Non–cooperative games

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Prisoner's dilemma

Two members of a criminal gang are arrested and imprisoned[1]. Each prisoner is in solitary confinement with no means of speaking to or exchanging messages with the other. The police admit they don't have enough evidence to convict the pair on the principal charge. They plan to sentence both to a year in prison on a lesser charge. Simultaneously, the police offer each prisoner a Faustian bargain.

Here's how it goes:

[1] Explanation taken from Wikipedia, http://en.wikipedia.org/wiki/Prisoner%27s_dilemma

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- If A and B both confess the crime, each of them serves 3 years in prison
- If A confesses but B denies the crime, A will serve only 1 year whereas B will serve 4 years in prison (and vice versa)
- If A and B both deny the crime, both of them will only serve only 2 years in prison

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Payoff matrix

		Prisoner B			
		confesses		stays silent	
Prisoner A	confesses	3	3	1	4
	stays silent	4	1	2	2

Numbers are years in prison.

What is the optimal strategy?

		Prisoner B			
		confesses		stays silent	
Prisoner A	confesses	3	3	1	4
	stays silent	4	1	2	2

From the A's viewpoint

- if B confesses, it is better to confess $3 < 4$.
- if B stays silent it is again better to confess $1 < 2$
- The *domimant* strategy is then always to confess.
- But, would there be a mutual agreement, ...
- ... well, this is the *dilemma*

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Advertise or not?

Numbers represent companies profit

		Company B			
		advertise		not advertise	
Company A	advertise	30	30	50	20
	not advertise	20	50	40	40

Restrict production or not

Numbers represent companies profit

		Company B			
		restrict		not restrict	
Company A	restrict	300	300	100	400
	not restrict	400	100	200	200

Lower prices or not?

Numbers represent companies profit

		Company B			
		lower		not lower	
Company A	lower	90	90	80	110
	not lower	110	80	100	100

Rock–paper–scissors

What is the payoff matrix?

Prisoner's dilemma, cooperate vs. defect

		Prisoner B			
		confess		stay silent	
Prisoner A	confess	3	3	1	4
	stay silent	4	1	2	2

To confess actually means to betray — *defect*. Staying silent on the other hand means to *cooperate*

		Prisoner B			
		defect		cooperate	
Prisoner A	defect	3	3	1	4
	cooperate	4	1	2	2

We see that the rational choice is *defect*
Really always?

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A bit more general view of the payoff matrix

		player B			
		defect		cooperate	
player A	defect	P	P	T	S
	cooperate	S	T	R	R

- P Punishment
- T Temptation
- S Sucker's payoff
- R Reward

Assume now, that the goal is to maximize profit and the numbers represent money. Can we derive P,T,S,R mutual relations that would justify the defect rationale?
For what P,T,S,R there is a dilemma?

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ok, defect is the rational choice. But the world is not that bad, is it?

Indeed, this was and unsolved contradiction.

- people often cooperate
- but why if it is not rational?
- does a crime pay off?
- do people cooperate only when it pays off?

This is indeed an essential problem. The game theory models human behavior. But people do not behave according the theory.

$\Longrightarrow$ Is the theory false? ???

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In real life we usually do not play one-round games.

- I begin cooperating, the opponent perhaps would do the same?
- Can I forgive?
- Is the opponent trully rational?
- ...

You can find more in the book The Origins of Virtue [1].

Basic terms

player make decisions (move)

strategy players' behavior

payoff output, consequence of the decision

dominant strategy the best player's strategy, regardless of the opponent's strategy

Let's play!

References

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