### GAME PLAYING

CHAPTER 6

## Outline

- $\diamondsuit \ \mathsf{Games}$
- $\diamondsuit$  Perfect play
  - minimax decisions
  - $\alpha \beta$  pruning
- $\diamondsuit$  Resource limits and approximate evaluation
- $\diamondsuit$  Games of chance
- $\diamondsuit$  Games of imperfect information

#### Games vs. search problems

"Unpredictable" opponent  $\Rightarrow$  solution is a strategy specifying a move for every possible opponent reply

Time limits  $\Rightarrow$  unlikely to find goal, must approximate

Plan of attack:

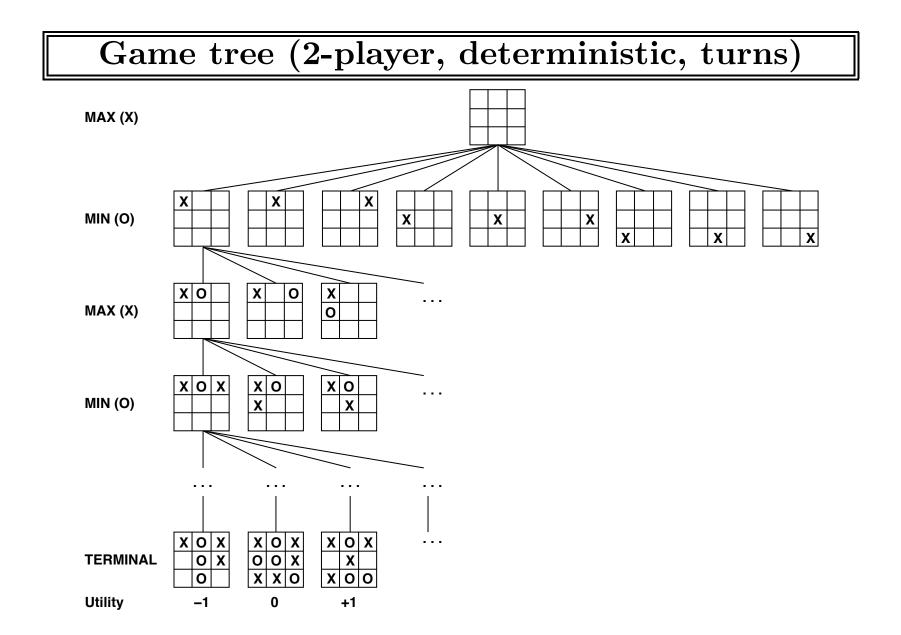
- Computer considers possible lines of play (Babbage, 1846)
- Algorithm for perfect play (Zermelo, 1912; Von Neumann, 1944)
- Finite horizon, approximate evaluation (Zuse, 1945; Wiener, 1948; Shannon, 1950)
- First chess program (Turing, 1951)
- Machine learning to improve evaluation accuracy (Samuel, 1952–57)
- Pruning to allow deeper search (McCarthy, 1956)

## Types of games

perfect information

imperfect information

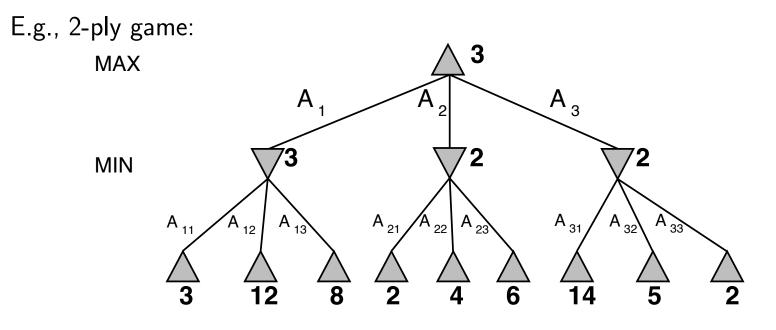
deterministic	chance
chess, checkers,	backgammon
go, othello	monopoly
battleships,	bridge, poker, scrabble
blind tictactoe	nuclear war



## Minimax

Perfect play for deterministic, perfect-information games

Idea: choose move to position with highest minimax value = best achievable payoff against best play



## Minimax algorithm

```
function MINIMAX-DECISION(state) returns an action
```

**inputs**: *state*, current state in game

```
return the a in ACTIONS(state) maximizing MIN-VALUE(RESULT(a, state))
```

```
function MAX-VALUE(state) returns a utility value
if TERMINAL-TEST(state) then return UTILITY(state)
v \leftarrow -\infty
for a, s in SUCCESSORS(state) do v \leftarrow MAX(v, MIN-VALUE(s))
return v
```

```
function MIN-VALUE(state) returns a utility value
if TERMINAL-TEST(state) then return UTILITY(state)
v \leftarrow \infty
for a, s in SUCCESSORS(state) do v \leftarrow MIN(v, MAX-VALUE(s))
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```

Complete??

<u>Complete</u>?? Only if tree is finite (chess has specific rules for this). NB a finite strategy can exist even in an infinite tree!

Optimal??

Complete?? Yes, if tree is finite (chess has specific rules for this)

Optimal?? Yes, against an optimal opponent. Otherwise??

Time complexity??

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Time complexity??  $O(b^m)$ 

Space complexity??

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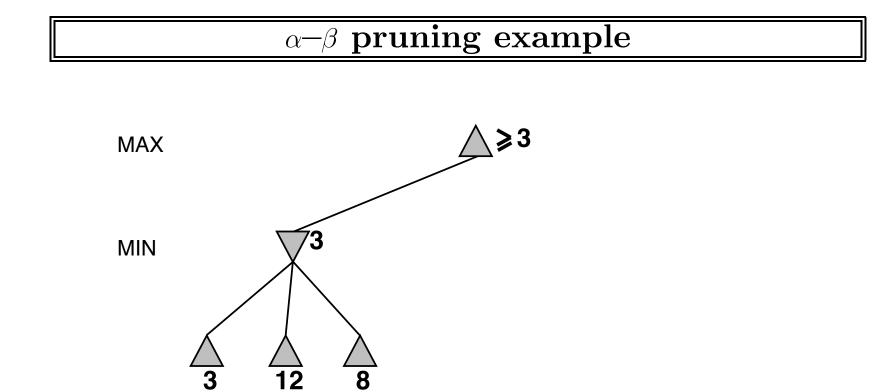
Optimal?? Yes, against an optimal opponent. Otherwise??

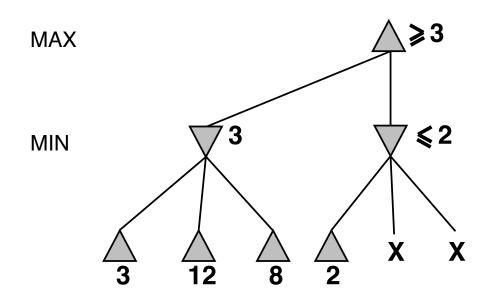
Time complexity??  $O(b^m)$ 

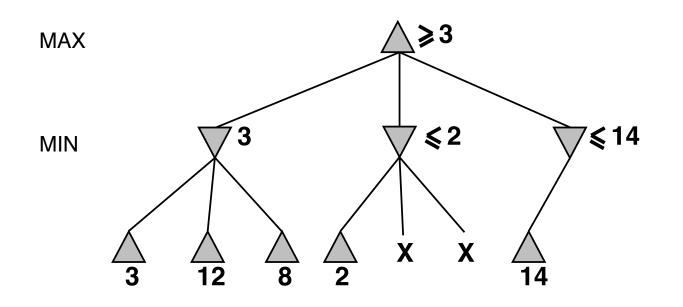
Space complexity?? O(bm) (depth-first exploration)

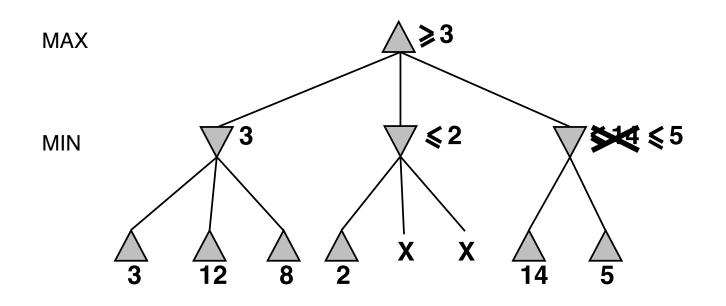
For chess,  $b \approx 35$ ,  $m \approx 100$  for "reasonable" games  $\Rightarrow$  exact solution completely infeasible

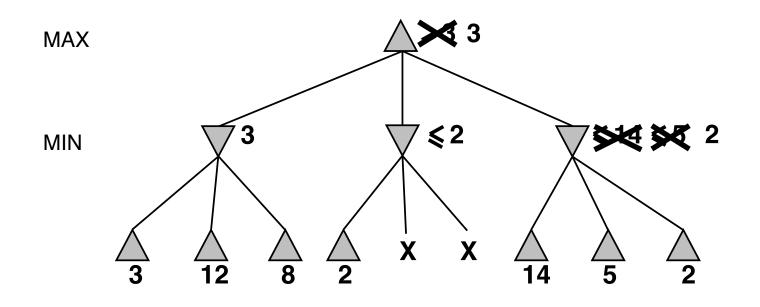
But do we need to explore every path?



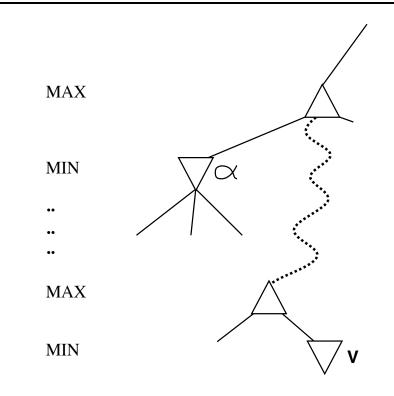








## Why is it called $\alpha - \beta$ ?



 $\alpha$  is the best value (to MAX) found so far off the current path If V is worse than  $\alpha$ , MAX will avoid it  $\Rightarrow$  prune that branch Define  $\beta$  similarly for MIN

## The $\alpha - \beta$ algorithm

```
function ALPHA-BETA-DECISION(state) returns an action
   return the a in ACTIONS(state) maximizing MIN-VALUE(RESULT(a, state))
function MAX-VALUE(state, \alpha, \beta) returns a utility value
   inputs: state, current state in game
            \alpha, the value of the best alternative for MAX along the path to state
            \beta, the value of the best alternative for MIN along the path to state
   if TERMINAL-TEST(state) then return UTILITY(state)
   v \leftarrow -\infty
   for a, s in SUCCESSORS(state) do
      v \leftarrow Max(v, MIN-VALUE(s, \alpha, \beta))
      if v \geq \beta then return v
      \alpha \leftarrow MAX(\alpha, v)
   return v
```

**function** MIN-VALUE(*state*,  $\alpha$ ,  $\beta$ ) **returns** *a utility value* same as MAX-VALUE but with roles of  $\alpha$ ,  $\beta$  reversed

### **Properties of** $\alpha - \beta$

Pruning does not affect final result

Good move ordering improves effectiveness of pruning

With "perfect ordering," time complexity =  $O(b^{m/2})$  $\Rightarrow$  **doubles** solvable depth

A simple example of the value of reasoning about which computations are relevant (a form of metareasoning)

Unfortunately,  $35^{50}$  is still impossible!

#### **Resource** limits

Standard approach:

• Use CUTOFF-TEST instead of TERMINAL-TEST e.g., depth limit (perhaps add quiescence search)

 $\bullet$  Use  $\mathrm{Eval}$  instead of  $\mathrm{UTILITY}$ 

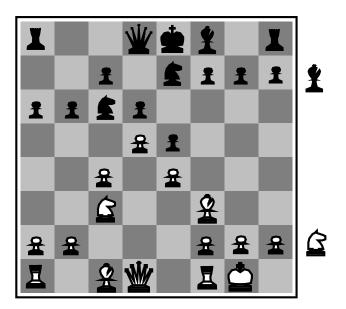
i.e., evaluation function that estimates desirability of position

Suppose we have 100 seconds, explore  $10^4$  nodes/second

 $\Rightarrow 10^6$  nodes per move  $\approx 35^{8/2}$ 

 $\Rightarrow \alpha - \beta$  reaches depth 8  $\Rightarrow$  pretty good chess program

## **Evaluation functions**



Black to move

White slightly better

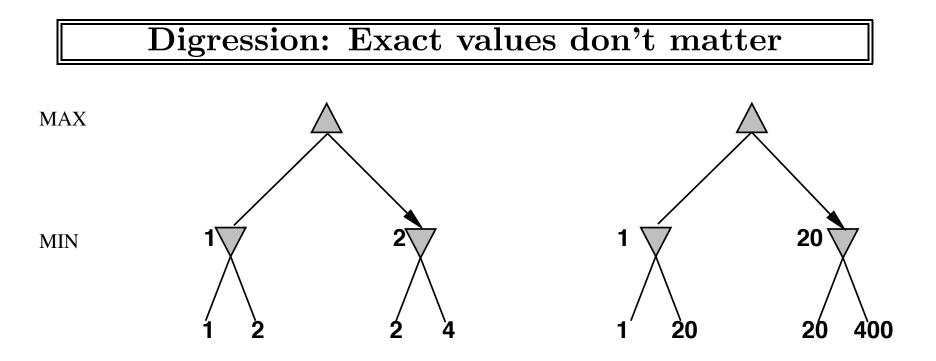
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> White to move Black winning

For chess, typically linear weighted sum of features

 $Eval(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$ 

e.g.,  $w_1 = 9$  with  $f_1(s) =$ (number of white queens) – (number of black queens), etc.



Behaviour is preserved under any monotonic transformation of EVAL

Only the order matters:

payoff in deterministic games acts as an ordinal utility function

### Deterministic games in practice

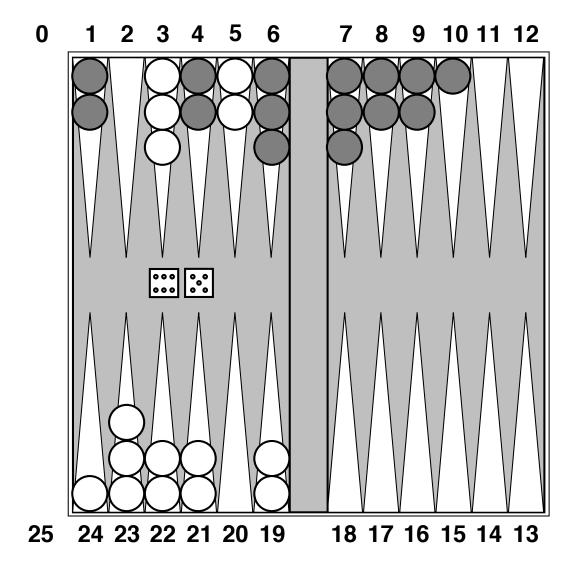
Checkers: Chinook ended 40-year-reign of human world champion Marion Tinsley in 1994. Used an endgame database defining perfect play for all positions involving 8 or fewer pieces on the board, a total of 443,748,401,247 positions.

Chess: Deep Blue defeated human world champion Gary Kasparov in a sixgame match in 1997. Deep Blue searches 200 million positions per second, uses very sophisticated evaluation, and undisclosed methods for extending some lines of search up to 40 ply.

Othello: human champions refuse to compete against computers, who are too good.

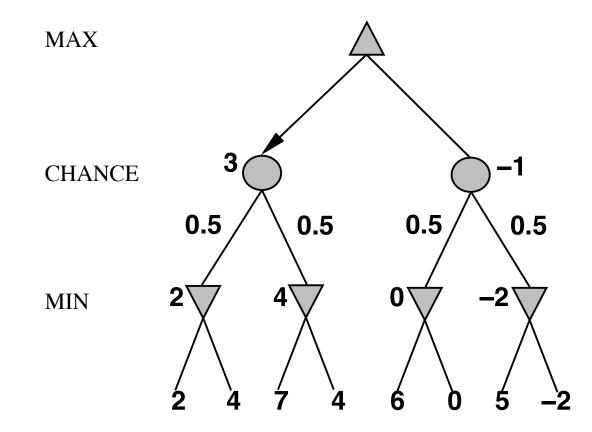
Go: human champions refuse to compete against computers, who are too bad. In go, b > 300, so most programs use pattern knowledge bases to suggest plausible moves.

Nondeterministic games: backgammon



Nondeterministic games in general

In nondeterministic games, chance introduced by dice, card-shuffling Simplified example with coin-flipping:



## Algorithm for nondeterministic games

EXPECTIMINIMAX gives perfect play

. . .

Just like  $\operatorname{MINIMAX}$ , except we must also handle chance nodes:

if state is a MAX node then
 return the highest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
if state is a MIN node then
 return the lowest EXPECTIMINIMAX-VALUE of SUCCESSORS(state)
if state is a chance node then
 return average of EXPECTIMINIMAX-VALUE of SUCCESSORS(state)

#### Nondeterministic games in practice

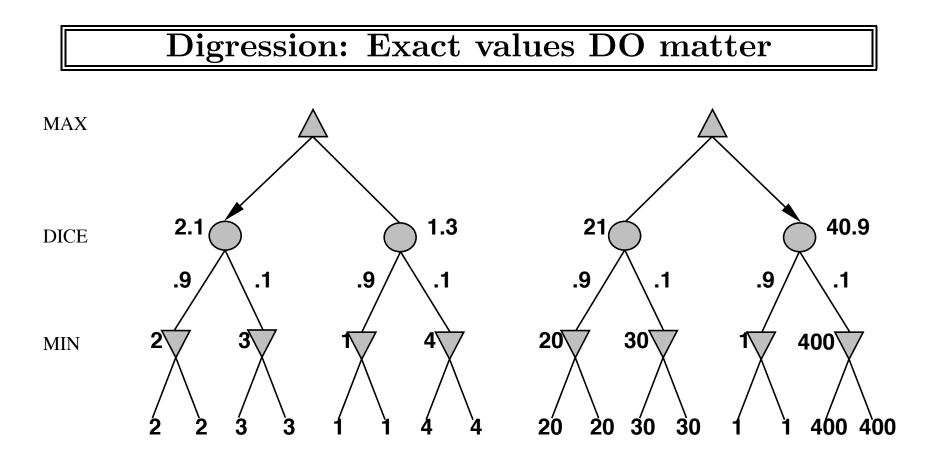
Dice rolls increase b: 21 possible rolls with 2 dice Backgammon  $\approx$  20 legal moves (can be 6,000 with 1-1 roll)

depth  $4 = 20 \times (21 \times 20)^3 \approx 1.2 \times 10^9$ 

As depth increases, probability of reaching a given node shrinks  $\Rightarrow$  value of lookahead is diminished

 $\alpha \text{-}\beta$  pruning is much less effective

$$\label{eq:total_total} \begin{split} TDGAMMON \text{ uses depth-2 search} + \text{very good } Eval \\ \approx \text{world-champion level} \end{split}$$



Behaviour is preserved only by positive linear transformation of  $\mathrm{EVAL}$ 

Hence  $\operatorname{Eval}$  should be proportional to the expected payoff

### Games of imperfect information

E.g., card games, where opponent's initial cards are unknown

Typically we can calculate a probability for each possible deal

Seems just like having one big dice roll at the beginning of the game  $^{\ast}$ 

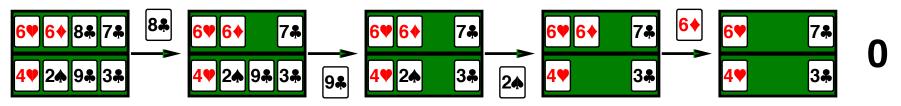
Idea: compute the minimax value of each action in each deal, then choose the action with highest expected value over all deals\*

Special case: if an action is optimal for all deals, it's optimal.\*

GIB, current best bridge program, approximates this idea by1) generating 100 deals consistent with bidding information2) picking the action that wins most tricks on average

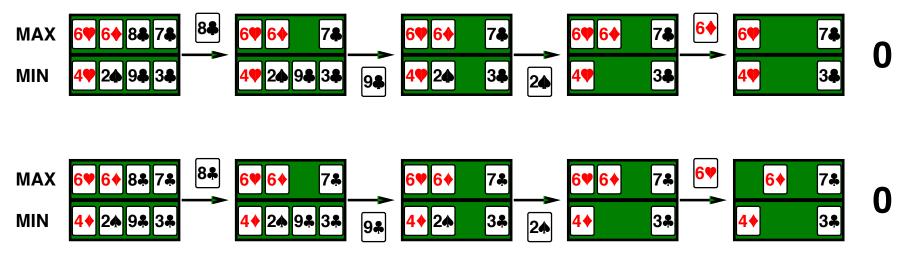
## Example

Four-card bridge/whist/hearts hand,  ${\rm MAx}$  to play first



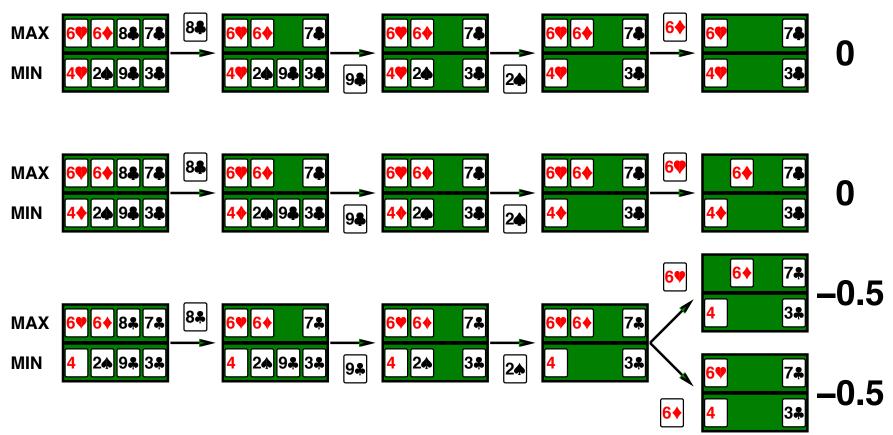
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#### Commonsense example

Road A leads to a small heap of gold pieces

Road B leads to a fork:

take the left fork and you'll find a mound of jewels; take the right fork and you'll be run over by a bus.

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Road B leads to a fork:

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Road A leads to a small heap of gold pieces Road B leads to a fork:

> guess correctly and you'll find a mound of jewels; guess incorrectly and you'll be run over by a bus.

#### Proper analysis

 $^{\ast}$  Intuition that the value of an action is the average of its values in all actual states is  $\mathbf{WRONG}$ 

With partial observability, value of an action depends on the information state or belief state the agent is in

Can generate and search a tree of information states

Leads to rational behaviors such as

- $\diamond$  Acting to obtain information
- $\diamond$  Signalling to one's partner
- $\diamondsuit$  Acting randomly to minimize information disclosure

### Summary

Games are fun to work on! (and dangerous)

They illustrate several important points about AI

- $\diamondsuit$  perfection is unattainable  $\Rightarrow$  must approximate
- $\diamondsuit$  good idea to think about what to think about
- $\diamondsuit$  uncertainty constrains the assignment of values to states
- $\diamond$  optimal decisions depend on information state, not real state

Games are to AI as grand prix racing is to automobile design