# Non–cooperative games

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# Prisoner's dilemma

Two members of a criminal gang are arrested and imprisoned. Each prisoner is in solitary confinement with no means of speaking to or exchanging messages with the other. The police admit they don't have enough evidence to convict the pair on the principal charge (burglary). They plan to sentence both to two years in prison on a lesser charge (possessing stolen property). Simultaneously, the police offer each prisoner a Faustian bargain.

Here's how it goes:
More detail can be found, e.g. in [2].

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- If A and B both confess (by testifying against each other) the crime, each of them serves 3 years in prison

- If A testifies against B who stays silent, A will serve only 1 year whereas B will serve 4 years in prison (and vice versa)

- If A and B both deny the crime, both of them will only serve only 2 years in prison for a lesser crime

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# Payoff matrix

| Prisoner A | | Prisoner B | | | |
|---|---|---|---|---|---|
| | | testifies | | stays silent | |
| | testifies | 3 | 3 | 1 | 4 |
| | stays silent | 4 | 1 | 2 | 2 |

Numbers are years in prison.

# What is the optimal strategy?

|  |  | Prisoner B | | | |
| --- | --- | --- | --- | --- | --- |
|  |  | testifies | | stays silent | |
| Prisoner A | testifies | 3 | 3 | 1 | 4 |
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From the A's viewpoint

- if B testifies, it is better to confess/testify $3 < 4$.
- if B stays silent it is better to testify $1 < 2$
- The *domimant* strategy is then always to testify.
- But, would there be a mutual agreement, ...
- ... well, this is the *dilemma*

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|            | stays silent | 4          | 1 | **2**        | **2** |

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# Advertise or not?

Numbers represent companies profit

| | | Company B | | | |
|---|---|---|---|---|---|
| | | advertise | | not advertise | |
| Company A | advertise | 30 | 30 | 50 | 20 |
| | not advertise | 20 | 50 | 40 | 40 |

# Restrict production or not

Numbers represent companies profit

| | | Company B | | not restrict | |
|---|---|---|---|---|---|
| | | restrict | | not restrict | |
| Company A | restrict | 300 | 300 | 100 | 400 |
| | not restrict | 400 | 100 | 200 | 200 |

# Lower prices or not?

Numbers represent companies profit

| | | Company B | | not lower | |
|---|---|---|---|---|---|
| | | lower | | not lower | |
| Company A | lower | 90 | 90 | 80 | 110 |
| | not lower | 110 | 80 | 100 | 100 |

# Rock–paper–scissors

What is the payoff matrix?

# Prisoner's dilemma, cooperate vs. defect

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To testify actually means to betray — *defect*. Staying silent on the other hand means to *cooperate*

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We see that the rational choice is *defect*
Really always?

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# A bit more general view of the payoff matrix

|  |  | player B | | | |
|---|---|---|---|---|---|
|  |  | defect | | cooperate | |
| player A | defect | P | P | T | S |
|  | cooperate | S | T | R | R |

P Punishment

T Temptation

S Sucker's payoff

R Reward

Assume now, that the goal is to maximize profit and the numbers
represent money. Can we derive P,T,S,R mutual relations that
would justify the defect rationale?
For what P,T,S,R there is a dilemma?

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# ok, defect is the rational choice. But the world is not that bad, is it?

Indeed, this was and unsolved contradiction.

- people often cooperate
- but why if it is not rational?
- does a crime pay off?
- do people cooperate only when it pays off?

This is indeed an essential problem. The game theory models human behavior. But people do not behave according the theory.

$\implies$ Is the theory false? ???

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# In real life we usually do not play one-round games.

- I begin cooperating, the opponent perhaps would do the same?
- Can I forgive?
- Is the opponent trully rational?
- . . .

You can find more in the book The Origins of Virtue [1].

# Basic terms

player   make decisions (move)

strategy   players' behavior

payoff   output, consequence of the decision

dominant strategy   the best player's strategy, regardless of the opponent's strategy

Let's play!

# References

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