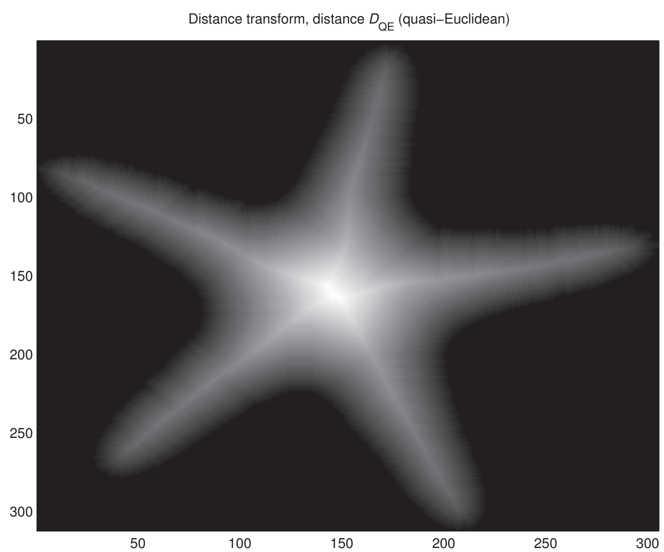
DT, starfish example, results



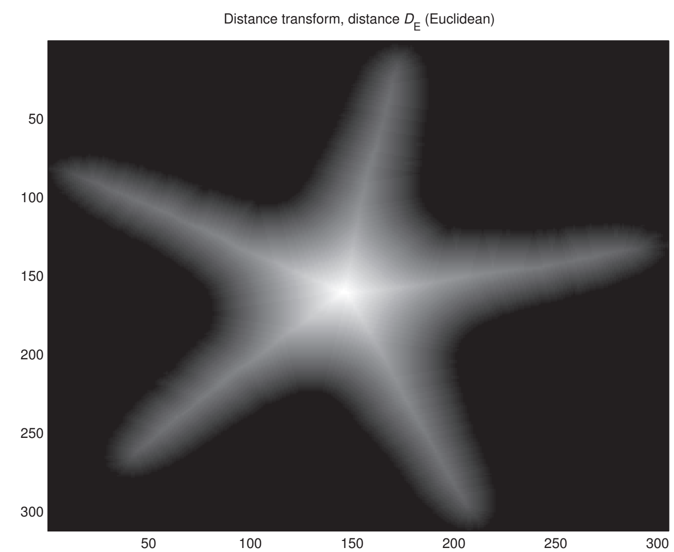
D4



D8



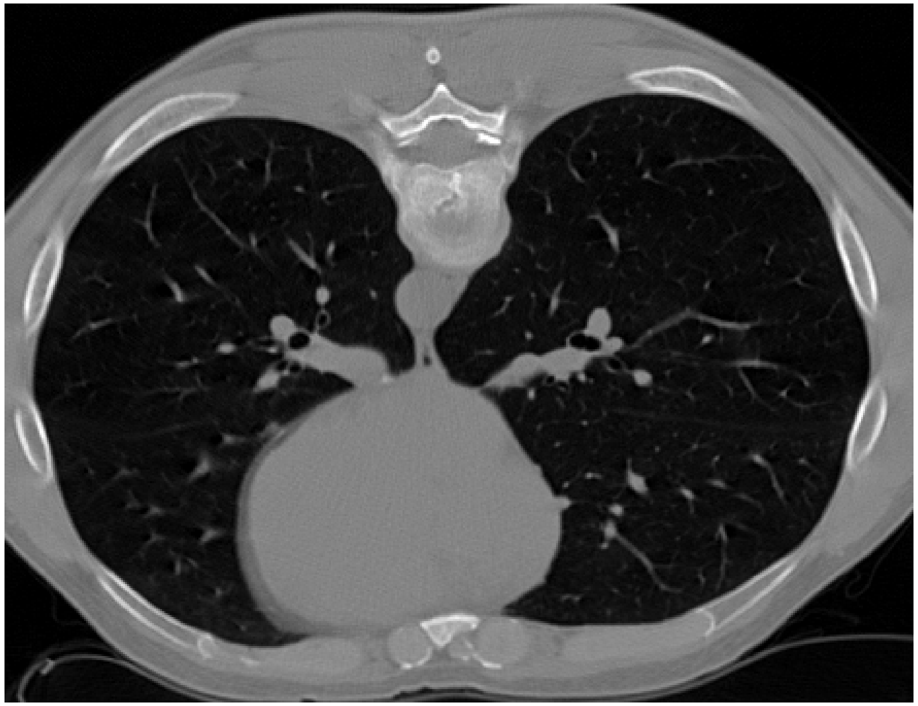
quazieuclidean



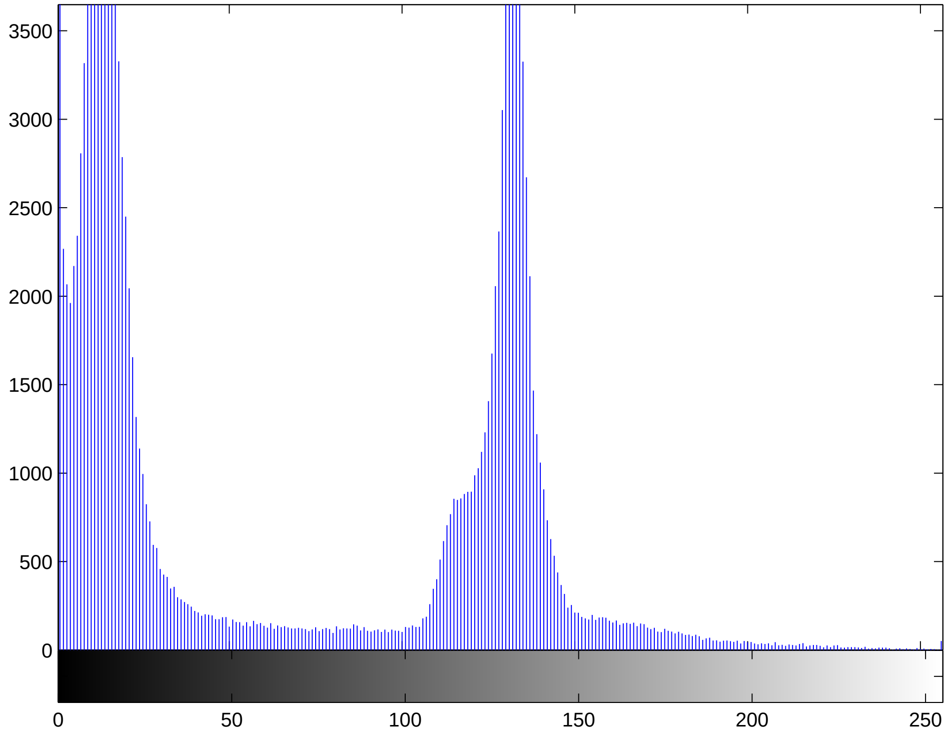
euclidean

Brightness histogram

Histogram of brightness values serves as the probability density estimate of a phenomenon, that a pixel has a definite brightness.



input image



its brightness histogram