# Evolutionary Algorithms: GA & GP

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# Evolutionary Algorithms: Characteristics

**EA are stochastic optimization algorithms**

- **Stochastic** – but not random search,
- **Use an analogy of natural evolution**
  - genetic inheritance (J.G. Mendel) – the basic principles of transference of hereditary factors from parent to offspring – genes, which present hereditary factors, are lined up on chromosomes.
  - strife for survival (Ch. Darwin) – the fundamental principle of natural selection – is the process by which individual organisms with favorable traits are more likely to survive and reproduce.
- **Not fast in some sense** – population-based algorithm,
- **Robust** – efficient in finding good solutions in difficult searches.

# EA: Vocabulary

Vocabulary borrowed from natural genetics

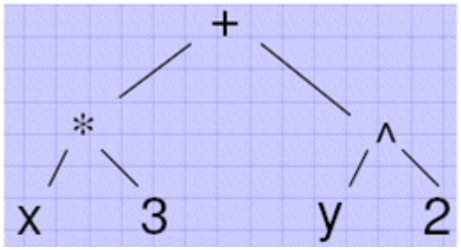
- **Individual** (chromosome + its quality measure "fitness value") – a solution to a problem.
- **Chromosome** – entire representation of the solution.
- **Fitness** – quality measure assigned to an individual, expresses how well it is adapted to the environment.
- **Gene** (also features, characters) – elementary units from which chromosomes are made.
  - each gene is located at certain place of the chromosome called locus (pl. loci),
  - a particular value for a locus is an allele.

    example: the "thickness" gene (which might be at locus 8) might be set to allele 2, meaning its second-thinnest value.
- **Genotype** – what's on the chromosome.
- **Phenotype** – what it means in the problem context (e.g., binary sequence may map to integers or reals, or order of execution, etc.).

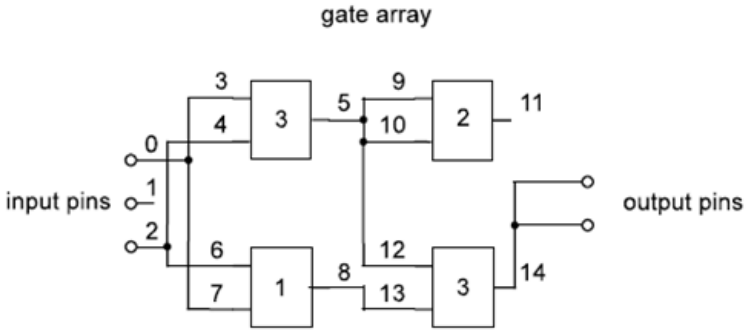
# Representation

Problem can be represented as

- **binary string** – 1 0 1 1 0 1 1 0 0 1 0 1 1 0 1
- **real-valued string** – 3,24 1,78 -2,61
- **string of chars** – D→E→A→C→B
- or as a **tree**




- or as a **graph**, and others.



# Evaluation Function

**Objective (Fitness) function**

- the only information about the sought solution the algorithm dispose of,
- must be defined for every possible chromosome.

**Fitness function may be**

- multimodal,
- discrete,
- multidimensional,
- nonlinear,
- noisy,
- multiobjective.

**Fitness does not have to be define analytically**

- simulation results,
- classification success rate.

**Fitness function should not be too costly!!!**

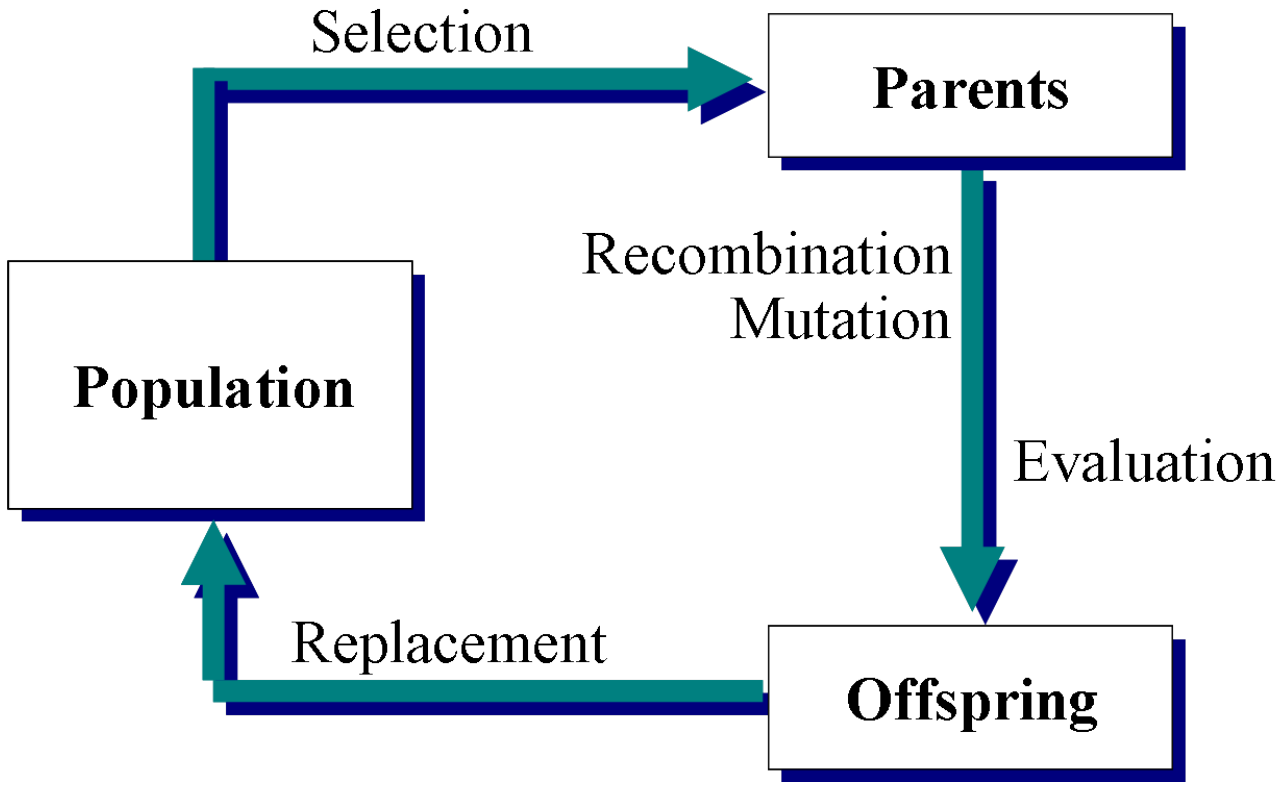
# Example: Coding & Evaluation

## Function optimization

- maximization of $f(x, y) = x^2 + y^2$,
- parameters $x$ and $y$ take on values from interval $< 0, 31 >$,
- and are code on 5 bits each.

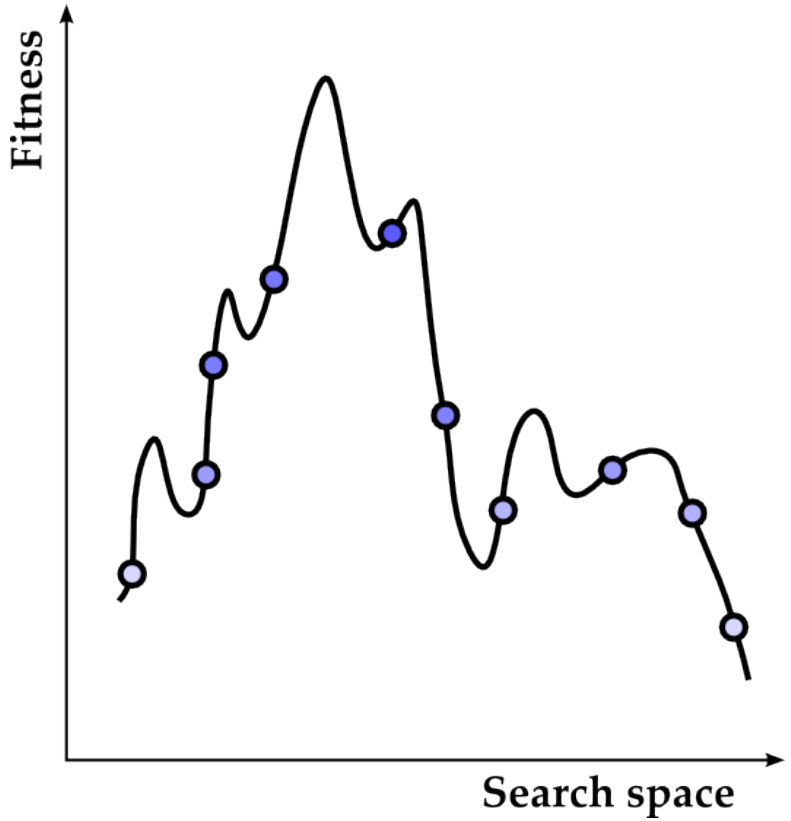
| genotype | phenotype | fitness |
|---|---|---|
| **0 0 0 0 0, 0 1 0 1 0** | **0, 10** | **100** |
| **0 0 0 0 1, 1 1 0 0 1** | **1, 25** | **625 + 1 = 626** |
| **0 1 0 1 1, 0 0 0 1 1** | **11, 3** | **121 + 9 = 130** |
| **1 1 0 1 1, 1 0 0 1 0** | **27, 18** | **729 + 324 = 1053** |

# Evolutionary Cycle

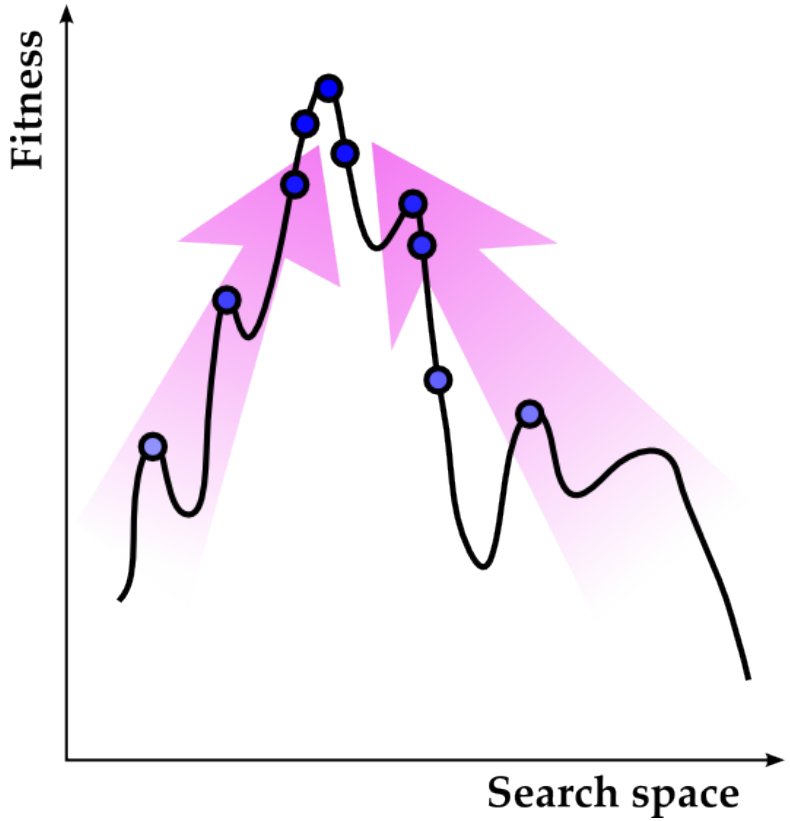


# Idealized Illustration of Evolution

- Uniformly sampled population.




- Population converged to promising regions.



# Initialization

## Random

- randomly generated solutions,
- no prior information about the shape of the sought solution,
- relies just on "lucky" sampling of the whole search space by a finite set of samples.

## Informed (pre-processing)

- (meta)heuristic routines used for seeding the initial population,
- biased random generator sampling regions of the search space that are likely to contain the sought solutions,
  - + may help to find better solutions,
  - + may speed up the search process,
  - − may cause irreversible focusing of the search process on regions with local optima.

# Reproduction

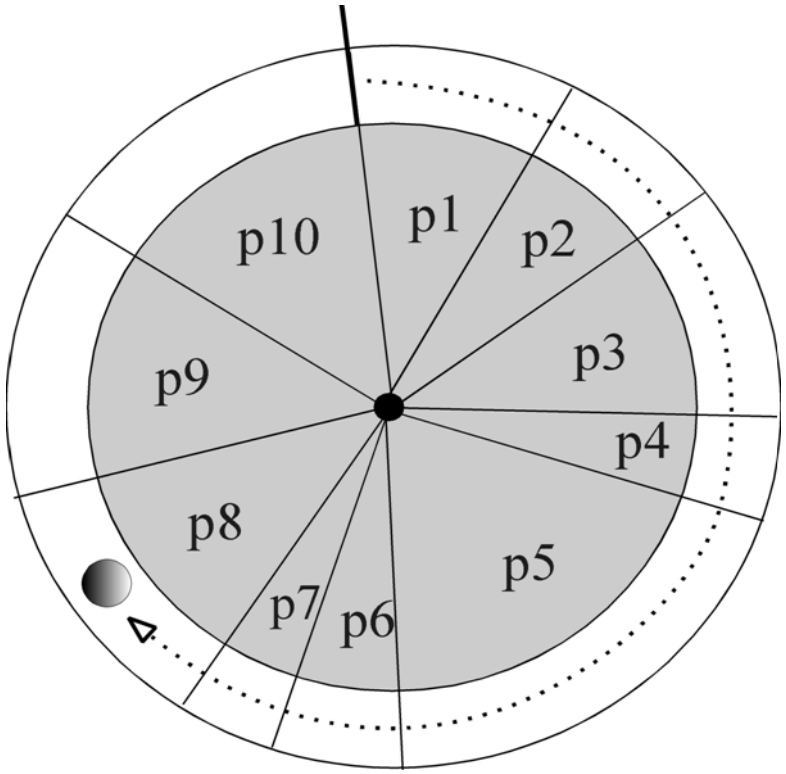
Models nature's survival-of-fittest principle

- prefers better individuals to the worse ones,
- still, every individual should have a chance to reproduce.

**Roulette wheel**

- probability of choosing some solution is directly proportional to its fitness value

$$P_i = \frac{f_i}{\frac{PopSize}{\sum_{j=1} f_j}}$$




Other methods

- Stochastic Universal Sampling,
- Tournament selection,
- Reminder Stochastic Sampling.

# Genetic Operators: Crossover

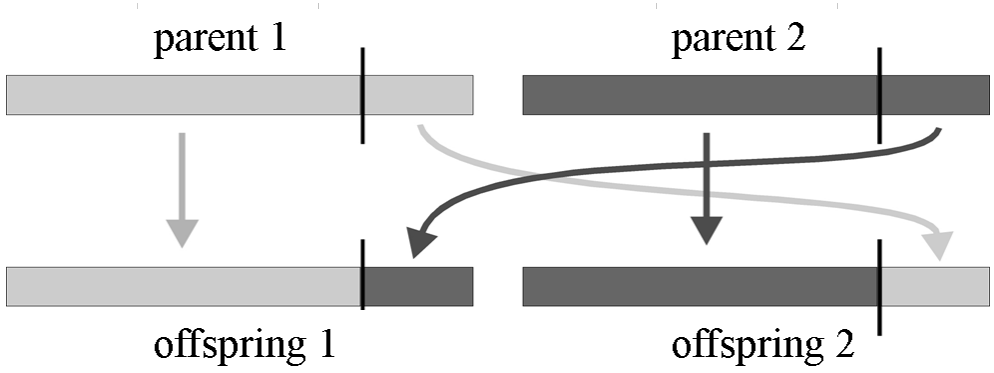
## Idea

- given two well-fit solutions to the given problem, it is possible to get a new solution by properly mixing the two that is even better than both its parents.

## Role of crossover

- sampling (exploration) of the search space

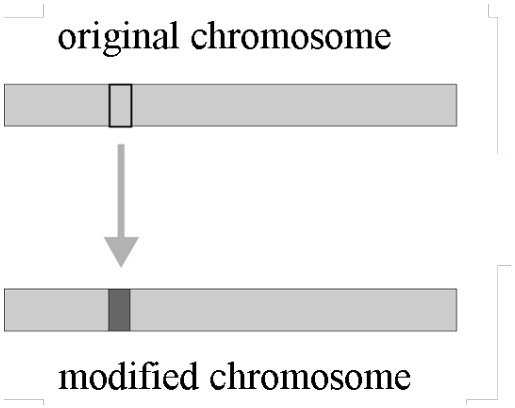
Example: 1-point crossover



# Genetic Operators: Mutation

## Role of mutation

- preservation of a population diversity,
- minimization of a possibility of loosing some important piece of genetic information.




Single bit-flipping mutation

| Population |
|---|
| 0 0 1 1 0 0 0 1 1 0 |
| 0 1 1 0 0 1 0 1 0 0 |
| 0 0 0 1 1 0 1 0 1 1 |
| 0 1 0 0 1 0 0 1 1 1 |
| 0 1 1 0 0 0 0 1 0 1 |
| . . . |
| 0 1 0 0 1 1 0 1 0 0 |

Example of missing genetic information

# Replacement Strategy

**Replacement strategy** defines

- how big portion of the current generation will be replaced in each generation, and
- which solutions in the current population will be replaced by the newly generated ones.

Two extreme cases

- **Generational** – the whole old population is completely rebuild in each generation (analogy of short-lived species).
- **Steady-state** – just a few individuals are replaced in each generation (analogy of longer-lived species).

# Application Areas of Evolutionary Algorithms

EAs are popular for their

- simplicity,
- effectiveness,
- robustness.

**Holland:** *"It's best used in areas where you don't really have a good idea what the solution might be. And it often surprises you with what you come up with."*

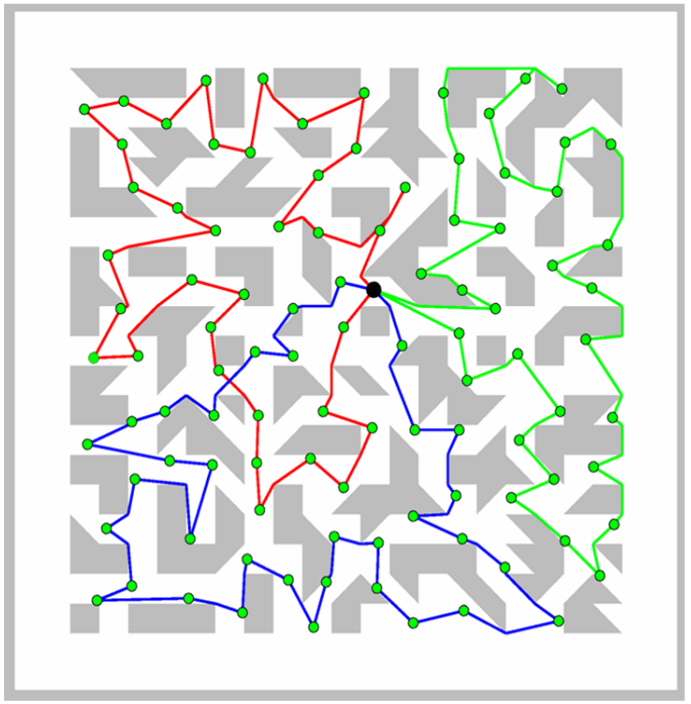
Applications

- control,
- engineering design,
- image processing,
- planning & scheduling,
- VLSI circuit design,
- network optimization & routing problems,
- optimal resource allocation,
- marketing,
- credit scoring & risk assessment,
- and many others.

# Multiple Traveling Salesmen Problem

**Rescue operations planning**

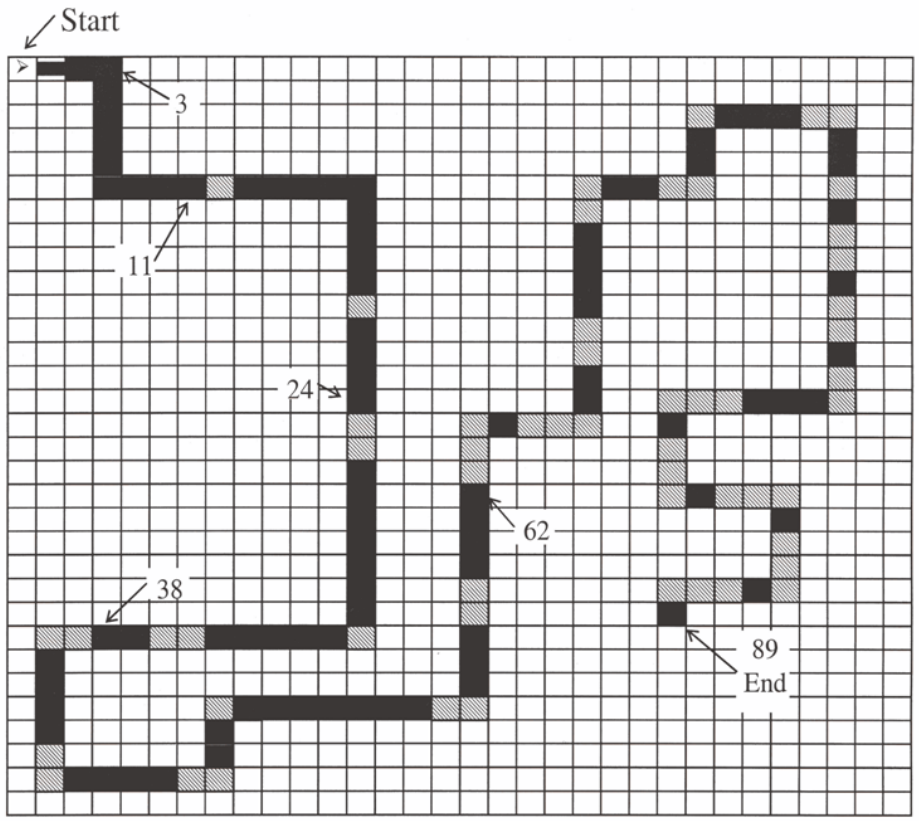
- Given $N$ cities and $K$ agents, find an optimal tour for each agent so that every city is visited exactly once.
- A typical criterion to be optimized is the overall time spent by the squad (i.e., the slowest team member) during the task execution.



# Artificial Ant Problem

## Santa Fe trail

- $32 \times 32$ **grid** with 89 food pieces.
- **Obstacles**
  - $1\times, 2\times$ strait,
  - $1\times, 2\times, 3\times$ right/left.




## Ant capabilities

- **detects** the food right in front of him in direction he faces.
- **actions** observable from outside
  - MOVE – makes a step and eats a food piece if there is some,
  - LEFT – turns left,
  - RIGHT – turns right,
  - NO-OP – no operation.

**Goal** is to find a strategy that would navigate an ant through the grid so that it finds all the food pieces in the given time (600 time steps).

# Artificial Ant Problem: GA Approach

**Collins a Jefferson 1991, standard GA using binary representation**

**Representation**

- strategy represented by finite state machine,
- table of transitions coded as binary chromosomes of fixed length.

Example: 4-state FSM, 34-bit long chromosomes

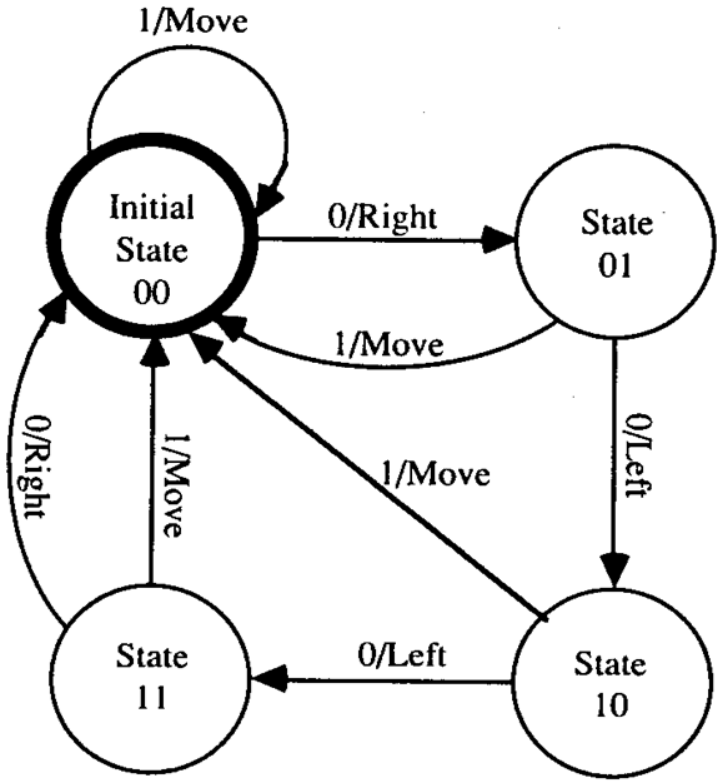
| | Current state | Input | New state | Operation |
|---|---|---|---|---|
| 1 | 00 | 0 | 01 | 10 = Right |
| 2 | 00 | 1 | 00 | 11 = Move |
| 3 | 01 | 0 | 10 | 01 = Left |
| 4 | 01 | 1 | 00 | 11 = Move |
| 5 | 10 | 0 | 11 | 01 = Left |
| 6 | 10 | 1 | 00 | 11 = Move |
| 7 | 11 | 0 | 00 | 10 = Right |
| 8 | 11 | 1 | 00 | 11 = Move |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 00 | 0110 | 0011 | 1001 | 0011 | 1101 | 0011 | 0010 | 0011 |

# Artificial Ant Problem: Example cont.

## Ant behavior

- What happens if the ant hits an obstacle?
- What is strange with transition from state $10$ to the initial state $00$?
- When does the ant succeed?
- Is the number of states sufficient to solve the problem?
- Do all of the possible 32-bit chromosomes represent a feasible solution?



# Artificial Ant Problem: GA result

**Representation**

- 32 states,
- $453 = 64 \times 7 + 5$ bits !!!

**Population size: 65.536 !!!**

**Number of generations: 200**

**Total number of samples tried:** $13 \times 10^6$ **!!!**

# Genetic Programming (GP)

**GP shares with GA** the philosophy of survival and reproduction of the fittest and the analogy of naturally occurring genetic operators.

**GP differs from GA** in a representation, genetic operators and a scope of applications.

**GP is extension of the conventional GA** in which the structures undergoing adaptation are trees of dynamically varying size and shape representing hierarchical computer programs.

**Applications**

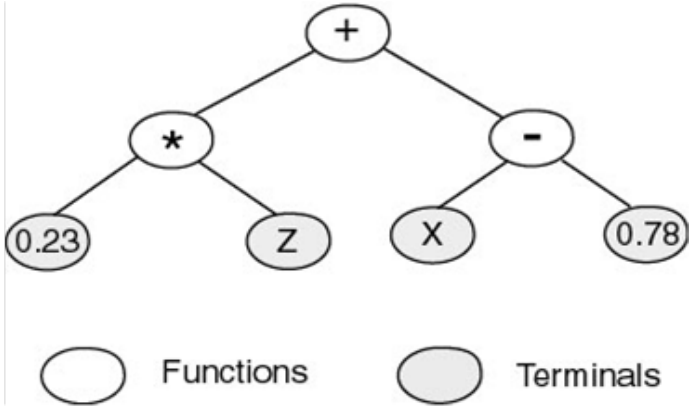
- learning programs,
- learning decision trees,
- learning rules,
- learning strategies,
- ...

# GP: Representation

All possible trees are composed of **functions** (inner nodes) and **terminals** (leaf nodes) appropriate to the problem domain

- **Terminals** – inputs to the programs (independent variables), real, integer or logical constants, actions.
- **Functions**
  - arithmetic operators (+, -, *, / ),
  - algebraic functions (sin, cos, exp, log),
  - logical functions (AND, OR, NOT),
  - conditional operators (If-Then-Else, cond?true:false),
  - and others.

**Example**: Tree representation of a LISP S-expression $0.23 * Z + X - 0.78$




**Closure** – each of the functions should be able to accept, as its argument, any value that may be returned by any function and any terminal.

# GP Initialisation: Common Methods

GP needs a good tree-creation algorithm to create trees for the initial population and subtrees for subtree mutation.

Required characteristics:

- Light computationally complex; optimally linear in tree size.
- User control over expected tree size.
- User control over specific node appearance in trees.

`GROW` method (each branch has $depth \leq D$):

- nodes at depth $d < D_{max}$ randomly chosen from $F \cup T$,
- nodes at depth $d = D_{max}$ randomly chosen from $T$.

`FULL` method (each branch has $depth = D$):

- nodes at depth $d < D$ randomly chosen from function set $F$,
- nodes at depth $d = D$ randomly chosen from terminal set $T$.

```
   GROW(depth d, max depth D)
   Returns: a tree of depth ≤ D − d
1  if (d = D) return a random terminal
2  else
3     choose a random func or term f
4     if (f is terminal) return f
5     else
6        for each argument a of f
7           fill a with GROW(d + 1, D)
8        return f
```

# GP Initialisation

Characteristics of GROW:

- does not have a size parameter – does not allow the user to create a desired size distribution,
- does not allow the user to define the expected probabilities of certain nodes appearing in trees,
- does not give the user much control over the tree structures generated.
- there is no appropriate way to create trees with either a fixed or average tree size or depth.

RAMPED HALF-AND-HALF – GROW & FULL method each deliver half of the initial population.
$D$ is chosen between 2 to 6,

# GP Initialisation

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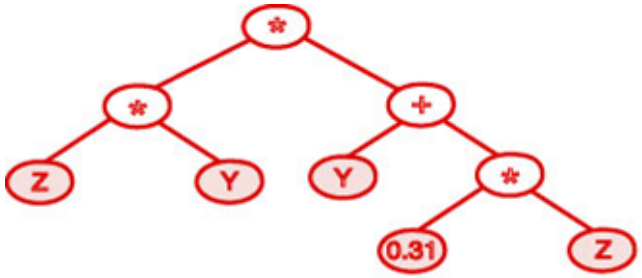
`RAMPED HALF-AND-HALF` – `GROW` & `FULL` method each deliver half of the initial population. $D$ is chosen between 2 to 6,

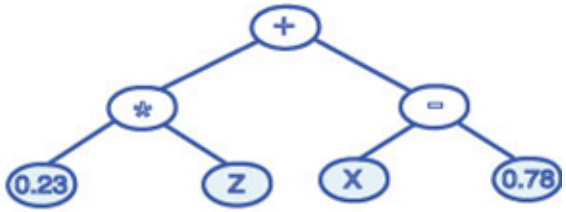
PTC1 is a modification of `GROW` that

- allows the user to define probabilities of appearance of functions within the tree,
- gives user a control over desired expected tree size, and guarantees that, on average, trees will be of that size.
- does not give the user any control over the variance in tree sizes,
- is fast, running in time near-linear in tree size.

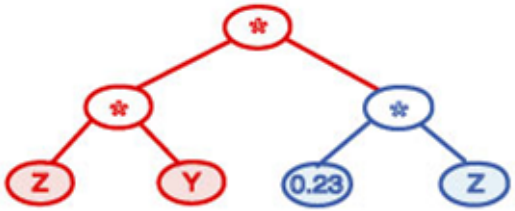
# GP: Standard Crossover



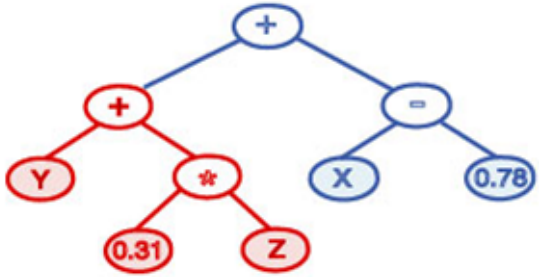

Parent 1: Z * Y * (Y + 0.31 * Z )




Parent 2: 0.23 * Z + X - 0.78



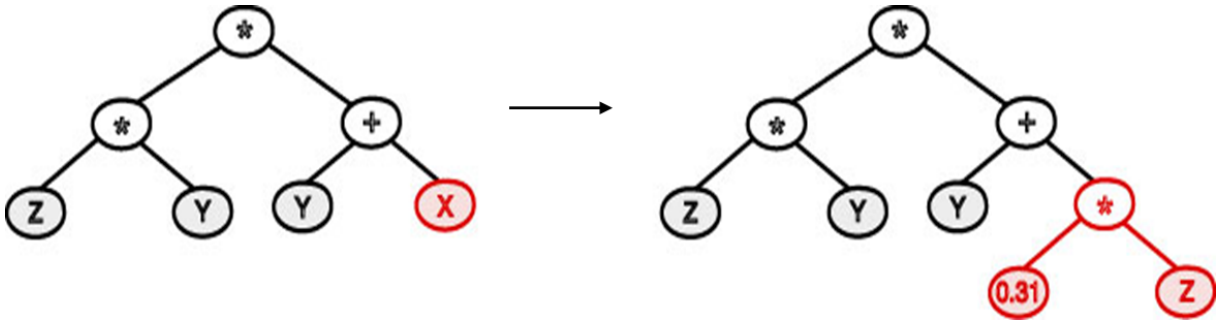

Child 1: 0.23 * Y * Z^2




Child 2: Y + 0.31 * Z + X - 0.78

# GP: Subtree-Replacing Mutation

**Mutation** replaces selected subtree with a randomly generated new one.



# GP: Selection

Commonly used are the fitness proportionate roulette wheel selection or the tournament selection.

**Greedy over-selection** is recommended for complex problems that require large populations ($> 1000$) – the motivation is to increase efficiency by increasing the chance of being selected to the fitter individuals in the population

- rank population by fitness and divide it into two groups:
  - group I: the fittest individuals that together accounting for $c = x\%$ of the sum of fitness values in the population,
  - group II: remaining less fit individuals.
- 80% of the time an individual is selected from group I in proportion to its fitness; 20% of the time, an individual is selected from group II.
- For population size $=$ 1000, 2000, 4000, 8000, $x = 32\%$, $16\%$, $8\%$, $4\%$.

  %'s come from rule of thumb.

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  %'s come from rule of thumb.

Example: Effect of greedy over-selection for the 6-multiplexer problem

| Population size | I(M,i,z) without over-selection | I(M,i,z) with over-selection |
|---|---|---|
| 1,000 | 343,000 | 33,000 |
| 2,000 | 294,000 | 18,000 |
| 4,000 | 160,000 | 24,000 |

# Artificial Ant Problem

## Santa Fe trail

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- **Obstacles**
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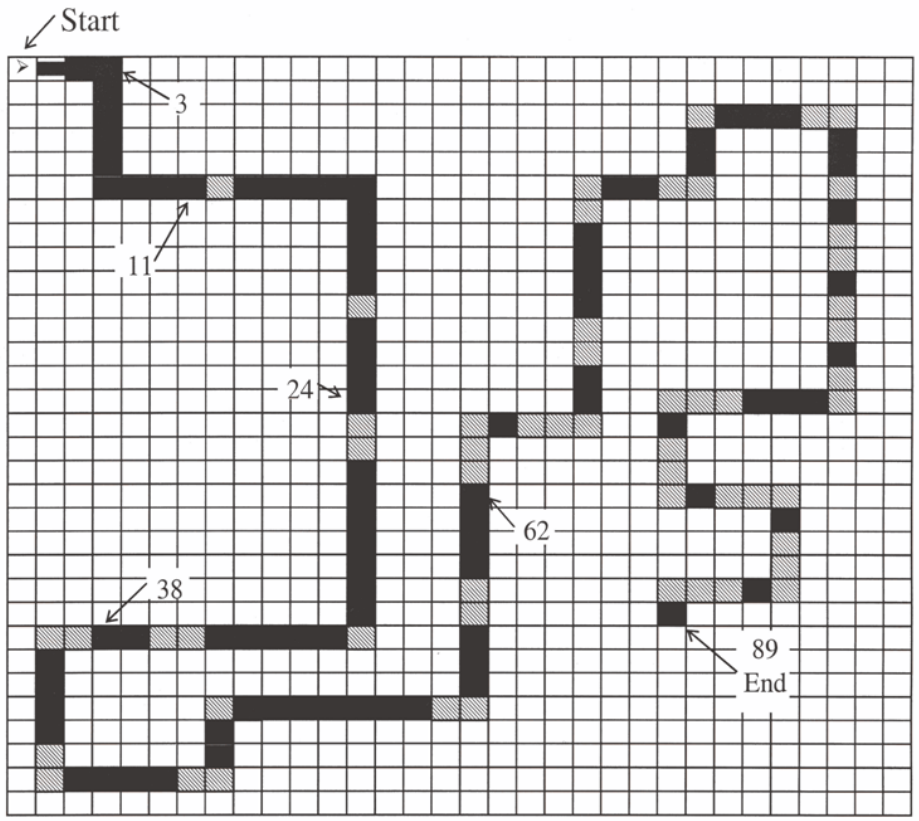



## Ant capabilities

- **detects** the food right in front of him in direction he faces.
- **actions** observable from outside
  - MOVE – makes a step and eats a food piece if there is some,
  - LEFT – turns left,
  - RIGHT – turns right,
  - NO-OP – no operation.

**Goal** is to find a strategy that would navigate an ant through the grid so that it finds all the food pieces in the given time (600 time steps).

# Artificial Ant Problem: GP Approach

**Terminals**

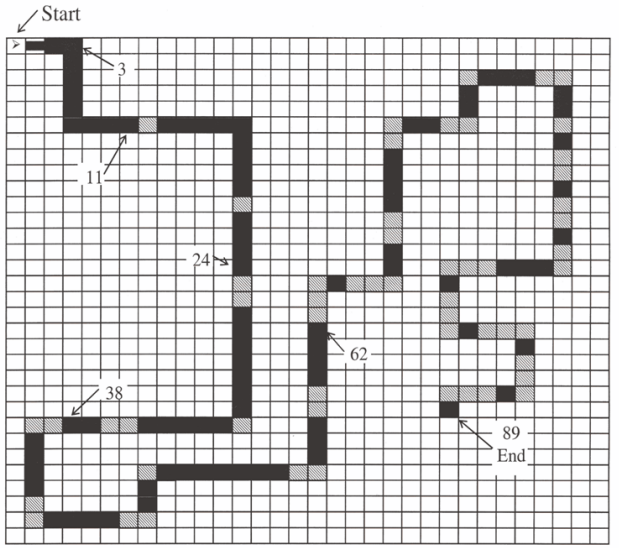
- motorial section,
- T = MOVE, LEFT, RIGHT.

**Functions**

- conditional IF-FOOD-AHEAD – food detection, 2 arguments (is/is_not food ahead),
- unconditional PROG2, PROG3 – sequence of 2/3 actions.

Ant repeats the program until time runs out (600 time steps) or all the food has been eaten.

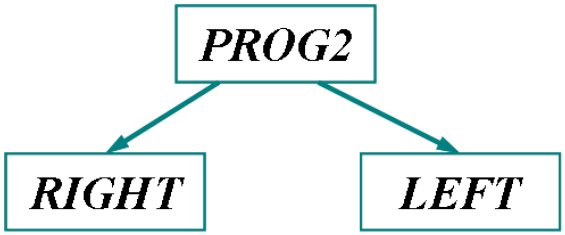
**Santa Fe trail**



# Artificial Ant Problem: GP Approach cont.

Typical solutions in the initial population

- this solution




  completely fails in finding and eating the food,

- similarly this one

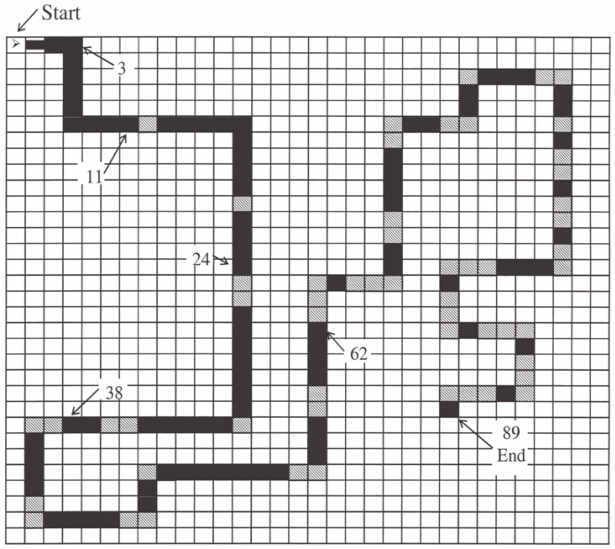
  (IF-FOOD-AHEAD (LEFT)(RIGHT)),

- this one

  (PROG2 (MOVE) (MOVE))

  just by chance finds 3 pieces of food.

**Santa Fe trail**



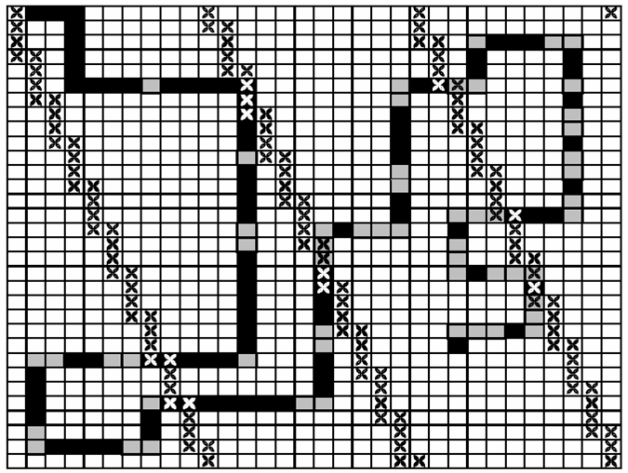
# Artificial Ant Problem: GP Approach cont.

More interesting solutions

- **Quilter** – performs systematic exploration of the grid,

```
(PROG3  (RIGHT)
        (PROG3  (MOVE) (MOVE) (MOVE))
        (PROG2  (LEFT) (MOVE)))
```

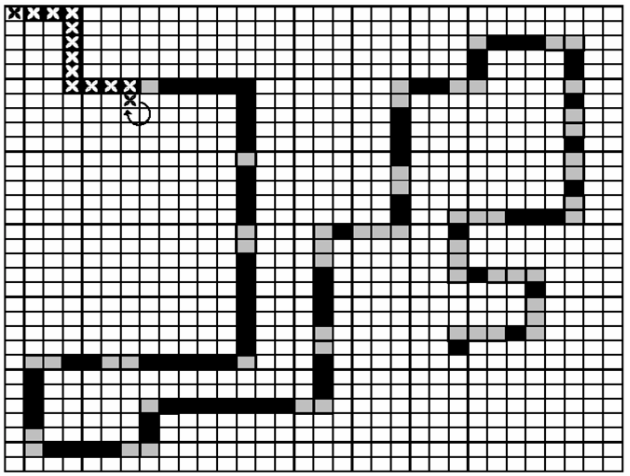
Quilter performance



- **Tracker** – perfectly tracks the food until the first obstacle occurs, then it gets trapped in an infinite loop.

```
(IF-FOOD-AHEAD (MOVE) (RIGHT))
```

Tracker performance

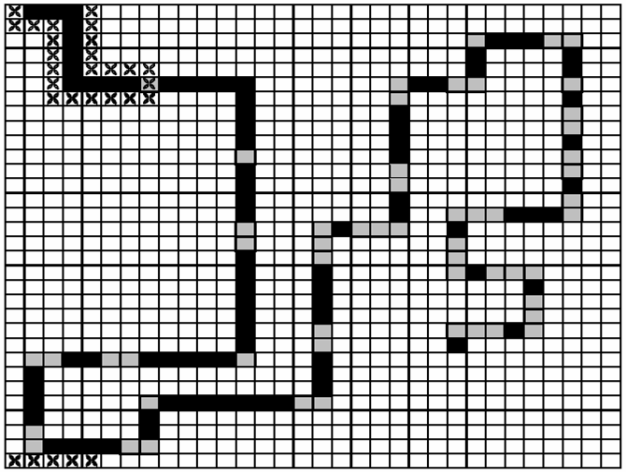


# Artificial Ant Problem: GP Approach cont.

- **Avoider** – perfectly avoids food!!!

```
(I-F-A  (RIGHT)
        (I-F-A      (RIGHT)
                    (PROG2 (MOVE) (LEFT))))
```

Avoider performance



Average fitness in the initial population is 3.5

# Artificial Ant Problem: GP result

In generation 21, the following solution was found that already navigates an ant so that he eats all 89 food pieces in the given time.

```
(I-F-A  (MOVE)
        (PROG3  (I-F-A      (MOVE)
                            (RIGHT)
                (PROG2      (RIGHT)
                            (PROG2      (LEFT)
                                        (RIGHT))))
                (PROG2      (I-F-A      (MOVE)
                                        (LEFT))
                            (MOVE))))
```

This program solves every trail with the obstacles of the same type as occurs in Santa Fe trail.

**Compare the computational complexity with the GA approach!!!**

GA approach: $65.536 \times 200 = 13 \times 10^6$ trials.

vs.

GP approach: $500 \times 21 = 10.500$ trials.

# Example of GP in Action: Trigonometric Identity

**Task** is to find an equivalent expression to $cos(2x)$.

**GP implementation:**

- **Terminal set** $T = \{x, 1.0\}$.
- **Function set** $F = \{+, -, *, \%, sin\}$.
- **Training cases**: 20 pairs $(x_i, y_i)$, where $x_i$ are values evenly distributed in interval $(0, 2\pi)$.
- **Fitness**: Sum of absolute differences between desired $y_i$ and the values returned by generated expressions.
- **Stopping criterion**: A solution found that gives the error less than 0.01.

# Example of GP in Action: Trigonometric Identity cont.

**1. run,** $13^{th}$ **generation**

```
(- (- 1 (* (sin x) (sin x))) (* (sin x) (sinx)))
```

which equals (after editing) to $1 - 2 * sin^2 x$.

**2. run,** $34^{th}$ **generation**

```
(- 1 (* (* (sin x) (sin x)) 2))
```

which is just another way of writing the same expression.

# Example of GP in Action: Trigonometric Identity cont.

**1. run,** $13^{th}$ **generation**

```
(- (- 1 (* (sin x) (sin x))) (* (sin x) (sinx)))
```

which equals (after editing) to $1 - 2 * sin^2 x$.

**2. run,** $34^{th}$ **generation**

```
(- 1 (* (* (sin x) (sin x)) 2))
```

which is just another way of writing the same expression.

**3. run,** $30^{th}$ **generation**

```
(sin (- (- 2 (* x 2))
        (sin (sin (sin (sin (sin (sin (* (sin (sin 1))
                                         (sin (sin 1))
                                         ))))))))
```

# Example of GP in Action: Trigonometric Identity cont.

**1. run,** $13^{th}$ **generation**

```
(- (- 1 (* (sin x) (sin x))) (* (sin x) (sinx)))
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which equals (after editing) to $1 - 2 * sin^2 x$.

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(- 1 (* (* (sin x) (sin x)) 2))
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which is just another way of writing the same expression.

**3. run,** $30^{th}$ **generation**

```
(sin (- (- 2 (* x 2))
        (sin (sin (sin (sin (sin (sin (* (sin (sin 1))
                                         (sin (sin 1))
                                         ))))))))
```

(2 minus the expression on the 2nd and 3rd rows) is almost $\pi/2$ so the discovered identity is

$$cos(2x) = sin(\pi/2 - 2x).$$

# EA Materials: Reading, Demos, Software

## Reading

- D. E. Goldberg: Genetic Algorithms in Search, Optimization, and Machine Learning, Addison-Wesley, 1989.
- Z. Michalewicz: Genetic Algorithms + Data Structures = Evolution Programs, Springer, 1998.
- Poli, R., Langdon, W., McPhee, N.F.: *A Field Guide to Genetic Programming*, 2008, http://www.gp-field-guide.org.uk/
- Koza, J.: Genetic Programming: On the Programming of Computers by Means of Natural Selection, MIT Press, 1992.

## HUMIES: Human-Competitive Results

- http://www.genetic-programming.org/hc2011/combined.html

## Demos

- M. Obitko: Introduction to genetic algorithms with java applets, http://cs.felk.cvut.cz/ xobitko/ga/

## Software

- ECJ 16 – A Java-based Evolutionary Computation Research System http://cs.gmu.edu/ eclab/projects/ecj/