Non-cooperative games

Tomáš Svoboda, svobodat@fel.cvut.cz

Department of Cybernetics, Center for Machine Perception

September 25, 2013

・ 同 ト ・ ヨ ト ・ ヨ ト

э

1/33

 $Tom\acute{a} \acute{s} voboda, \ svobodat@fel.cvut.cz \ / \ Department \ of \ Cybernetics, \ CMP \ / \ Non-cooperative \ games$

Prisoner's dilemma

Two members of a criminal gang are arrested and imprisoned¹. Each prisoner is in solitary confinement with no means of speaking to or exchanging messages with the other. The police admit they don't have enough evidence to convict the pair on the principal charge. They plan to sentence both to a year in prison on a lesser charge. Simultaneously, the police offer each prisoner a Faustian bargain.

Here's how it goes:

Prisoner's dilemma

Two members of a criminal gang are arrested and imprisoned¹. Each prisoner is in solitary confinement with no means of speaking to or exchanging messages with the other. The police admit they don't have enough evidence to convict the pair on the principal charge. They plan to sentence both to a year in prison on a lesser charge. Simultaneously, the police offer each prisoner a Faustian bargain.

Here's how it goes:



How it goes

- If A and B both confess the crime, each of them serves 3 years in prison
- If A confesses but B denies the crime, A will serve only 1 year whereas B will serve 4 years in prison (and vice versa)
- If A and B both deny the crime, both of them will only serve only 2 years in prison

イロト イポト イヨト イヨト

3

4/33

 $Tom\acute{a} \acute{s} voboda, \ svobodat@fel.cvut.cz \ / \ Department \ of \ Cybernetics, \ CMP \ / \ Non-cooperative \ games$

How it goes

- If A and B both confess the crime, each of them serves 3 years in prison
- If A confesses but B denies the crime, A will serve only 1 year whereas B will serve 4 years in prison (and vice versa)
- If A and B both deny the crime, both of them will only serve only 2 years in prison

3

How it goes

- If A and B both confess the crime, each of them serves 3 years in prison
- If A confesses but B denies the crime, A will serve only 1 year whereas B will serve 4 years in prison (and vice versa)
- If A and B both deny the crime, both of them will only serve only 2 years in prison

3

4 A >

Payoff matrix

		Prisoner B			
		confesses stays silen		silent	
Driconor A	confesses	3	3	1	4
Prisoner A	stays silent	4	1	2	2

・ロト ・四ト ・ヨト ・ヨト

э

7/33

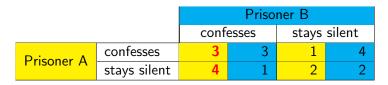
Numbers are years in prison.

Tomáš Svoboda, svobodat@fel.cvut.cz / Department of Cybernetics, CMP / Non-cooperative games

		Prisoner B			
		confesses stays silent			silent
Dricopor A	confesses	3	3	1	4
Prisoner A	stays silent	4	1	2	2

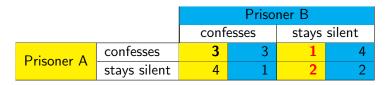
From the A's viewpoint

- ▶ if B confesses, it is better to confess 3 < 4.</p>
- ▶ if B stays silent it is again better to confess 1 < 2</p>
- The dominant strategy is then always to confess.
- But, would there be a mutual agreement,
- ... well, this is the dilemma



From the A's viewpoint

- if B confesses, it is better to confess 3 < 4.
- ▶ if B stays silent it is again better to confess 1 < 2</p>
- The domimant strategy is then always to confess.
- But, would there be a mutual agreement, ...
- ... well, this is the dilemma

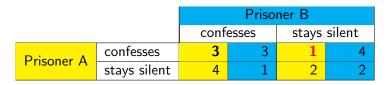


-

10/33

From the A's viewpoint

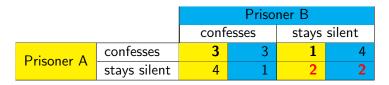
- if B confesses, it is better to confess 3 < 4.
- ▶ if B stays silent it is again better to confess 1 < 2
- The dominant strategy is then always to confess.
- But, would there be a mutual agreement, ...
- ... well, this is the dilemma



From the A's viewpoint

- ▶ if B confesses, it is better to confess 3 < 4.
- ▶ if B stays silent it is again better to confess 1 < 2
- The *domimant* strategy is then always to confess.
- But, would there be a mutual agreement,
- ... well, this is the dilemma

-



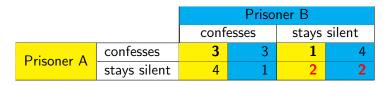
From the A's viewpoint

- ▶ if B confesses, it is better to confess 3 < 4.
- if B stays silent it is again better to confess 1 < 2
- The *domimant* strategy is then always to confess.
- But, would there be a mutual agreement, ...

... well, this is the dilemma

-

A B M A B M



From the A's viewpoint

- if B confesses, it is better to confess 3 < 4.
- if B stays silent it is again better to confess 1 < 2
- The *domimant* strategy is then always to confess.
- But, would there be a mutual agreement, ...
- ... well, this is the dilemma

-

A B > A B >

Numbers represent companies profit

		Company B			
		advertise not adver		vertise	
Company A	advertise	30	30	50	20
Company A	not advertise	20	50	40	40

イロン イヨン イヨン イヨン

э

14/33

Tomáš Svoboda, svobodat@fel.cvut.cz / Department of Cybernetics, CMP / Non-cooperative games

Restrict production or not

Numbers represent companies profit

		Company B			
		restrict not restri		estrict	
Company A	restrict	300	300	100	400
Company A	not restrict	400	100	200	200

Tomáš Svoboda, svobodat@fel.cvut.cz / Department of Cybernetics, CMP / Non-cooperative games

э

(< ≥) < ≥)</p>

< 口 > < 同 >

Numbers represent companies profit

		Company B			
		lower not lower			ower
Company	lower	90	90	80	110
Company A	not lower	110	80	100	100

Tomáš Svoboda, svobodat@fel.cvut.cz / Department of Cybernetics, CMP / Non-cooperative games

э

<ロ> <同> <同> < 回> < 回>

Rock-paper-scissors

What is the payoff matrix?

Tomáš Svoboda, svobodat@fel.cvut.cz / Department of Cybernetics, CMP / Non-cooperative games

		Prisoner B			
		confess stay silent			silent
Prisoner A	confess	3	3	1	4
r fisolier A	stay silent	4	1	2	2

To confess actually means to betray — *defect*. Staying silent on the other hand means to *cooperate*

э

18 / 33

< 🗇 🕨

We see that the rational choice is *defect* Really always?

 $\label{eq:comparison} Tomáš \ Svoboda, \ svobodat@fel.cvut.cz \ / \ Department \ of \ Cybernetics, \ CMP \ / \ Non-cooperative \ games$

			Priso	ner B	
		confess stay silent			silent
Prisoner A	confess	3	3	1	4
r fisolier A	stay silent	4	1	2	2

To confess actually means to betray — *defect*. Staying silent on the other hand means to *cooperate*

			Priso	ner B	
		defect cooperate			erate
Prisoner A	defect	3	3	1	4
r fisofier A	cooperate	4	1	2	2

We see that the rational choice is *defect* Really always?

Tomáš Svoboda, svobodat@fel.cvut.cz / Department of Cybernetics, CMP / Non-cooperative games

3

				Prisoner B			
		confess stay silent			silent		
Prisoner A	confess	3	3	1	4		
r fisolier A	stay silent	4	1	2	2		

To confess actually means to betray — *defect*. Staying silent on the other hand means to *cooperate*

			Priso	ner B	
		defect cooperate			erate
Prisoner A	defect	3	3	1	4
r fisofier A	cooperate	4	1	2	2

We see that the rational choice is *defect* Really always?

3

(B)

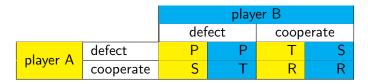
			Priso	ner B	
		confess stay silent			silent
Prisoner A	confess	3	3	1	4
r fisolier A	stay silent	4	1	2	2

To confess actually means to betray — *defect*. Staying silent on the other hand means to *cooperate*

		Prisoner B			
		defect		cooperate	
Prisoner A	defect	3	3	1	4
	cooperate	4	1	2	2

We see that the rational choice is *defect* Really always?

A bit more general view of the payoff matrix



- P Punishment
- T Temptation
- S Sucker's payoff
- R Reward

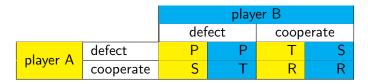
Assume now, that the goal is to maximize profit and the numbers represent money. Can we derive P,T,S,R mutual relations that would justify the defect rationale? For what P,T,S,R there is a dilemma?



22 / 33

4 A >

A bit more general view of the payoff matrix



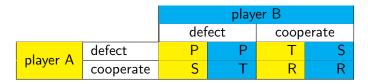
- P Punishment
- T Temptation
- S Sucker's payoff
- R Reward

Assume now, that the goal is to maximize profit and the numbers represent money. Can we derive P,T,S,R mutual relations that would justify the defect rationale?

For what P,T,S,R there is a dilemma?

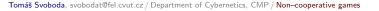
< = > < = >

A bit more general view of the payoff matrix



- P Punishment
- T Temptation
- S Sucker's payoff
- R Reward

Assume now, that the goal is to maximize profit and the numbers represent money. Can we derive P,T,S,R mutual relations that would justify the defect rationale? For what P,T,S,R there is a dilemma?



< ∃> < ∃>

Indeed, this was and unsolved contradiction.

- people often cooperate
- but why if it is not rational?
- does a crime pay off?
- do people cooperate only when it pays off?

This is indeed an essential problem. The game theory models human behavior. But people do not behave according the theory.

 \implies Is the theory false? ???

Indeed, this was and unsolved contradiction.

- people often cooperate
- but why if it is not rational?
- does a crime pay off?
- do people cooperate only when it pays off?

This is indeed an essential problem. The game theory models human behavior. But people do not behave according the theory.

(B)

26 / 33

 \implies Is the theory false? ???

Indeed, this was and unsolved contradiction.

- people often cooperate
- but why if it is not rational?
- does a crime pay off?
- do people cooperate only when it pays off?

This is indeed an essential problem. The game theory models human behavior. But people do not behave according the theory.

 \implies Is the theory false? ???

-

(B)

Indeed, this was and unsolved contradiction.

- people often cooperate
- but why if it is not rational?
- does a crime pay off?
- do people cooperate only when it pays off?

This is indeed an essential problem. The game theory models human behavior. But people do not behave according the theory.

きょうきょう

28 / 33

 \implies Is the theory false? ???

Indeed, this was and unsolved contradiction.

- people often cooperate
- but why if it is not rational?
- does a crime pay off?
- do people cooperate only when it pays off?

This is indeed an essential problem. The game theory models human behavior. But people do not behave according the theory.

きょうきょう

29/33

 \implies Is the theory false? ???

In real life we usually do not play one-round games.

- I begin cooperating, the opponent perhaps would do the same?
- Can I forgive?
- Is the opponent trully rational?
- ▶ ...

You can find more in the book The Origins of Virtue [1].

3

player make decisions (move) strategy players' behavior payoff output, consequence of the decision dominant strategy the best player's strategy, regardless of the opponent's strategy

伺い くさい くさい

3

31/33

Tomáš Svoboda, svobodat@fel.cvut.cz / Department of Cybernetics, CMP / Non-cooperative games

Let's play!

erative games 32/33

Tomáš Svoboda, svobodat@fel.cvut.cz / Department of Cybernetics, CMP / Non-cooperative games

References



Matt Ridley.

The Origins of Virtue, Human Instincts and the Evolution of Cooperation. Viking (Penguin Books), 1998. Also in Czech, Původ ctnosti, Portál 2010.

3

33 / 33

