Image registration

Jan Kybic

2010–2013

Image registration

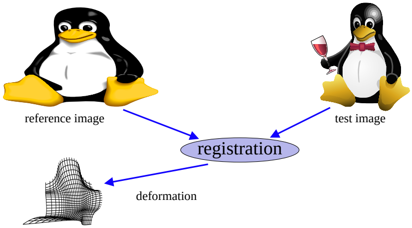
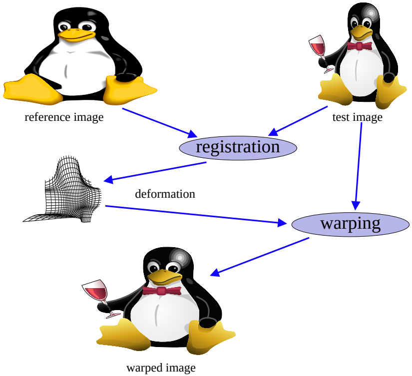


Image registration



(Biomedical) applications

. . . of image registration

- Comparing images
  - Different times
  - Different methods
  - Different subjects
- Analyzing sequences
  - Motion estimation
  - Segmentation

Qualitative and quantitative information.

Other applications of image registration

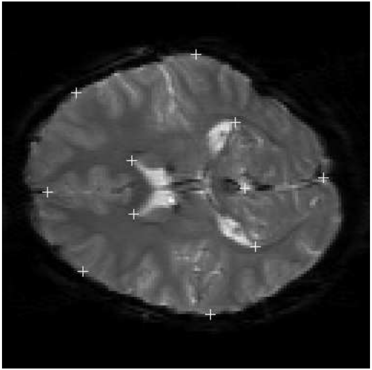
- video stabilization
- video compression
- image mosaicking
- stereo matching
- structure from motion

Motion analysis example

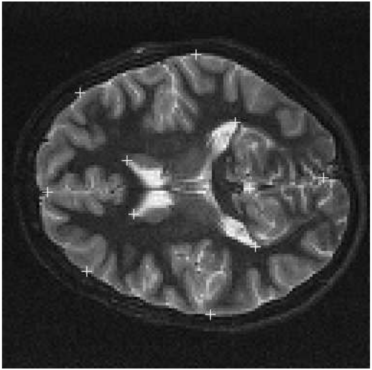
- heart ultrasound sequence (2C,4C)

```
mplayer -fs -loop 10
```

Registration example



EPI MRI



anatomical MRI

Image alignment

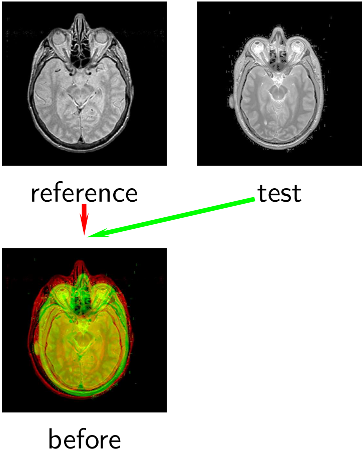
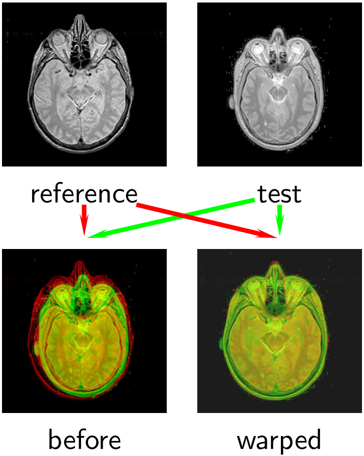
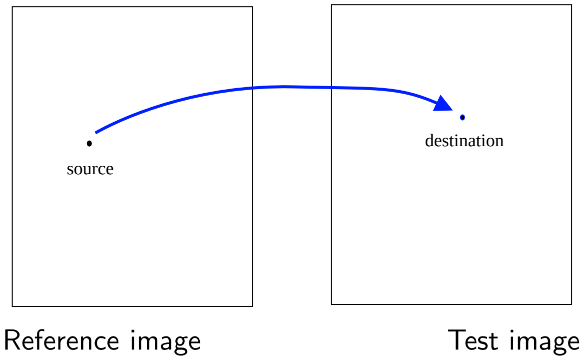


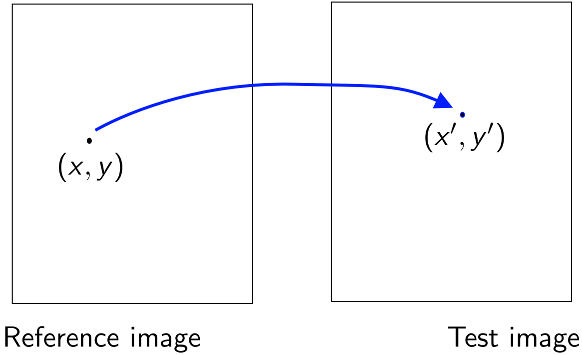
Image alignment



Correspondence function

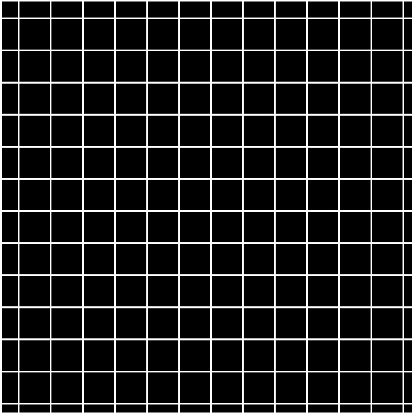


Correspondence function



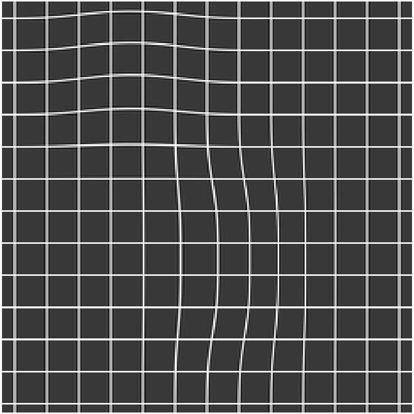
$$\mathbf{g}\left([x\ y]^T\right) = [x'\ y']^T$$

Deformation field



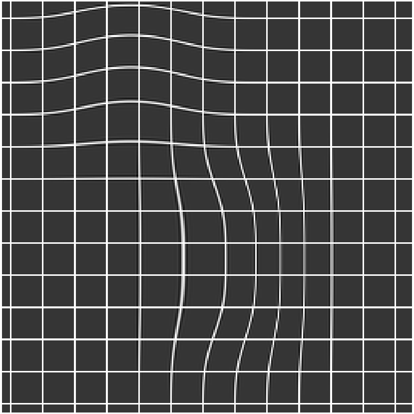
0 % deformation

Deformation field



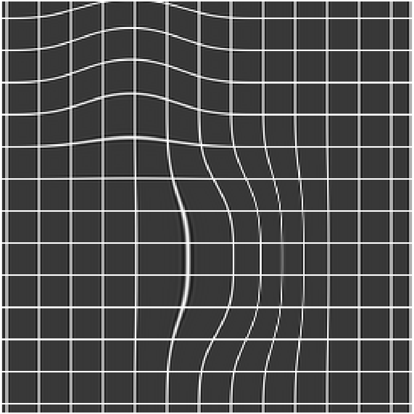
25 % deformation

Deformation field



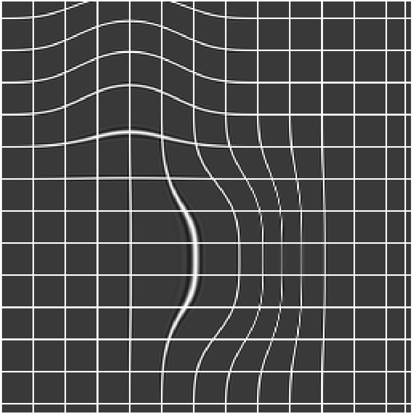
50 % deformation

Deformation field



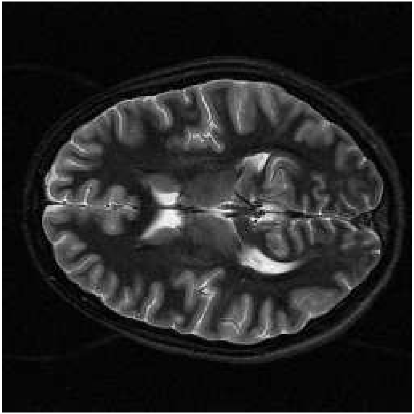
75 % deformation

Deformation field



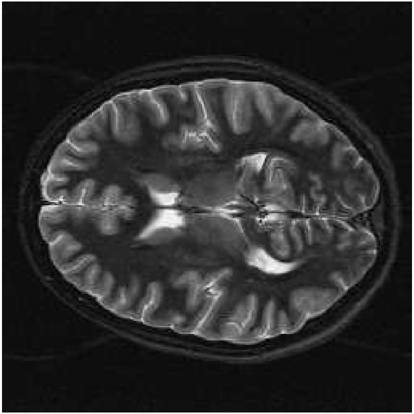
100 % deformation

Image warping



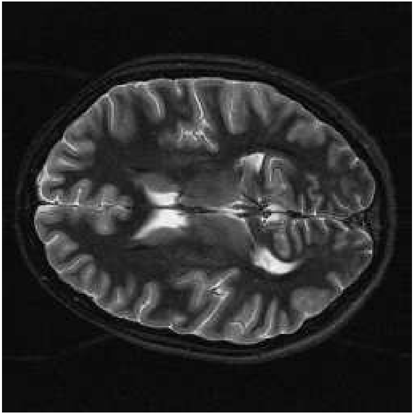
0 % deformation

Image warping



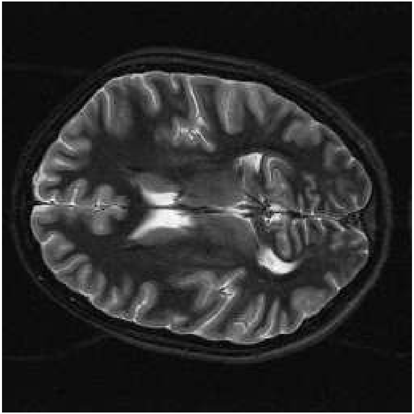
25 % deformation

Image warping



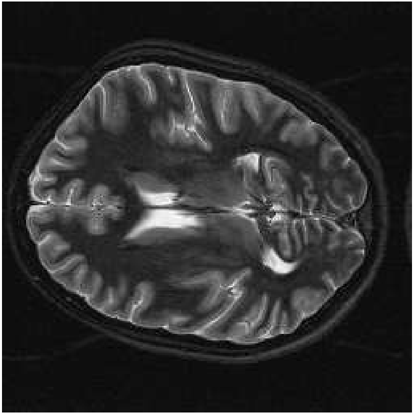
50 % deformation

Image warping



75 % deformation

Image warping

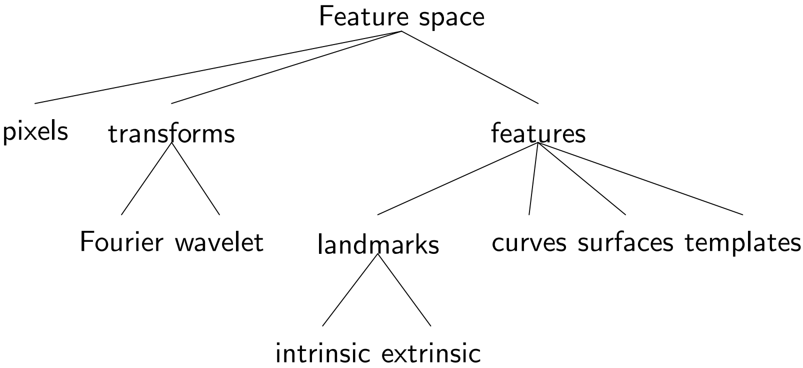


100 % deformation

Classification of registration methods

- **Feature space** — intermediate data extracted from image
- **Search space** — representation of the deformation
- **Similarity metric** — measuring the dissimilarity
- **Search strategy** — how to find the minimum
- **User interaction level**

Registration methods – Feature space



Registration methods – Search space

- Local
  - Variational
  - PDE
- Semi-local
  - (Quad)tree
  - B-splines
  - Wavelets
- Global
  - linear
  - polynomial
  - harmonic
- RBF, krigging
- Image dependent models (e.g. adaptive quadtrees)

Similarity metrics

- **Data term for pixel based criteria**
  - l_2 norm (SSD)
  - l_1 norm
  - correlation, normalized correlation
  - mutual information, normalized mutual information
- **Other data terms**
  - image interpolation — important
  - feature-based methods — distance
  - template-based methods — windowed pixel-based criteria
  - transform-based methods — norm in the transform domain
  - *preprocessing* — filtering, histogram equalization,...
- **Regularization**
  - Norm (l_p) of the derivatives
  - Implicit regularization (constrained model)
  - Smoothing

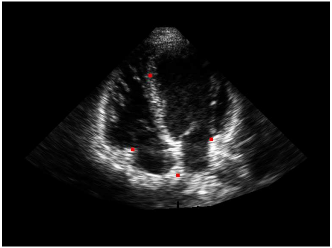
Search strategy

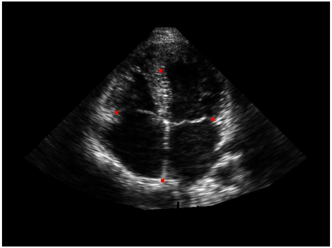
- Direct solution
- Exhaustive search
- Dynamic programming
- PDE evolution
- Multidimensional optimisation
  - gradient descent
  - Newton-like methods, exact/estimated Hessian, Marquardt-Levenberg, conjugated gradients, BFGS, ...
- Multiresolution

User interaction level

- Manual
- Automatic
- Semi-automatic

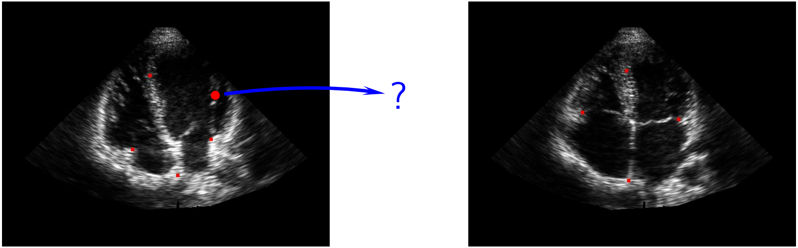
Manual registration





- Landmark identification

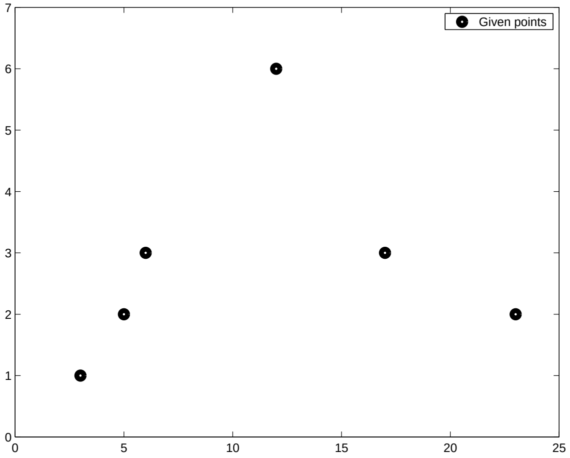
Manual registration



- Landmark identification
- Landmark interpolation

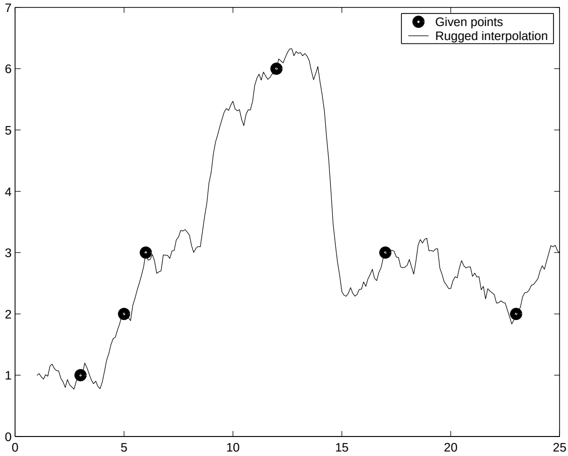
Variational reconstruction

Find a function



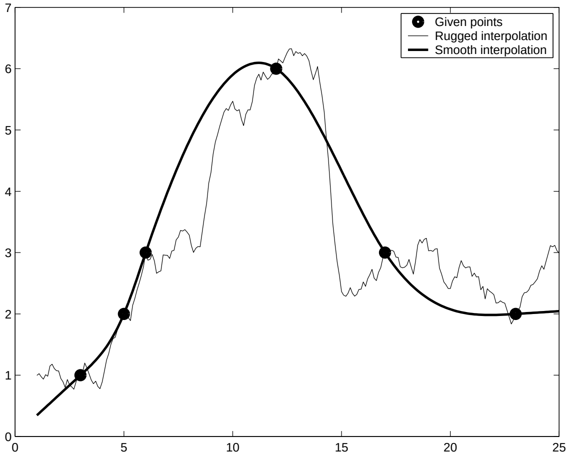
Variational reconstruction

Find a function

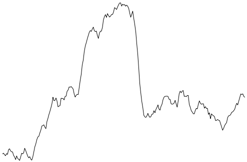


Variational reconstruction

Find a function

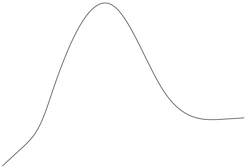


Rank functions



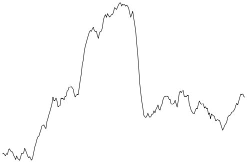
ugliest

. . .



prettiest

Rank functions



ugliest

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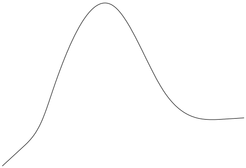


prettiest

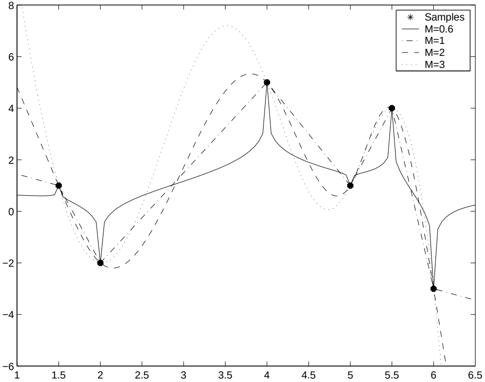
Variational criterion

$$J : \left(\mathbb{R}^m \to \mathbb{R}^n\right) \to \mathbb{R}_0^+$$
$$J(\mathbf{f}) \geq 0$$

Variational reconstruction

Find the best function
satisfying the constraints.

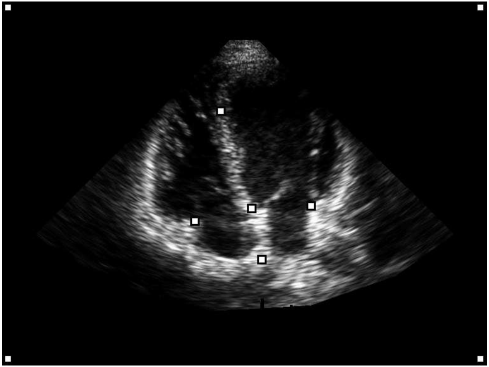
Tunable 1D interpolation



$$J(f) = \left\| \frac{\partial^M f}{\partial x^M} \right\|^2$$

Tunable 2D interpolation

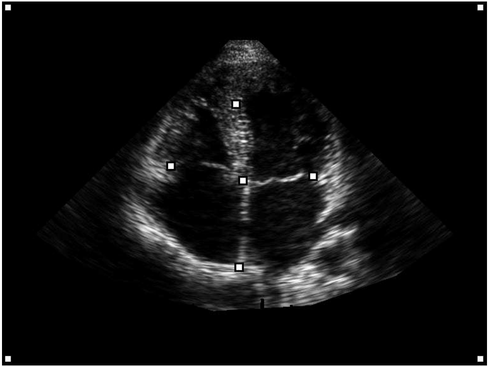
Reference



reference

Tunable 2D interpolation

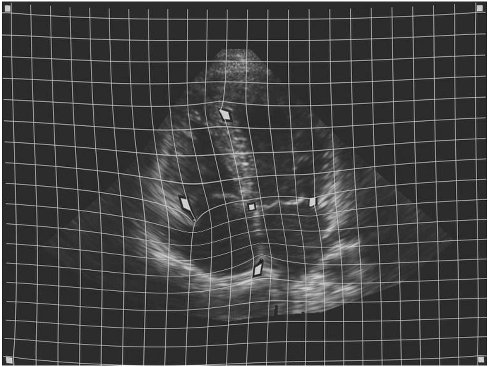
Test



test

Tunable 2D interpolation

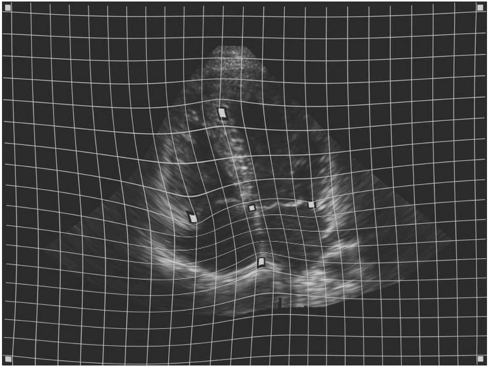
alpha=0.5



$$\int \|\nabla^{0.5} g(\mathbf{x})\|^2 \mathrm{d}\mathbf{x}$$

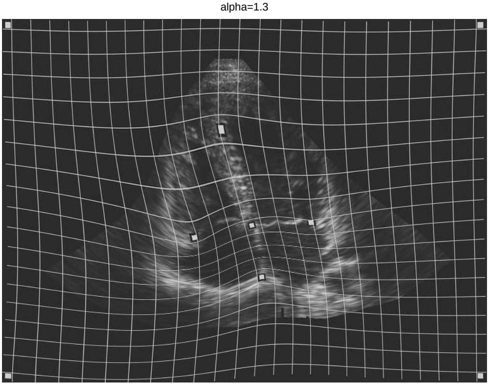
Tunable 2D interpolation

alpha=0.9



$$\int \|\nabla^{0.9} g(\mathbf{x})\|^2 \mathrm{d}\mathbf{x}$$

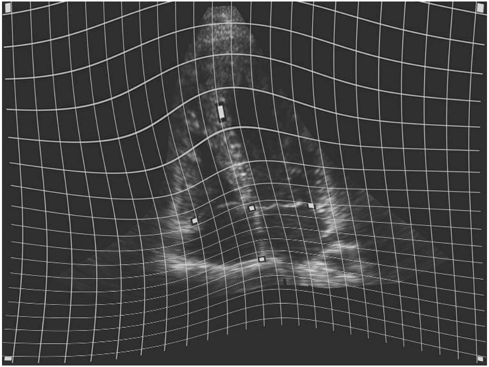
Tunable 2D interpolation



$$\int \|\nabla^{1.3} g(\mathbf{x})\|^2 \mathrm{d}\mathbf{x}$$

Tunable 2D interpolation

alpha=2.5



$$\int \|\nabla^{2.5} g(\mathbf{x})\|^2 \mathrm{d}\mathbf{x}$$

Landmark interpolation (2)

- **Constraints**
  - Hard constraints

$$\mathbf{g}(\mathbf{x}_i) = \begin{bmatrix} g_x(\mathbf{x}_i) \\ g_y(\mathbf{x}_i) \end{bmatrix} = \mathbf{z}_i = (x_i', y_i') \quad \text{for all } i \in \{1, \dots, N\}$$

  - Soft constraints

$$\sum_{i=1}^{N} \|\mathbf{g}(\mathbf{x}_i) - \mathbf{z}_i\|^2 \le \varepsilon$$

- **Properties**
  - invariance to scale, shifts, rotations
  - representability of linear transforms

Thin-plate splines

- Minimize an energy

$$J(g) = \int \left(\frac{\partial^2 g}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2 g}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 g}{\partial y^2}\right)^2 \mathrm{d}x\mathrm{d}y$$

$$J(\mathbf{g}) = J(g_x) + J(g_y)$$

under constraints

$$g(x_i, y_i) = z_i$$

Thin-plate splines

- Minimize an energy

$$J(g) = \int \left(\frac{\partial^2 g}{\partial x^2}\right)^2 + 2\left(\frac{\partial^2 g}{\partial x \partial y}\right)^2 + \left(\frac{\partial^2 g}{\partial y^2}\right)^2 \mathrm{d}x\mathrm{d}y$$

$$J(\mathbf{g}) = J(g_x) + J(g_y)$$

under constraints

$$g(x_i, y_i) = z_i$$

- Solution (for g_x only)

$$g_x(x, y) = \sum_{i=1}^{N} \lambda_i \varrho(\|\mathbf{x} - \mathbf{x}_i\|) + a_0 x + a_1 y + a_2$$

with $\quad \|\mathbf{x} - \mathbf{x}_i\| = \sqrt{(x - x_i)^2 + (y - y_i)^2} = r$

where $\varrho(r)$ is a *radial basis function* and $\quad \varrho(r) = r^2 \log r$

Thin-plate spline calculations

$$g_x(x, y) = \sum_{i=1}^{N} \lambda_{x,i} \varrho(\|\mathbf{x} - \mathbf{x}_i\|) + a_0 x + a_1 y + a_2$$
$$g_y(x, y) = \sum_{i=1}^{N} \lambda_{y,i} \varrho(\|\mathbf{x} - \mathbf{x}_i\|) + a_3 x + a_4 y + a_5$$

We have the data constraints:

$$g_x(x_i, y_i) = x_i' \qquad g_y(x_i, y_i) = y_i'$$

and the orthogonality constraints

$$\sum\nolimits_i \lambda_{x,i} = 0 \qquad \sum\nolimits_i \lambda_{y,i} = 0$$
$$\sum\nolimits_i \lambda_{x,i} x_i = 0 \qquad \sum\nolimits_i \lambda_{y,i} x_i = 0$$
$$\sum\nolimits_i \lambda_{x,i} y_i = 0 \qquad \sum\nolimits_i \lambda_{y,i} y_i = 0$$

Thin-plate spline calculations (2)

Matrix form (for g_x only)

$$\underbrace{\begin{bmatrix} \mathrm{A} & \mathrm{Q} \\ \mathrm{Q}^T & 0 \end{bmatrix}}_{\mathrm{B}} \begin{bmatrix} \boldsymbol{\lambda} \\ \mathbf{a} \end{bmatrix} = \begin{bmatrix} \mathbf{s} \\ \mathbf{0} \end{bmatrix}$$

with

$$\begin{aligned} \boldsymbol{\lambda} &= [\lambda_1 \dots \lambda_N] \\ \mathbf{a} &= [a_0 \; a_1 \; a_2]^T \\ \mathbf{s} &= [x'_1 \; x'_2 \dots x'_N]^T \\ (\mathrm{A})_{i,j} &= \varrho(\|\mathbf{x}_i - \mathbf{x}_j\|) \\ (Q)_{i,:} &= [x_i \; y_i \; 1] \end{aligned}$$

Thin-plate spline approximation

Matrix form (for g_x only)

$$\underbrace{\begin{bmatrix} \mathsf{A} + \gamma^{-1} I & \mathsf{Q} \\ \mathsf{Q}^T & 0 \end{bmatrix}}_{\mathsf{B}} \begin{bmatrix} \boldsymbol{\lambda} \\ \mathbf{a} \end{bmatrix} = \begin{bmatrix} \mathbf{s} \\ \mathbf{0} \end{bmatrix}$$

γ is a regularization weight and a Lagrange coefficient corresponding to the allowed error ε.

Least squares fitting

Assume that the transformation $\mathbf{g}$ is parameterized by a small number of parameters $\boldsymbol{\theta}$.

Minimize the landmark registration error:

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} \sum_i \|\mathbf{g}(\mathbf{x}_i) - \mathbf{z}_i\|^2$$

In simple cases, closed form solution is available, otherwise minimize iteratively.

Aligning two shapes

$$\mathbf{x}^{(1)} = (x_1^{(1)}, y_1^{(1)}, x_2^{(1)}, y_2^{(1)}, \ldots, x_N^{(1)}, y_N^{(1)})^T$$
$$\mathbf{x}^{(2)} = (x_1^{(2)}, y_1^{(2)}, x_2^{(2)}, y_2^{(2)}, \ldots, x_N^{(2)}, y_N^{(2)})^T$$

Find a transformation (rotation, translation, scaling) of $\mathbf{x}^{(2)}$

$$\mathcal{T}(\mathbf{x}^{(2)}) = s\,R \begin{bmatrix} x_i^{(2)} \\ y_i^{(2)} \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} = \begin{bmatrix} x_i^{(2)} s\cos\theta - y_i^{(2)} s\sin\theta \\ x_i^{(2)} s\sin\theta + y_i^{(2)} s\cos\theta \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

such that a sum of squared distances is minimized

$$E = \sum_{i=1}^{M} w_i \left\| s \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_i^{(2)} \\ y_i^{(2)} \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} - \begin{bmatrix} x_i^{(1)} \\ y_i^{(1)} \end{bmatrix} \right\|^2$$

Aligning two shapes

$$E = \sum_{i=1}^{M} w_i \left\| s \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x_i^{(2)} \\ y_i^{(2)} \end{bmatrix} + \begin{bmatrix} t_x \\ t_y \end{bmatrix} - \begin{bmatrix} x_i^{(1)} \\ y_i^{(1)} \end{bmatrix} \right\|^2$$

Minimize $E(\theta, s, t_x, t_y)$ as $\min_\theta \min_{s,t_x,t_y} E_\theta(s, t_x, t_y)$

- **Inner minimization wrt** s, t_x, t_y

$$\frac{\partial E}{\partial t_x} = 0\,, \qquad \frac{\partial E}{\partial t_y} = 0\,, \qquad \frac{\partial E}{\partial s} = 0\,,$$

Aligning two shapes

- **Inner minimization wrt** s, t_x, t_y

$$\frac{\partial E}{\partial t_x} = 0\,, \qquad \frac{\partial E}{\partial t_y} = 0\,, \qquad \frac{\partial E}{\partial s} = 0\,,$$

leads to linear equations:

$$s \textstyle\sum_{i=1}^{M} w_i\, q(y_i, -x_i, \theta) - N\, t_x = -\textstyle\sum_{i=1}^{M} w_i\, x_i'$$
$$s \textstyle\sum_{i=1}^{M} w_i\, q(-x_i, -y_i, \theta) - N\, t_y = -\textstyle\sum_{i=1}^{M} w_i\, y_i'$$

$$s \textstyle\sum_{i=1}^{M} w_i^2 \Big(q^2(y_i, -x_i, \theta) + q^2(x_i, y_i, \theta)\Big) - t_x \textstyle\sum_{i=1}^{M} w_i\, q(y_i, -x_i, \theta)$$
$$- t_y \textstyle\sum_{i=1}^{M} w_i\, q(-x_i, -y_i, \theta)$$
$$= -\sum_{i=1}^{M} w_i\, x_i'\, q(y_i, -x_i, \theta) + \sum_{i=1}^{M} w_i\, y_i'\, q(x_i, -y_i, \theta)$$

where $q(a, b, \theta) = a \sin\theta + b\cos\theta$.

Aligning two shapes

- **Inner minimization wrt** s, t_x, t_y
- **Outer minimization wrt** θ
  One dimensional functional minimization, e.g. Brent's routine or golden section search. (Alternatively, Horn's absolute orientation method can be used.)

Automatic landmark registration

- **Feature point detection**
  - Harris corner detector (smoothed derivatives $\rightarrow$ local structure matrix $\rightarrow$ eigenvalues $\rightarrow$ corner response function)
- **Feature point matching**
  - Template correlation
  - Invariant descriptors, e.g. **S**cale **I**nvariant **F**eature **T**ransform (gradient direction $\rightarrow$ rotation invariance, scale-space image $\rightarrow$ scale invariance, 4×4 $8-$bin histograms of orientations and magnitudes in a neighborhood, normalization $\rightarrow$ 128-element descriptor)
- **Outlier pruning**
  - **RAN**dom **S**ampling **A**nd **C**onsensus
- **Landmark fitting** (interpolation)

Medical applications of landmark registration

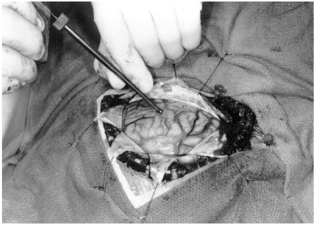
Overview

- Point- and surface based registration
  - Image-to-image registration
  - Image-to-physical space registration
- Image guided surgery (IGS) applications
  - Pre-operative imaging
    - Can be slow
  - Intra-operative imaging
    - Must be fast
  - Tracking of surgical instruments and alignment of pre/intra-operative data with physical space of patient lying on operating table
    - Must be near real-time and very accurate

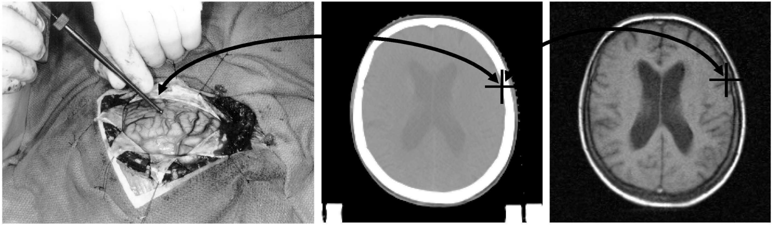
Why Image-Guided Surgery?

Amputation at St. Thomas' Hospital 1775
(source: The Old Operating Theatre Museum, St. Thomas' Street, London SE1 9RT)

Why Image-Guided Surgery?



Why Image-Guided Surgery?



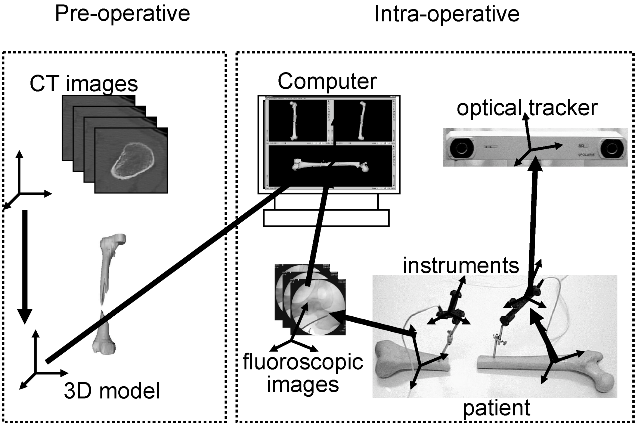
Registration of pre-operative CT and MRI to intra-operative scene

Why Image-Guided Surgery?

Why Image-Guided Surgery?

Augmented Reality

Registration Chain for IGS



Generic Feature-Based Registration Procedure

1. Distortion correction and camera calibration for each modality

while *dissimilarity* > 0 **and** *improvement* **do**

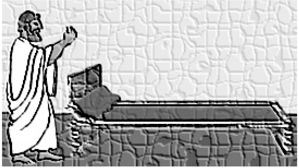
2. Feature extraction
3. Feature pairing
4. Similarity formulation and outlier removal
5. Dissimilarity reduction (optimization)

Great differences in each step
depending on images and task!

Point-Based Registration

- Orthogonal Procrustes problem:
  - Named after a robber in Greek mythology, who would offer travellers the opportunity to stay the night in a perfectly fitting bed.
  - Unfortunately, it was the guest who was altered to fit the bed, rather than the bed to fit the guest!
  - Short visitors were stretched to fit, and tall visitors had parts of their body cut off so that they would fit, with invariably fatal results.

The hero Theseus stopped this unpleasant practice by subjecting Procrustes to his own method.



Point-Based Registration

- Orthogonal Procrustes problem:
  - “Procrustes” became the criticism for the practice of unjustifiably forcing data to look like they fit another set.
  - Now Procrustes statistics has lost is negative associations and is used in shape analysis.
  - The Procrustes problem is an optimal fitting problem of least square type.

Point-based registration

- Optimal fitting problem of least square type:
  - Closed-form solution for rigid case. Non-rigid registration requires iterative solution
  - Given two sets of *N* corresponding points $\{\mathbf{x}_i\}$ and $\{\mathbf{y}_i\}$, find the rigid-body transformation (rotation matrix **R** and translation vector **t**) that minimizes the mean squared distance between the points:

$$\frac{1}{N}\sum_{i=1}^{N}|R\mathbf{x}_i + \mathbf{t} - \mathbf{y}_i|^2$$

Point-based registration

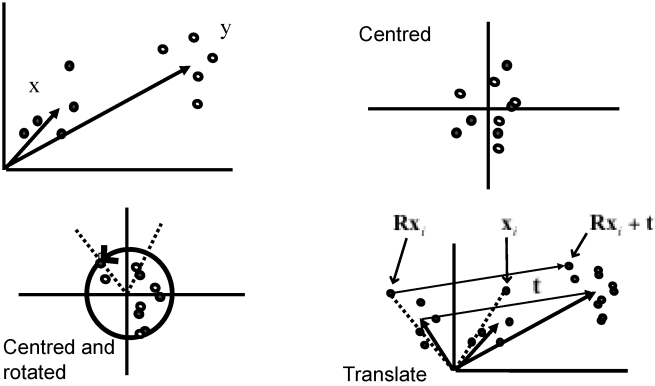
- Centre points:
  - Compute mean: $\bar{\mathbf{x}} = \sum_{i=1}^{N} \mathbf{x}_i \qquad \bar{\mathbf{y}} = \sum_{i=1}^{N} \mathbf{y}_i$
  - Centre wrt mean: $\tilde{\mathbf{x}}_i = \mathbf{x}_i - \bar{\mathbf{x}} \qquad \tilde{\mathbf{y}}_i = \mathbf{y}_i - \bar{\mathbf{y}}$
- Determine rotation matrix R via singular value decomposition (SVD) of correlation matrix H:

$$\mathbf{H} = \sum_{i=1}^{N} \tilde{\mathbf{x}}_i \tilde{\mathbf{y}}_i^T \qquad \Lambda = diag(\lambda_1, \lambda_2, \lambda_3), \lambda_1 \geq \lambda_2 \geq \lambda_3$$

$$\mathbf{H} = \mathbf{U}\Lambda\mathbf{V}^T, \mathbf{U}^T\mathbf{U} = \mathbf{V}^T\mathbf{V} = \mathbf{I} \qquad \mathbf{R} = \mathbf{V}\mathbf{D}\mathbf{U}^T, \mathbf{D} = diag(1,1,\det(\mathbf{V}\mathbf{U}^T))$$

- Determine translation vector: $\mathbf{t} = \bar{\mathbf{y}} - \mathbf{R}\bar{\mathbf{x}}$

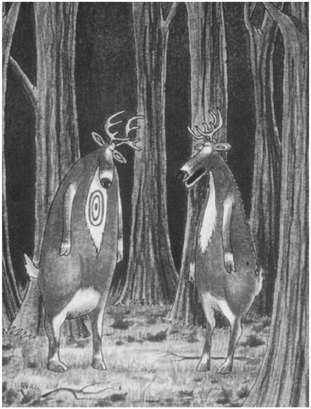
Point-based registration



Points for Registration

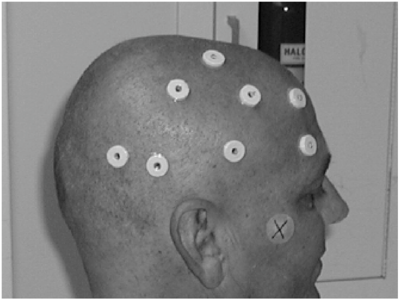
- Anatomic landmarks
- Skin-affixed markers
- Bone-implanted markers
- Advantages of markers
  - Fiducial is independent of anatomy
  - Automatic algorithms for locating fiducial markers can take advantage of marker's shape and size in order to accurately and robustly compute the fiducial point

“Bummer of a birthmark, Hal!”



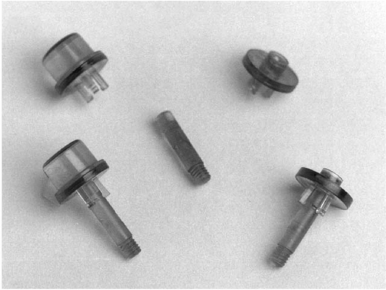
Skin-Affixed Markers

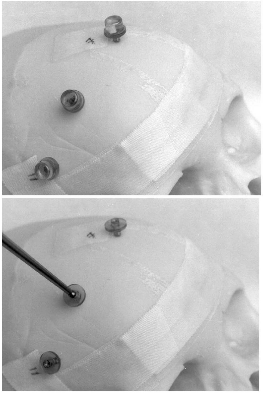
- Advantage: non-invasive
- Disadvantage: can move due to mobility of skin



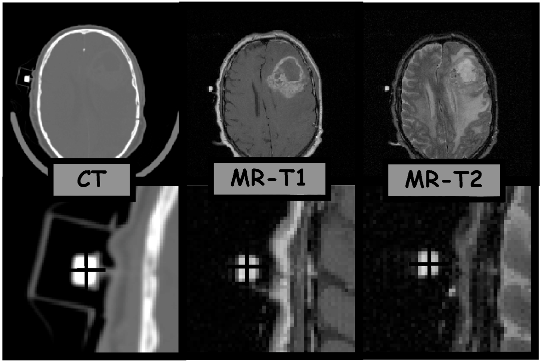
Bone-Implanted Markers

- Advantage: cannot move
- Disadvantage: invasive



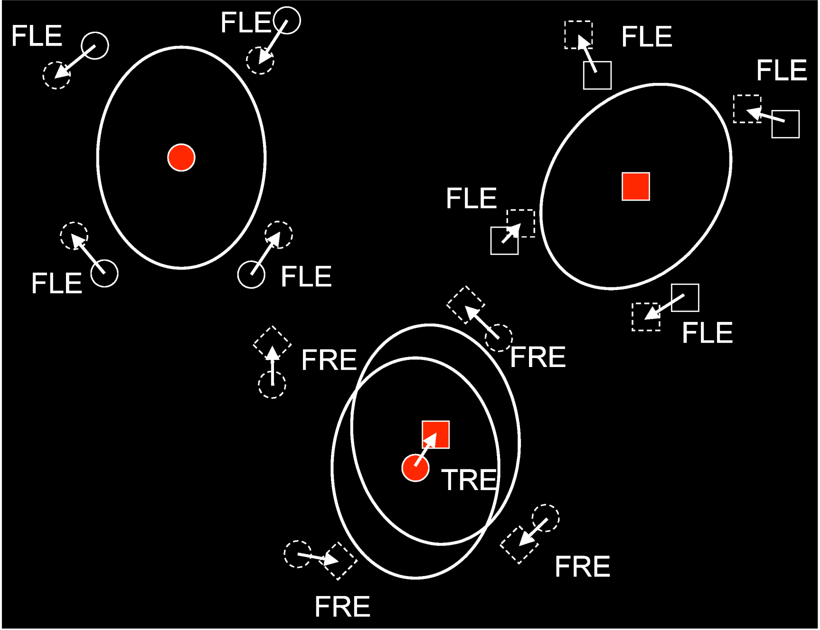


Bone-Implanted Markers



Registration Error Measures

- Fiducial localization error (FLE)
  - Error of localizing fiducials
- Fiducial registration error (FRE)
  - Distance between points (fiducials) used for registration
  - Minimum value of cost function
- Target registration error (TRE)
  - Distance between points (targets) not used for registration
  - This is the clinically relevant error



FLE
FLE
FLE
FLE
FLE
FLE
FLE
FLE
FRE
FRE
TRE
FRE
FRE

Point-Based Registration Error: Theory

- Expected value of FRE and TRE has been estimated using perturbation theory
- FRE = f(FLE, N)
- TRE = f(FLE, N, shape of fiducial configuration, target position)

Point-Based Registration Error: Theory

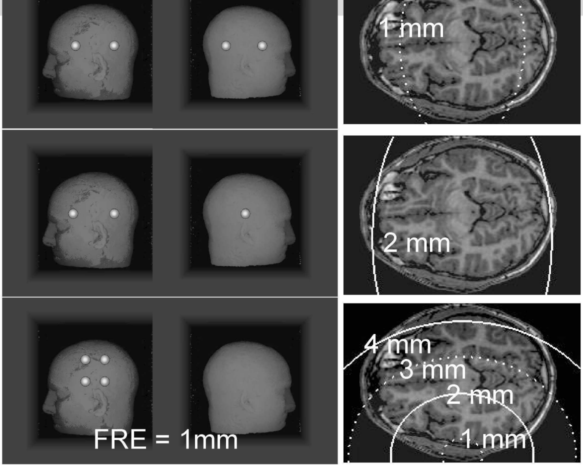
$$\left\langle \mathrm{FRE}^2 \right\rangle = \frac{N-2}{N} \left\langle \mathrm{FLE}^2 \right\rangle$$

$$\left\langle \mathrm{TRE}^2(\mathbf{r}) \right\rangle = \frac{\left\langle \mathrm{FLE}^2 \right\rangle}{N} \left\{ 1 + \frac{1}{3} \sum_{k=1}^{3} \frac{d_k^2}{f_k^2} \right\}$$

Characteristics of TRE

- TRE proportional to FLE
- TRE inversely proportional to $N^{1/2}$
- TRE depends on target position **r**
- TRE has its minimum value at fiducial configuration centroid (translational component)
- TRE increases as the distance of the target point from the principal axes increases
- Iso-error TRE contours are ellipsoidal

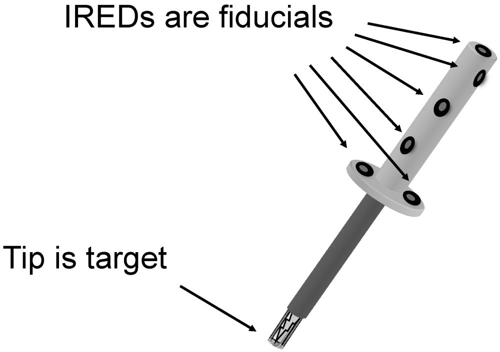
Marker Placement



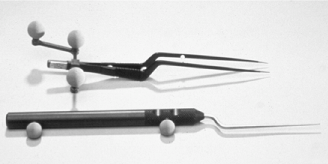
Guidelines for Fiducial Marker Placement in Cranial IGS

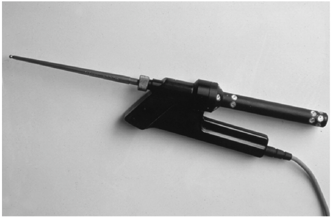
- Use as many markers as feasible
- Place markers so that the centroid of their configuration is near the regions that are most critical during surgery
- Distribute the markers as far apart as is possible
- Avoid linear, or almost linear, fiducial configurations
- Avoid placing markers on mobile areas of the scalp

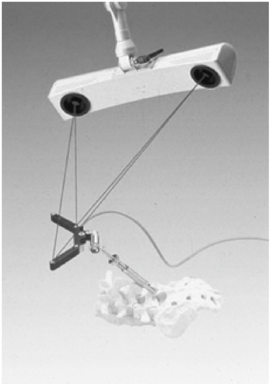
Design of Optically Tracked Probes



Optical Tracking of Instruments: Point-based Registration







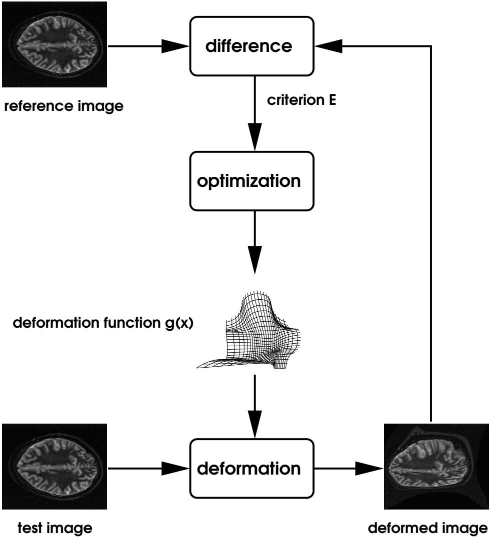
Automatic rigid registration

- Look for rigid (euclidean or affine) transformation
- To compensate different position, scale
- ...or to simplify a more complicated problem

Registration based on voxel similarity

- Registration based on geometrical features requires
  - the extraction of points, lines or surfaces
  - registration error is affected by localization errors during the feature extraction stage
- Registration based on voxel similarity measures
  - uses some measure derived from the intensity of the voxels directly
  - assumes that there is a relationship between the image intensities of both images if the images are registered
  - does not require any feature extraction, thus registration error is not affected by any localization errors

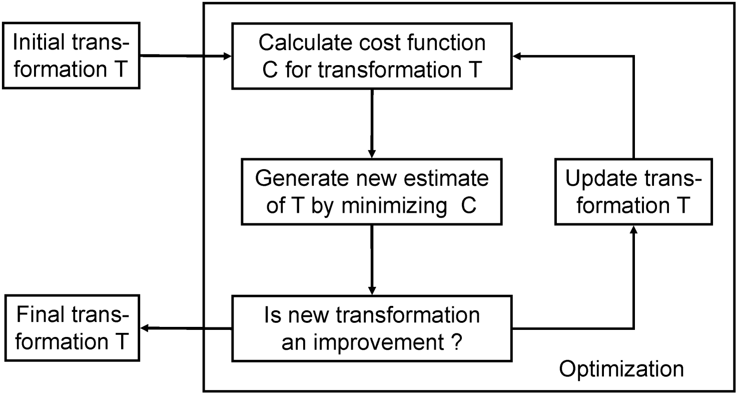
Registration as minimization



Registration based on voxel similarity

- Optimal transformation $\mathbf{T}$ is determined iteratively by maximizing a voxel-based similarity measure C.
- Voxel-similarity measure C is a function of
  - image A (target or reference image)
  - image B (source image)
  - transformation $\mathbf{T}$

Registration algorithm



Registration based on voxel similarity

- Registration based on geometric features is independent of the modalities from which the features have been derived
- Registration based on voxel similarity measures features must make a distinction between
  - monomodality registration:
    - CT-CT, MR-MR, PET-PET, etc
  - multimodality registration
    - MR-CT, MR-PET, CT-PET, etc

Mono-modal image registration

- Sums of Squared Differences (SSD)

$$C = \frac{1}{N}\sum_{i}(I_A(\mathbf{p}_i) - I_B(\mathbf{T}(\mathbf{p}_i)))^2$$

For all voxels i

  - assumes an identity relationship between image intensities in both images
  - optimal measure if the difference between both images is Gaussian noise
  - sensitive to outliers

Mono-modal image registration

- Robust statistics can be used to reduce the influence of outliers on the registration
- Sum of absolute differences (SAD)

$$C = \frac{1}{N} \sum_i \left| I_A(\mathbf{p}_i) - I_B(\mathbf{T}(\mathbf{p}_i)) \right|$$

  – assumes an identity relationship between image intensities
  – less sensitive to outliers

Mono-modal image registration

- Correlation

$$C = \frac{1}{N} \sum_{i} I_A(\mathbf{p}_i) \cdot I_B(\mathbf{T}(\mathbf{p}_i))$$

  - assumes a linear relationship between image intensities
  - sensitive to large intensity values
  - useful if images have been acquired with different intensity windowing

Mono-modal image registration

- Normalized Cross Correlation (CC)

$$C = \frac{\sum \left(I_A(\mathbf{p}) - \mu_A\right)\left(I_B(\mathbf{T}(\mathbf{p})) - \mu_B\right)}{\sqrt{\left(\sum \left(I_A(\mathbf{p}) - \mu_A\right)^2\right)\left(\sum \left(I_B(\mathbf{T}(\mathbf{p})) - \mu_B\right)^2\right)}}$$

  - μ_A average intensity in image A
  - μ_B average intensity in image B
  - assumes a linear relationship between image intensities
  - useful if images have been acquired with different intensity windowing

Mono-modality image registration

- Ratio of Image Uniformity (RIU , Woods et al. 1992)

$$R(\mathbf{p}) = \frac{I_A(\mathbf{p})}{I_B(\mathbf{T}(\mathbf{p}))} \qquad \bar{R} = \frac{1}{N}\sum R(\mathbf{p})$$

$$RIU = \frac{\sqrt{1/N \sum \left(R(\mathbf{p}) - \bar{R}\right)^2}}{\bar{R}}$$

aims to maximize the uniformity of the ratio between intensities in image A and B which is measured in form of their standard deviation

Registration basis: Image intensity

- Mono-modal image registration
  - Image intensities are related by some simple function
    - identity: Use SSD or SAD
    - linear: Use CC or RIU
- Multi-modal image registration
  - Image intensities are related by some unknown function or statistical relationship
  - Relationship between intensities is not known a-priori
  - Relationship between intensities can be viewed by inspecting a 2D histogram or co-occurrence matrix

Multi-modality image registration

- Intensity remapping (van den Elsen et al. 1993)
  - specific to MR-CT registration
  - CT intensities are remapped so that
    - soft tissue is bright
    - bone and air are dark
    - CT looks like MR
  - use correlation
- Gradient correlation (van den Elsen et al. 1993)
  - extract features like gradients, ridges or creases
  - use correlation

Multi-modality image registration

- Partitioned Image Uniformity (PIU, Woods et al. 1993)

$$PIU_B = \sum_a \frac{n_a}{N} \frac{\sigma_B(a)}{\mu_B(a)} \qquad \text{or} \qquad PIU_A = \sum_b \frac{n_b}{N} \frac{\sigma_A(b)}{\mu_A(b)}$$

- PIU_B - measures the sum of the normalized standard deviation of voxel values in image B for each intensity level in image A
- PIU_A - measures the sum of the normalized standard deviation of voxel values in image A for each intensity level in image B

Multi-modality image registration

- Partitioned Image Uniformity (PIU)

$$PIU_B = \sum_a \frac{n_a}{N} \frac{\sigma_B(a)}{\mu_B(a)}$$

N is the total number of voxels

n_a is the total number of voxels with intensity a

$$\mu_B(a) = \frac{1}{n_a} \sum_a h(a,b)$$

is the average intensity in image B corresponding to intensity *a* image A

$$\sigma_B(a) = \frac{1}{n_a} \sum_a \left(h(a,b) - \mu(a)\right)^2$$

is the standard deviation in image B corresponding to intensity *a* image A

Multi-modality image registration

- Correlation Ratio (CR, Roche et al. 1998)

$$CR_B = \frac{1}{\sigma_B^{\,2}} \sum_a \frac{n_a}{N} \sigma_B(a)$$

or

$$CR_A = \frac{1}{\sigma_A^{\,2}} \sum_b \frac{n_b}{N} \sigma_A(b)$$

$$\sigma_B = \frac{1}{N} \sum_a \sum_b \left(h(a,b) - \mu_B\right)^2$$

is the standard deviation in image B

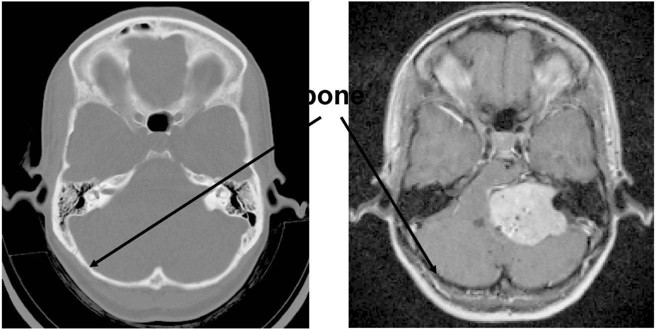
$$\sigma_B(a) = \frac{1}{n_a} \sum_a \left(h(a,b) - \mu(a)\right)^2$$

is the standard deviation in image B corresponding to intensity *a* in image A

Images as probability distributions

- Images can be viewed as probability distributions p(a)
  - marginal probability p(a) of a pixel having intensity a
  - joint probability p(a, b) of a pixel having intensity a in one image and intensity b in another image
- Probability distribution of an image can be estimated using
  - parzen windowing
  - histograms
- Histograms require “binning”
  - usually use 32 to 256 bins per image

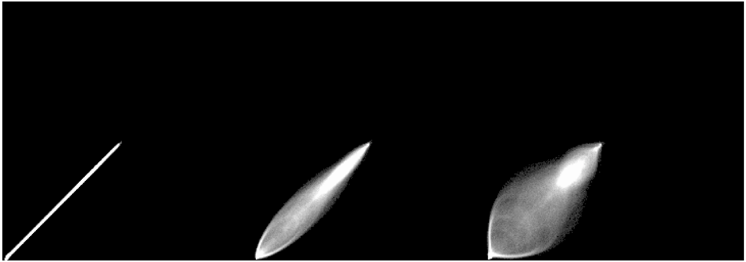
Registration based on voxel similarity: Problems



CT **MR**

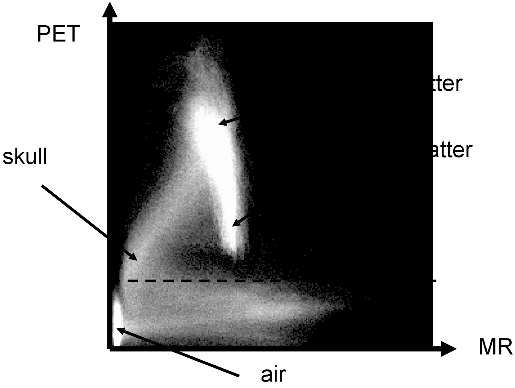
2D Histograms

MR/MR



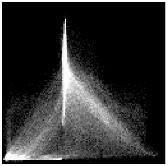
registered | misregistered by 2mm | misregistered by 5mm

2D Histograms

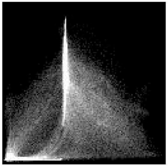


2D Histograms

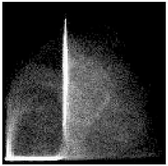
MR/CT



registered

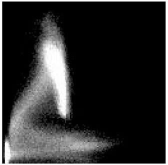


misregistered by 2mm

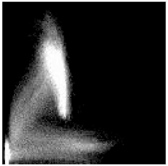


misregistered by 5mm

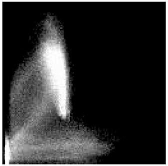
MR/PET



registered

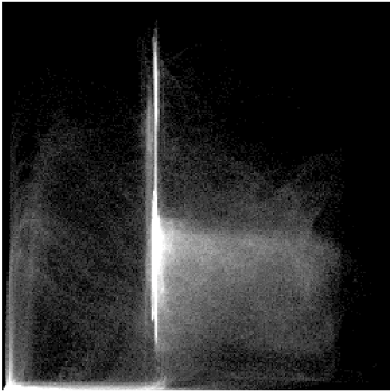


misregistered by 2mm

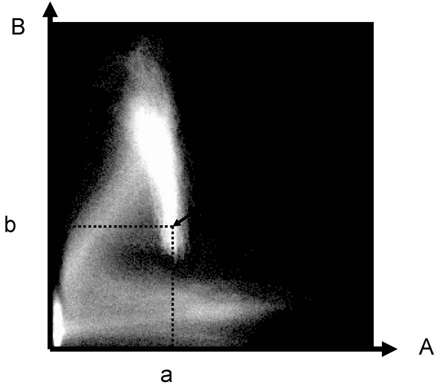


misregistered by 5mm

2D Histograms



2D Histograms



Images as probability distributions

- Frequency of corresponding intensity pairs can be interpreted in terms of probabilities

$$p(a,b) = \frac{h(a,b)}{N}$$

is the joint probability of a voxel having greyvalue a in the first image and greyvalue b in the second image.

$$p(a) = \sum_b p(a,b)$$

is the marginal probability of a voxel in the first image having greyvalue a

$$p(b) = \sum_a p(a,b)$$

is the marginal probability of a voxel in the second image having greyvalue b

Voxel similarity based on information theory

- Entropy (Shannon-Wiener)

$$H(A) = -\sum_{a} p(a) \log p(a)$$

describes the amount of information in image A.

  - The information content of an image is maximal (in the information theoretic sense) if all intensities have equal probability.
  - The information content of an image is minimal (in the information theoretic sense) if one intensity a has a probability of one, i.e. $p(a) = 1$.

Voxel similarity based on information theory

- Joint Entropy (Hill et al., 1994)

$$H(A,B) = -\sum_a \sum_b p(a,b) \log p(a,b)$$

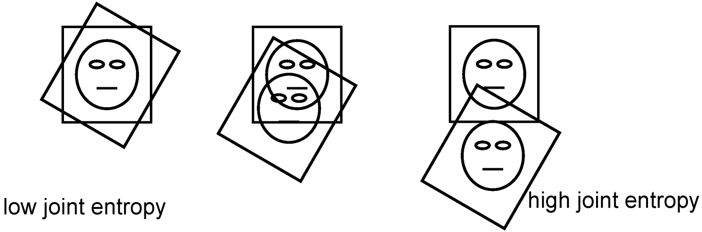
  describes the amount of information in the combined images A and B.
  - If A and B are totally unrelated, the joint entropy will be the sum of the entropies of A and B
  - If A and B are related, the joint entropy will be smaller, i.e.

$$H(A,B) \leq H(A) + H(B)$$

- Registration can be achieved my minimizing the joint entropy between both images

Voxel similarity based on information theory

- Interpretation of Joint Entropy



- Joint Entropy is highly sensitive to the overlap of the two images

Voxel similarity based on information theory

- Mutual Information (Viola et al., 1995 and Collignon et al., 1995)

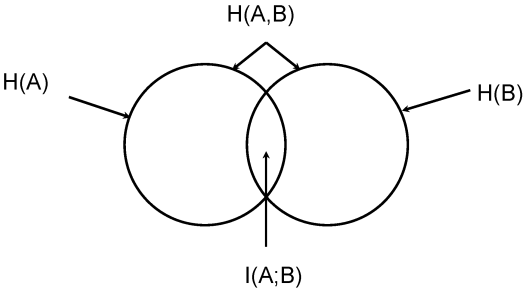
$$I(A,B) = H(A) + H(B) - H(A,B)$$

  describes how well one image can be explained by another image.
- Mutual Information can be expressed in terms of marginal and joint probability distributions:

$$I(A,B) = -\sum_a \sum_b p(a,b) \log \frac{p(a,b)}{p(a)p(b)}$$

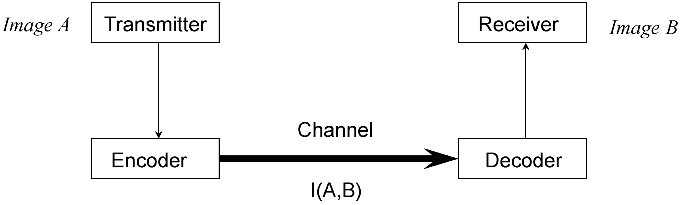
Voxel similarity based on information theory

- Venn diagram:



Voxel similarity based on information theory

- Communication model:



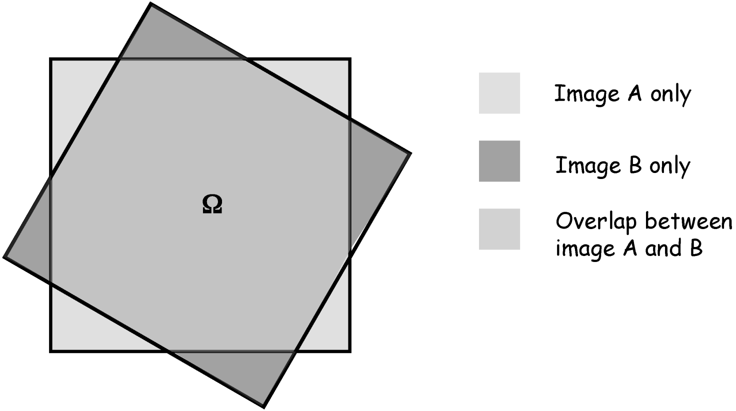
Voxel similarity based on information theory

- Mutual Information is still sensitive to the overlap of the two images
- Normalized Mutual Information (Studholme et al, 1999)

$$I(A,B) = \frac{H(A) + H(B)}{H(A,B)}$$

  can be shown to be independent of the amount of overlap between images.
- Registration can be achieved by maximizing (Normalized) Mutual Information between both images

Evaluation of voxel-similarity measures

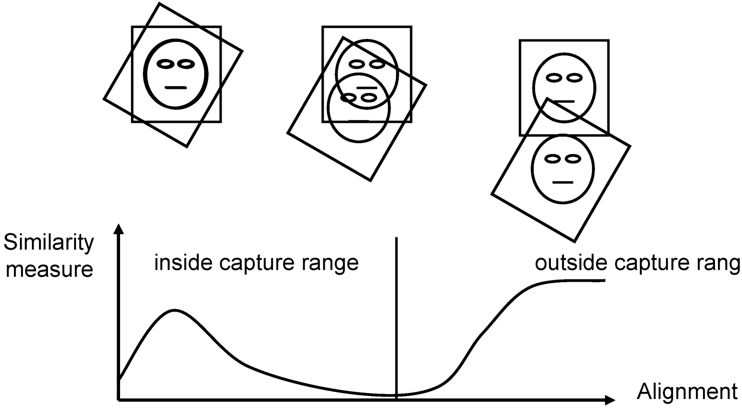


Optimization of voxel-similarity measures

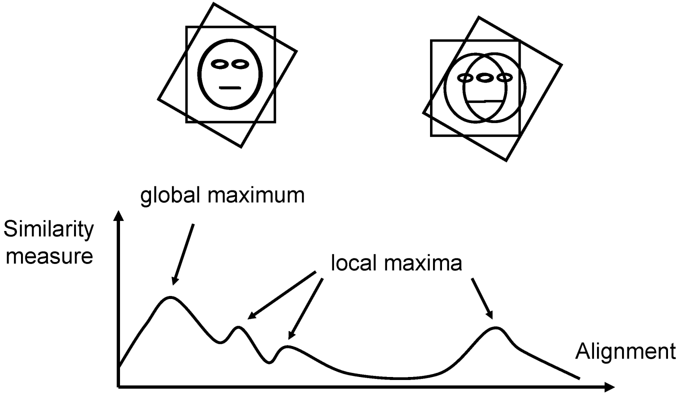
- Optimization of voxel-similarity measures normally requires iterative techniques, i.e.
  - downhill descent
  - gradient descent
  - see Numerical Recipes for a description of various optimization schemes
- Global optimization schemes are not feasible for image registration
- Local optimization schemes are much more efficient but will get trapped in local optima

$\Rightarrow$ Registration has a limited capture range

Optimization of voxel-similarity measures

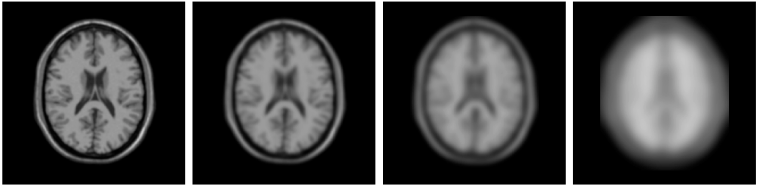


Optimisation of voxel-similarity measures



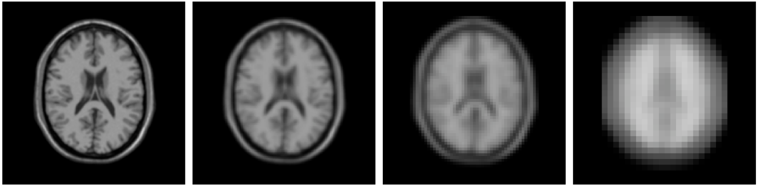
Multi-resolution optimization

- Capture range can be increased by using multi-scale techniques:

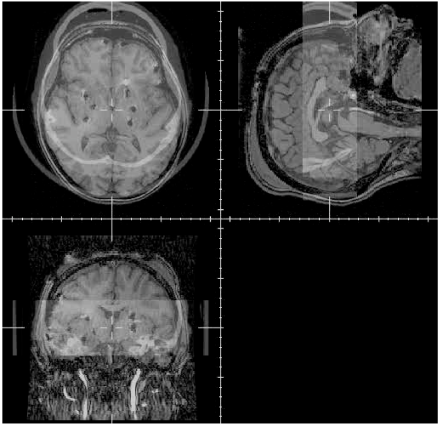


Multi-resolution optimization

- Registration can be accelerated by using multi-resolution techniques:



Optimization of voxel-similarity measures

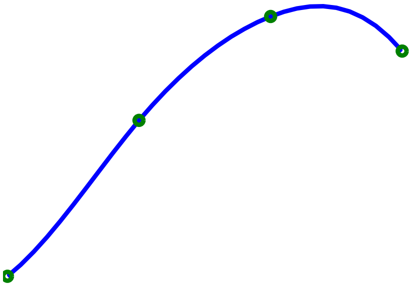


ITK (Insight Registration and segmentation toolbox)

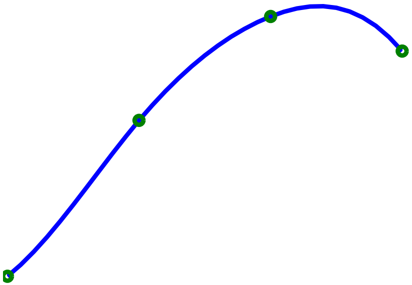
Automatic elastic B-spline registration

- Look for elastic (non-linear) transformation
- Smooth deformation wanted
- Semi-local model with many parameters

(Uniform) splines



(Uniform) splines



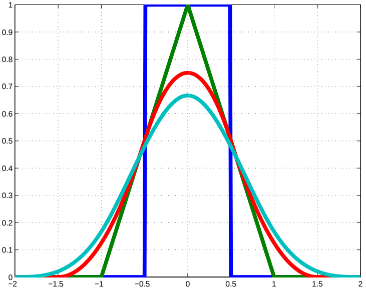
- Piecewise polynomial of degree n
- Continuous $(n-1)^{\text{th}}$ derivative
- (Uniform) knots

Non-uniform splines (1D)

- Polynomial in each interval
- Continuous derivatives
- Boundary conditions (natural)
- $\longrightarrow$ band system of linear equations
- Example: cubic splines

Uniform B-splines

Haar	β_0
linear	β_1
quadratic	β_2
cubic	β_3



Uniform B-splines

Haar	β_0
linear	β_1
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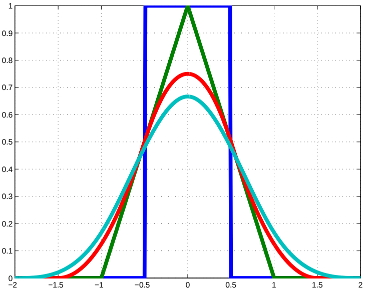


- Generation: $\beta_{n+1} = \beta_n * \beta_0$
- Basis for splines: $s(x) = \sum_i c_i \, \beta(x - i)$

Practical B-splines

- Separability ➢ speed
- B-spline transform (finding coefficients) fast through IIR filtering
- Interpolation fast (small support)
- Extension to n-D by Cartesian product. Separability.

Software

Splines Matlab, Numerical Recipes, ...

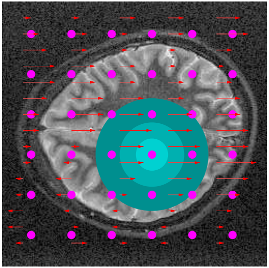
B-splines Unser, Thevenaz, `bigwww.epfl.ch`

B-spline image interpolation

$$f(x, y) = \sum_{i,j} c_{ij} \beta_n(x - i) \beta_n(x - j)$$

- B-spline interpolation is global, while P0,P1,P2,P3... are local
- Pixels on a regular grid ➢ B-spline coefficients can be precalculated (IIR filtr)
- Evaluation β_3 as fast as for P3, better quality.
- Boundary conditions (zero, periodic, mirror)
- Higher orders lead to Gibbs artifacts (ringing).

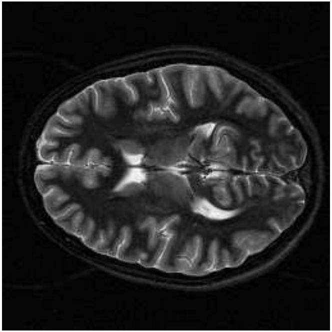
Spline based warping

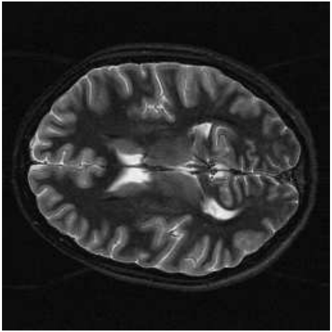


- Approximation properties ➢ precision
- Short support ➢ speed
- Scalability
- Representability of linear transforms

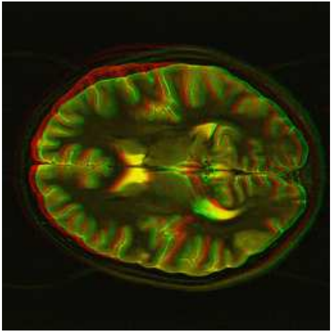
$$\mathbf{g}(\mathbf{x}) = \mathbf{x} + \sum_{\mathbf{i}\in\mathbb{Z}^2} \mathbf{c}(\mathbf{i})\,\beta(\mathbf{x}/\mathbf{h} + \mathbf{d} - \mathbf{i})$$

Evaluating the difference

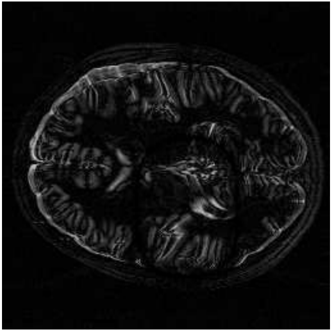




Evaluating the difference



Evaluating the difference



$$E = (1/N)\sum_{\mathbf{i}}\big(f_t^c(\mathbf{g}(\mathbf{i})) - f_r(\mathbf{i})\big)^2$$

Multiresolution

32×32

Multiresolution

64×64

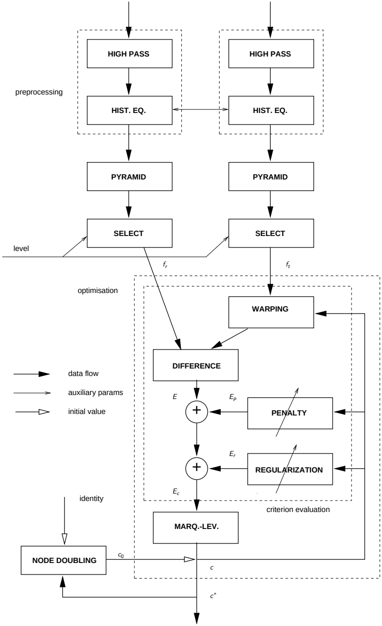
Multiresolution

128×128

Multiresolution

256×256

Multiresolution algorithm

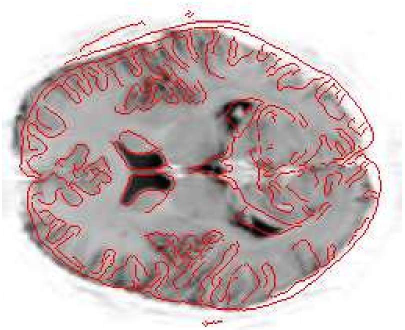


video of the registration

```
xanim -Ss2
```

Applications

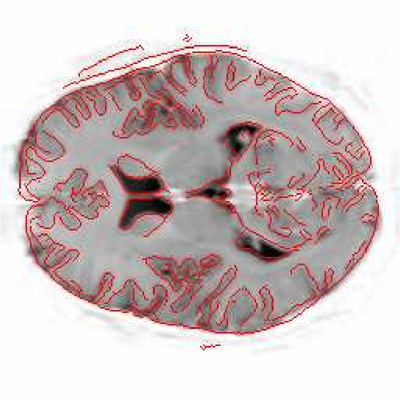
- EPI distortion



Before (with Arto Nirkko)

Applications

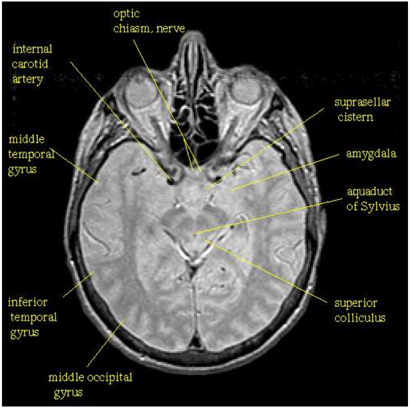
- EPI distortion



After

Applications

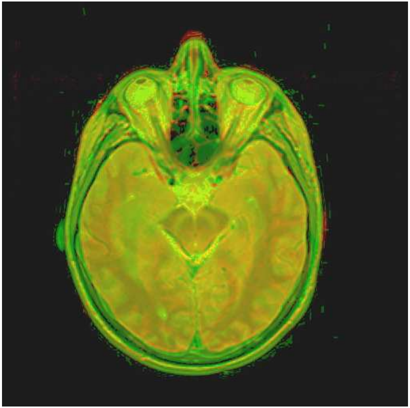
- EPI distortion
- MRI atlas



Atlas

Applications

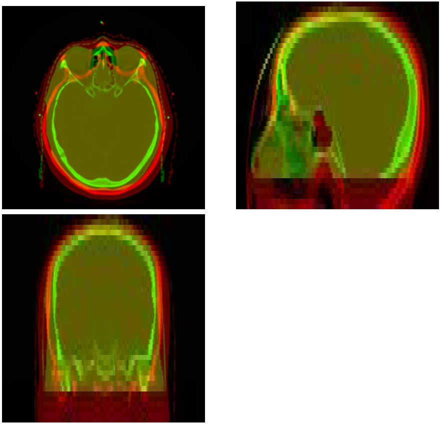
- EPI distortion
- MRI atlas



Aligned

Applications

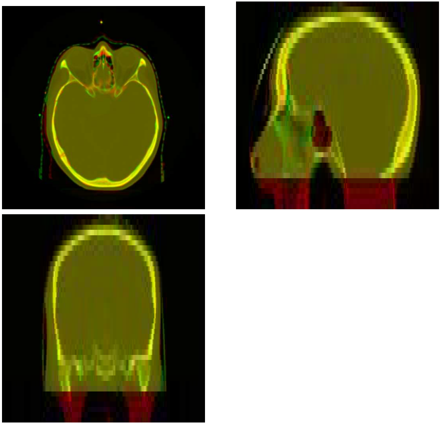
- EPI distortion
- MRI atlas
- CT alignment



Before

Applications

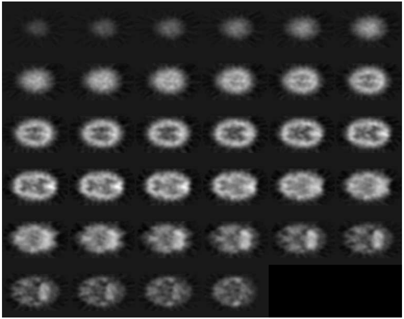
- EPI distortion
- MRI atlas
- CT alignment



After

Applications

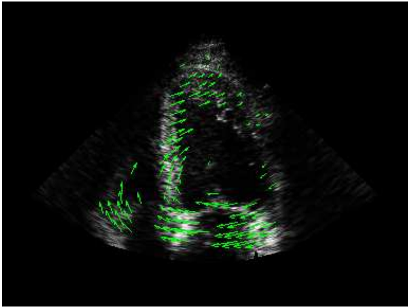
- EPI distortion
- MRI atlas
- CT alignment
- SPECT atlas



(with University Hospital in Geneva)

Applications

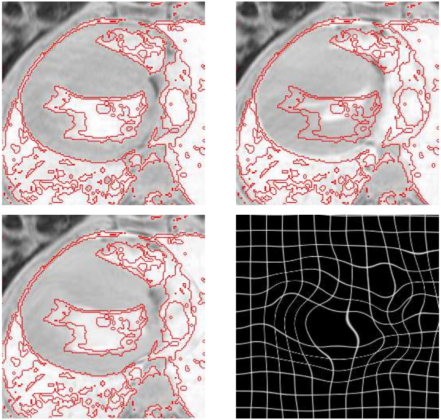
- EPI distortion
- MRI atlas
- CT alignment
- SPECT atlas
- Ultrasound



velocity (with Mara J. Ledesma-Carbayo)

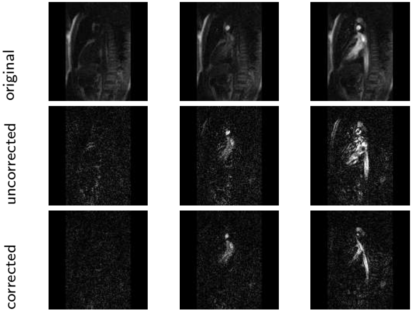
Applications

- EPI distortion
- MRI atlas
- CT alignment
- SPECT atlas
- Ultrasound
- MRI heart sequence



Applications

- EPI distortion
- MRI atlas
- CT alignment
- SPECT atlas
- Ultrasound
- MRI heart sequence
- MRI perfusion



Validation of registration

- Before an image registration can enter clinical practice, it must undergo thorough validation
  - Technical validation
    - Speed, robustness, accuracy, reliability,…
  - Clinical validation
    - Usefulness, improved clinical diagnosis and patient management in health care
  - FDA approval, incorporation into commercial system
    - Liability

Validation of rigid registration

- Robustness:
  - Measurement precision
- Consistency:
  - Circular (invertible) transformations
- Visual assessment:
  - subtraction images, overlays, landmarks
- Gold standard:
  - Implanted/attached markers, landmarks
- Simulation of a ground truth:
  - misregistration followed by motion recovery

Robustness

- Robustness can be measured as the discrepancy of registration transformations if different starting estimates are given or the images are perturbed:
  - Misregistration of an image by a known amount, e.g. by upto +-30mm or +- 30 degrees
  - Adding noise, inhomogeneity
- Deviation from identity transformation which forms the ground truth
  - Individual deviations of DOFs (tx,ty,tz,rx,ry,rz,…)
  - Target registration error (TRE) at landmarks or within volume overlap

Consistency

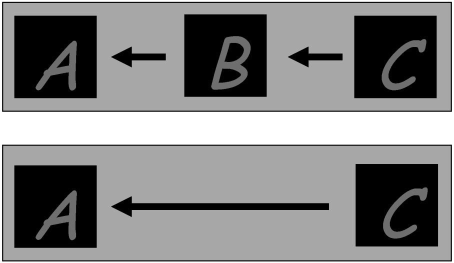
- Involves registration of an image triple, A to B, B to C, and C to A
- Ideally, the resulting transformations form a circular registration:

$$\mathbf{T_{AB}\ T_{BC}\ T_{CA}\ (x) = I(x) = x}$$

- This can be tested as for robustness via individual transformation parameters or via TRE
- Consistency is not a measure for registration accuracy!

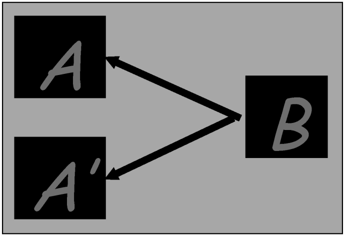
Consistency 1

- Can individual registrations be composed ?



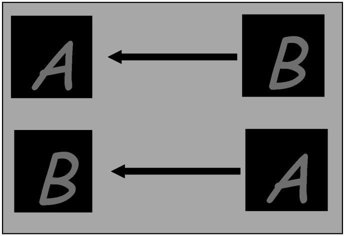
Consistency 2

- Scan-Rescan Consistency



Consistency 3

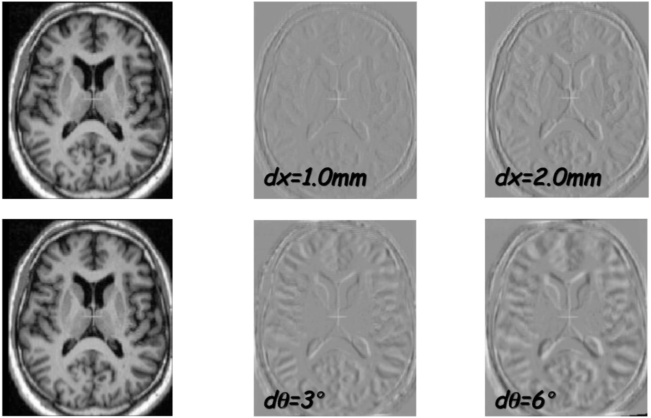
- Forward-reverse registration consistency.



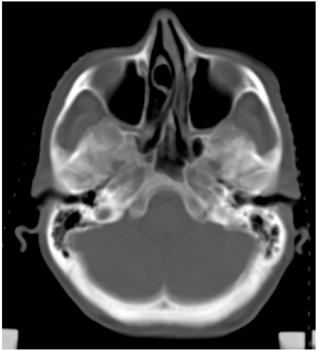
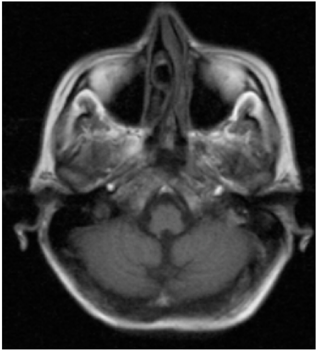
Visual assessment

- View image pair before and after registration
  - Subtraction mode (single-modal images)
  - Overlay mode
    - Iso-contour, colour
    - Chess-board, morphing
    - Horizontal or vertical “shutters”
  - At interesting anatomical positions
- Image reslicing using intensity interpolation
  - Nearest-neighbour, trilinear, b-spline, sinc
  - Blurring artefacts vs computational complexity
- Only a qualitative validation tool!

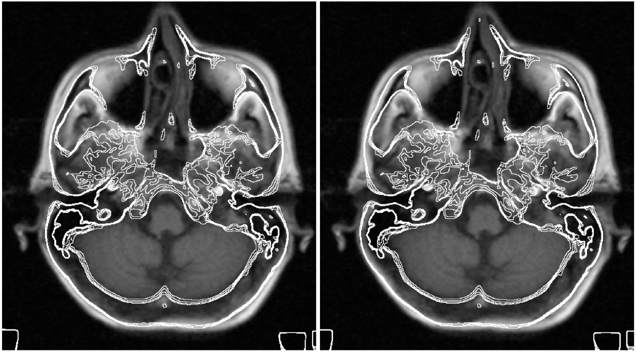
Visual Assessment : Subtraction



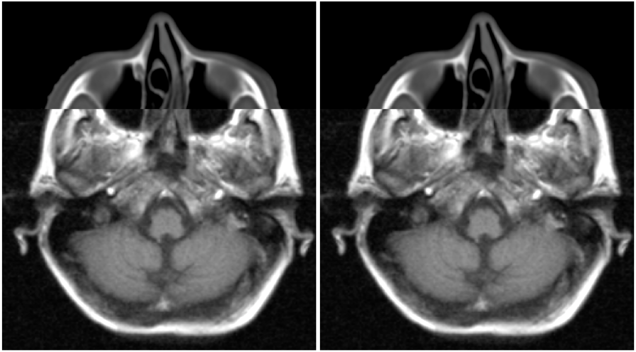
Visual Assessment

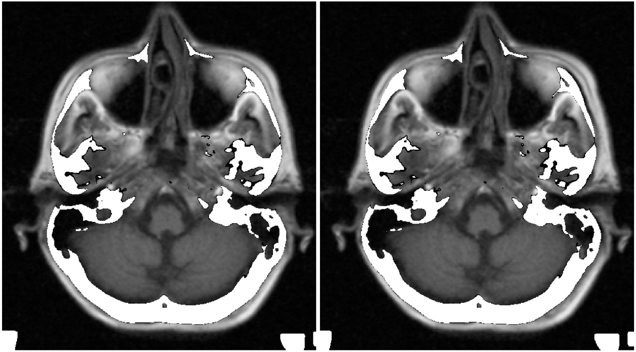
Visual Assessment (cont'd)



Visual Assessment (cont'd)



Visual Assessment (cont'd)



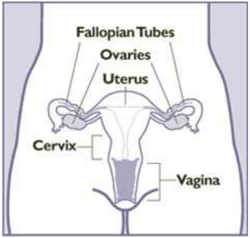
Automatic dense PDE-based registration

- Look for elastic (non-linear) transformation
- General motion (vector) field is sought
- Criteria formulated in the continuous domain
- Regularization to impose smoothness

Some facts about cervical cancer

- Cervical cancer is the second most common cancer among women worldwide
- Nearly 380,000 new cases are diagonosed yearly
- When detected early, cervical neoplasia is nearly 100% curable
- Papanicolau test (Pap Smear) and Colposcopy are the most widespread tests for cancer screening

Female Anatomy



Diagnosis: Colposcopy

- Colposcopy visually inspects inspects the cervix area at low magnification
- The application of acetic-acid will temporally alter the appearence of cancerous tissue
- Colposcopists must subjectively asses appearence changes in small areas over prolonged periods of time

60 seconds

300 seconds

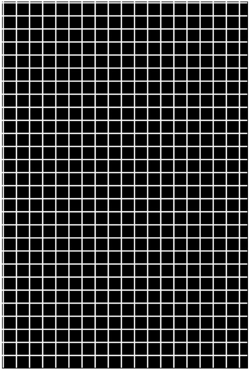
Deformation as a Vector Field

We represent correspondence function H as a dense vector field

$$H([x, y]) = [x', y']$$

Original Image

Deformation Field



Deformed Image

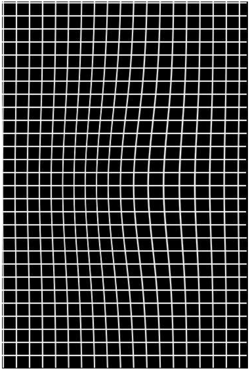
Deformation as a Vector Field

We represent correspondence function H as a dense vector field

$$H([x, y]) = [x', y']$$

Original Image

Deformation Field



Deformed Image

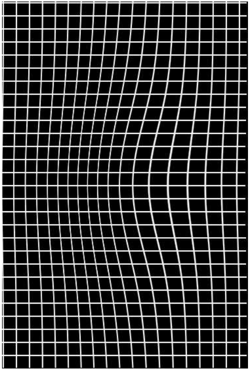
Deformation as a Vector Field

We represent correspondence function H as a dense vector field

$$H([x, y]) = [x', y']$$

Original Image1

Deformation Field



Deformed Image

Deformation as a Vector Field

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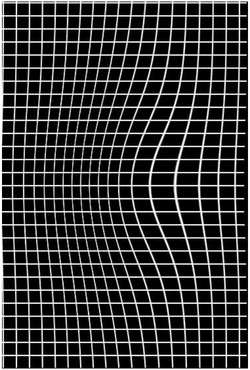
Original Image1

Deformation Field



Deformed Image

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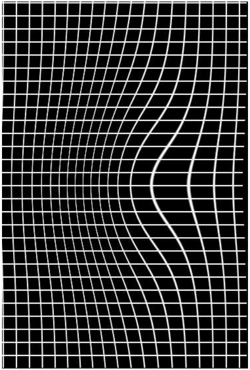
Original Image1

Deformation Field



Deformed Image

Registration as optimization

- Correspondence function H and vector field $\mathbf{h}$ are related by:

$$H([i,j]) = [i,j] + \mathbf{h}(i,j) \tag{1}$$

- The problem is then formulated as the minimization of a criterion J with respect to vector field $\mathbf{h}$:

$$\mathbf{h}^* = \arg\min_{\mathbf{h}}(J(\mathbf{f} \circ \mathbf{h}, \mathbf{g}, \mathbf{h}\ )) \tag{2}$$

  where $\mathbf{h}^*$ is the optimal solution, $\mathbf{f}$ and $\mathbf{g}$ are the images to be registered and J is a cost function measuring the dissimilarity between the images and the likelihood of the transformation.

- Cost function J is divided into a data and a regularization term multiplied by a proportionality constant:

$$J(\mathbf{f}, \mathbf{g}, \mathbf{h}) = J_D(\mathbf{f} \circ \mathbf{h}, \mathbf{g}) + \alpha J_R(\mathbf{h}) \tag{3}$$

Similarity criteria

- The data term J_D is the sum of squared differences (SSD) between the template image $\mathbf{g}$ and the moving image $\mathbf{f}$ deformed by $\mathbf{h}$:

$$J_D(\mathbf{f}\circ\mathbf{h},\mathbf{g}) = \int_{(x,y)\subset\Omega} (\mathbf{f}(\mathbf{h}(x,y)+[x,y]) - \mathbf{g}(x,y))^2 \,\mathrm{d}x\,\mathrm{d}y \quad (4)$$

Discretized version:

$$J_D(\mathbf{f}\circ\mathbf{h},\mathbf{g}) = \sum_{(i,j)\subset\Omega} (\mathbf{f}(\mathbf{h}(i,j)+[i,j]) - \mathbf{g}(i,j))^2 \quad (5)$$

Regularization

- Regularization term penalizes un-smooth deformations and makes the optimization of J a well-posed problem
- Regularization criterion J_R is chosen so its gradient coincides with the linearized 2D elasticity operator describing equilibrium in an elastic material.

$$\nabla J_R(\mathbf{h}) = \xi \Delta \mathbf{h} + (1-\xi)\nabla(\nabla \cdot \mathbf{h}) \tag{6}$$

$$J_R(\mathbf{h}) = \frac{1}{2} \int_{(x,y)\subset\Omega} \left[\xi\left(\partial_x h_x\right)^2 + (1-\xi)\left(\left(\partial_x h_x\right)^2 + \partial_x h_x \cdot \partial_y h_y\right)\right] + \left[\xi\left(\partial_y h_y\right)^2 + (1-\xi)\left(\left(\partial_y h_y\right)^2 + \partial_x h_x \cdot \partial_y h_y\right)\right]$$

Gradient descent optimization

On every iteration:

- Calculate the new deformation field

$$\mathbf{h}' = \mathbf{h}_i - \lambda(\nabla J(\mathbf{f}, \mathbf{g}, \mathbf{h}_i)) \tag{8}$$

- If the step is succesful, then the step is accepted and the step size is increased

$$\lambda \leftarrow 2\lambda, \mathbf{h}_{i+1} \leftarrow \mathbf{h}', J_{i+1} \leftarrow J' \tag{9}$$

- Otherwise the step size is reduced

$$\lambda \leftarrow \lambda/10 \tag{10}$$

- We iterate until convergence (given by a suitable threshold).

Other implementation details

- Multi-resolution was used
- ROI masks were automatically generated
- Images were rigidly pre-registred
- Green color channel only

Experiments

- Algorithm tested with 45 image pairs
- Images taken before and 60 seconds after acetic-acid application
- Cross-polarization filters used to reduce the glint
- Uncompressed 1125x750 pixel 16-bit images were used

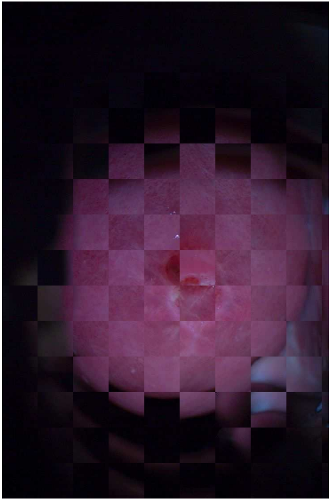
Results

Template

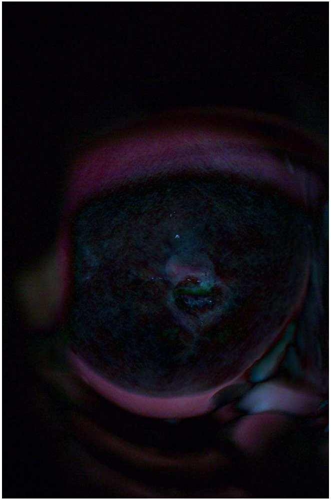
Moving

Results

Unregistered Checkerboard

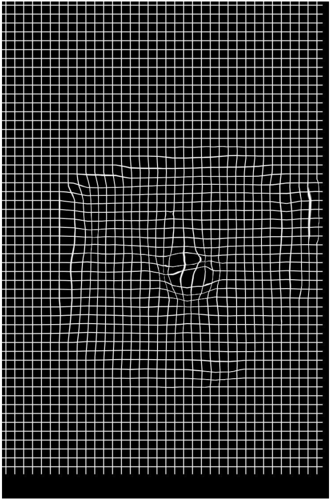


Unregistered Difference



Results

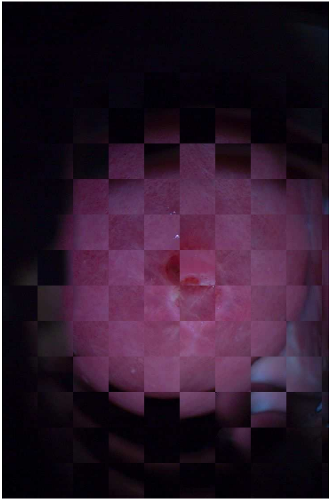
Deformation Field



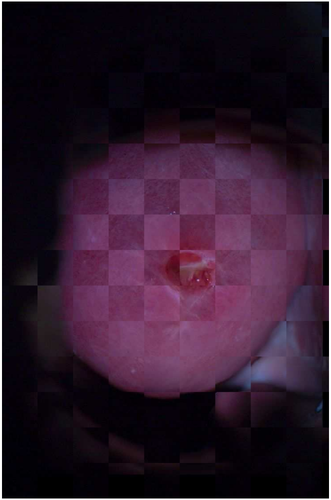
Registered

Results

Unregistered Checkerboard

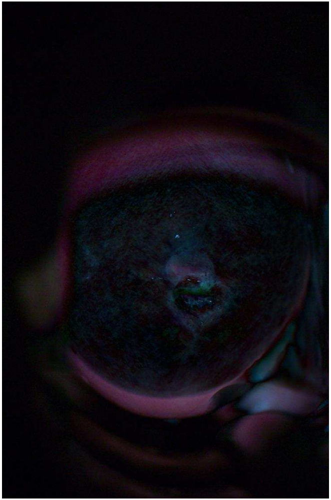


Registered Checkerboard

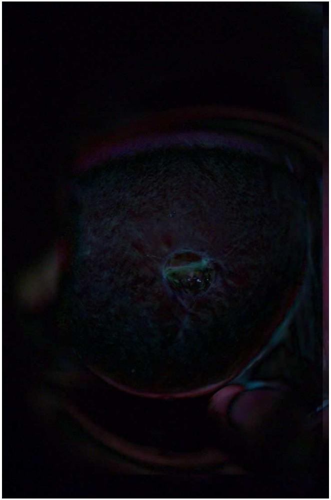


Results

Unregistered Difference

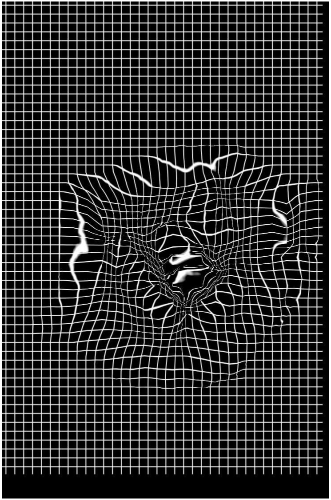


Registered Difference

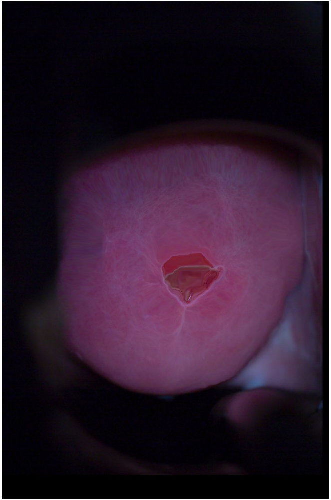


Insufficient Regularization

Deformation Field



Registered



Video cervix registration

Surface-Based Registration

- Surface-based registration:
  - Generally aligns a large number of points
  - 3D correspondence of anatomy or pathology is often not known or unavailable
  - The 3D boundary of an object is an intuitive and easily characterized geometrical feature that can be used for registration
  - Surface-based methods involve determining corresponding surfaces in different images and/or physical space and finding transformation that best aligns these surfaces

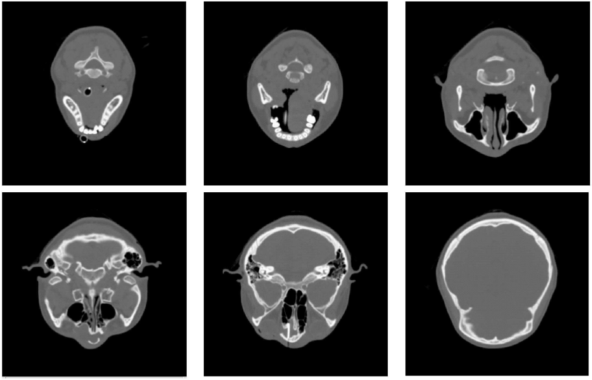
Surfaces

- Skin surface (air-tissue interface)
- Bone surface (tissue-bone interface)
- Representations
  - Point set (collection of points on the surface)
  - Faceted surface, e.g., triangle set approximating surface
  - Implicit surface
  - Parametric surface, e.g., B-spline surface

Surface Feature Extraction

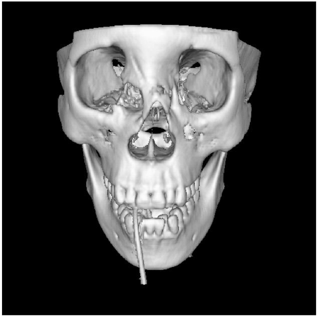
- Images
  - Isointensity contour extraction (Marching Cubes)
  - Deformable models
- Physical space
  - Laser range finders
  - Stereo video systems (photogrammetry)
  - Localizers
    - Articulated mechanical
    - Magnetic
    - Active and passive optical
  - Tracked ultrasound for bone surface

Marching Cubes: Example



Marching Cubes: Example

Skin surface



Bone surface

Surface-Based Registration

- Given a set of N surface points $\{\mathbf{x_i}\}$ and a surface $\mathbf{Y}$, find the rigid-body transformation (rotation matrix **R** and translation vector **t**) that minimizes the mean squared distance between the points and the surface:

$$d(T) = \frac{1}{N}\sum_{j=1}^{N} | T(\mathbf{x}_j) - \mathbf{y}_j |^2$$

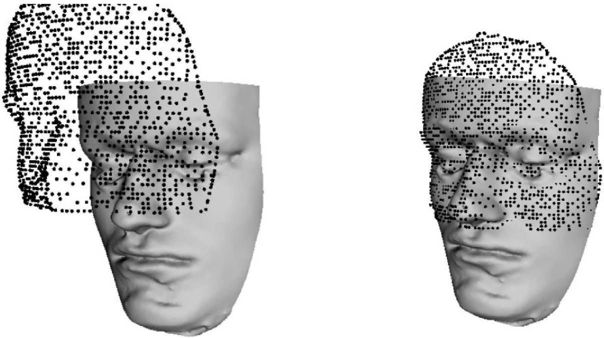
$$\mathbf{y}_j = C(T(\mathbf{x}_j), Y)$$

where y_j denotes the closest (rather than corresponding) point on Y.

Surface-Based Registration
Iterative Closest Point (ICP)

- [Besl & McKay, PAMI 1992]
- To register data shape **X** to model shape **Y**, decompose **X** into point set $\{\mathbf{x}_i\}$, then
  - Compute closest points $\{\mathbf{y}_i\}$ on ***Y***
  - Register points $\{\mathbf{x}_i\}$ to points $\{\mathbf{y}_i\}$
  - Apply resulting transformation to points $\{\mathbf{x}_i\}$
  - Repeat until convergence

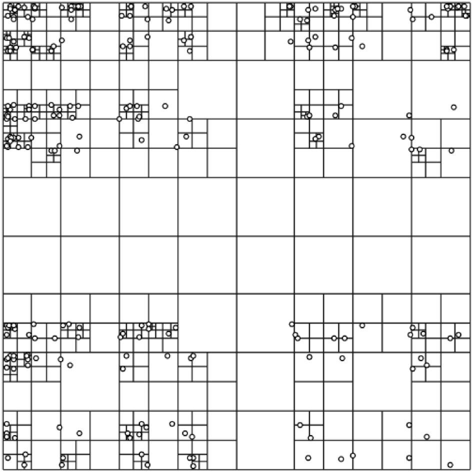
Surface-Based Registration



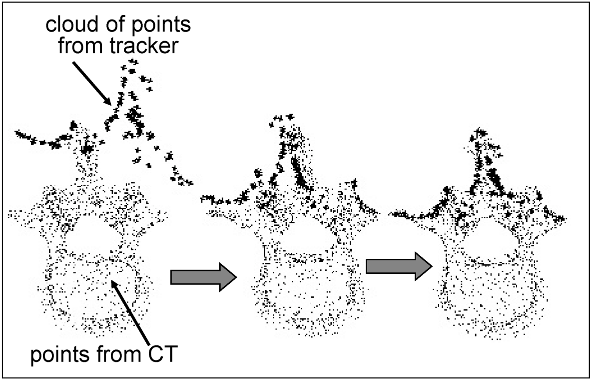
Point-to-Surface Distance

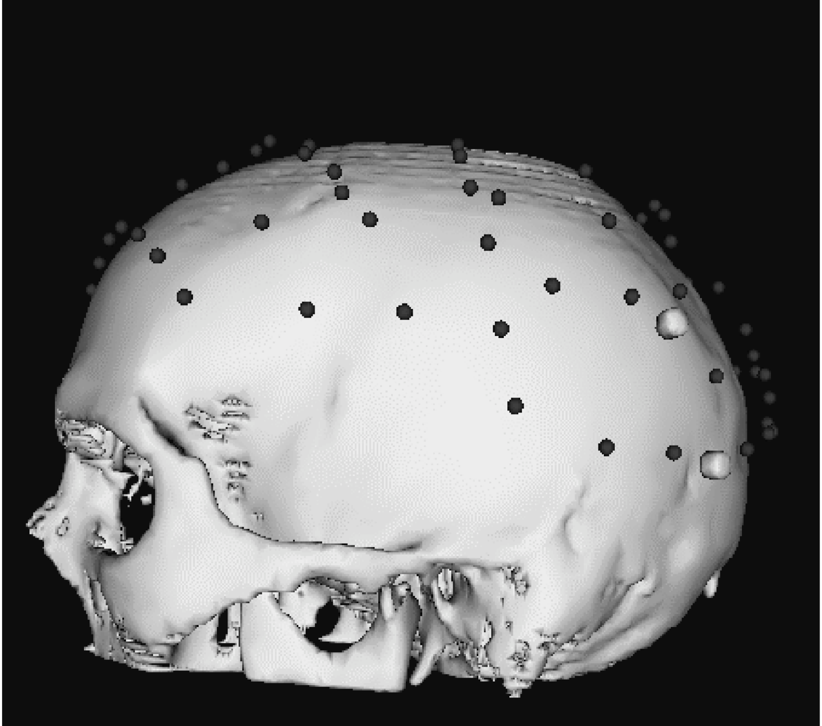
- The calculation of point-to-surface distance is computationally intensive
- Special data structures
  - Octree
  - k-d binary search tree
- Distance transforms
  - Assignment to each voxel **v** of the distance between **v** and the closest feature voxel
  - If feature voxels are surface voxels, then DT provides method for precomputing and storing point-to-surface distance

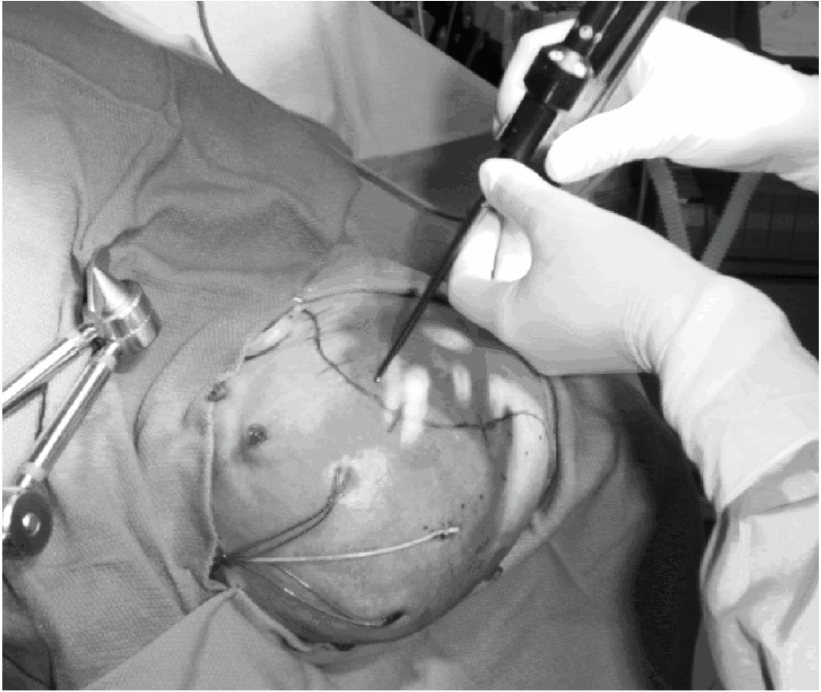
Quadtrees/Octrees



3D/3D CT-to-Physical Registration







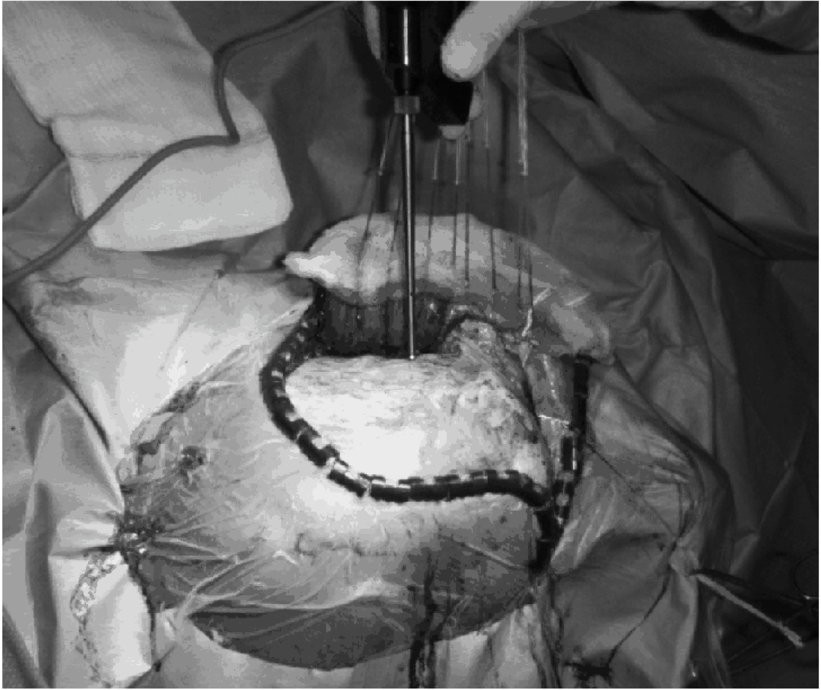
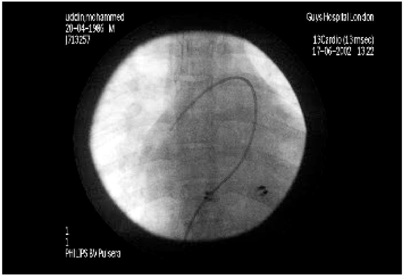


Image-Physical Registration Accuracy

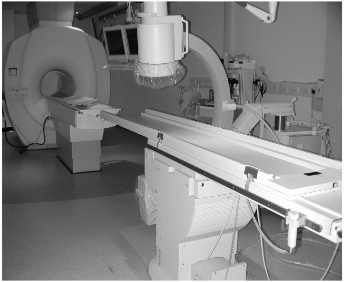
- Stereotactic frame systems (~ 1-2 mm)
- Point-based registration
  - External anatomic landmarks (~ 2-4 mm)
  - Skin-affixed markers (~ 2-4 mm)
  - Bone-implanted markers (~ 1 mm)
- Surface-based registration
  - Skin surface (~ 2-5 mm)
  - Bone surface (~ 1 mm)

2D-3D registration – Example: Image-guided cardiac interventions



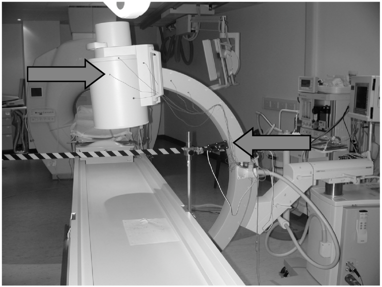
2D-3D registration – Example: XMR System

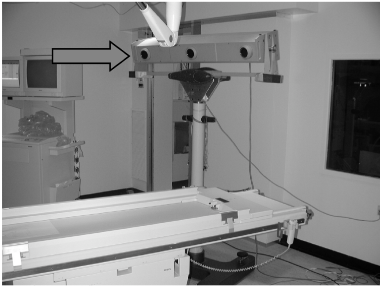
- XMR = X-Ray + MR in same room
- Common sliding patient table
- Provides path to MR-guided intervention



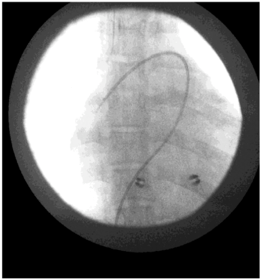
2D-3D registration – Example: Image-Guided Cardiac Interventions

- Registration by optical tracking
- X-ray table & c-arm are tracked by Optotrak
- Sliding patient table is tracked by MR system

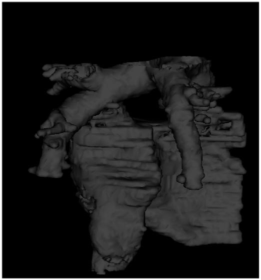




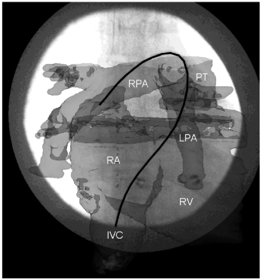
2D-3D registration – Example: Image-Guided Cardiac Interventions



x-ray



MR rendering



x-ray + MR rendering

Rhode et al.
IEEE TMI 2003

2D-3D registration

- So far we have assumed that registration for IGS is based either on
  - points
  - surfaces

  which are identified in the pre-operative image and intra-operative scene
- Registration for IGS can also involve registration of pre-operative image to intra-operative images
  - ultrasound
  - video
  - fluoroscopy

Perspective Transformations

- Subset of projective transformations
- Describe image formation for many modalities
  - Photography (pinhole camera) including video
  - X-ray projection
  - Microscopy
  - Endoscopy

2D-3D registration

$$\mathbf{T}_{projective}(x,y,z) = \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = T_{intrinsic} \cdot T_{extrinsic} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

fixed, e.g. via calibration

unknown, to be estimated by the registration

2D-3D registration

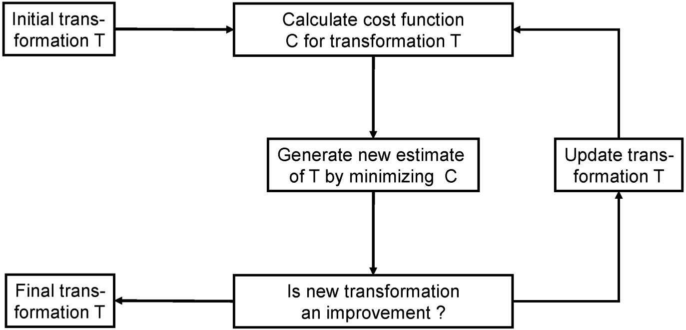
$$\mathbf{T}_{projective}(x,y,z) = \begin{pmatrix} x' \\ y' \\ 1 \end{pmatrix} = T_{intrinsic} \cdot T_{extrinsic} \cdot \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

Simplest form of projection:

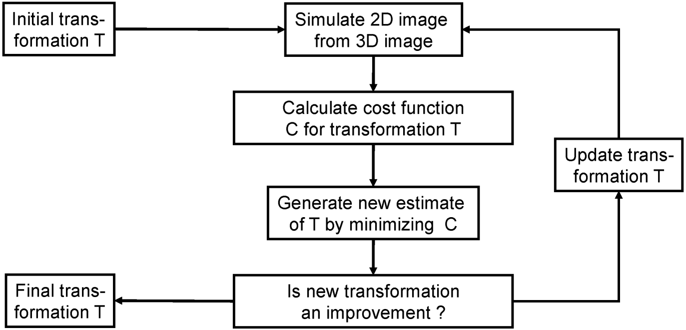
$$T_{intrinsic} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{z} & 0 \end{pmatrix}$$

can be extended to take more physical properties of the imaging into account, e.g. focal length, primary point, distortion, etc

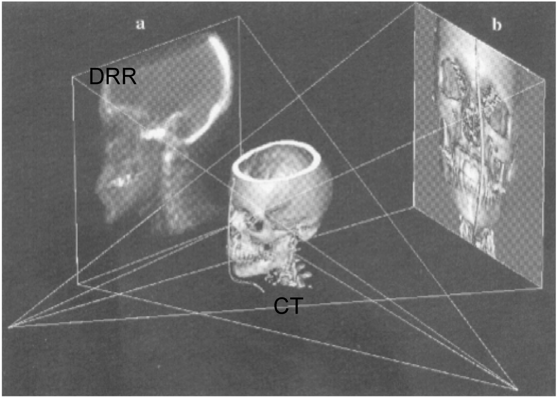
3D-3D Registration



2D-3D Registration

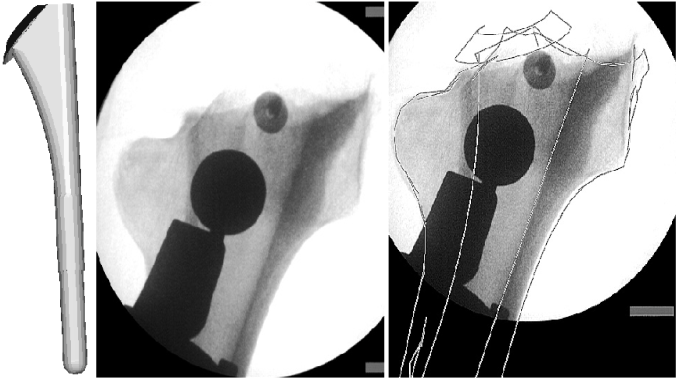


2D-3D Registration Via Digitally Reconstructed Radiograph (DRR)

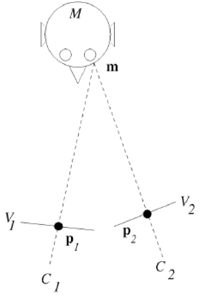


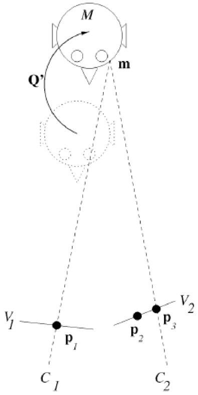
L. Lemieux et al. Med. Phys. Volume 21, Issue 11, pp. 1749-1760, 1994

2D-3D Registration of Bone Fluoroscopy to CT via DRR Construction



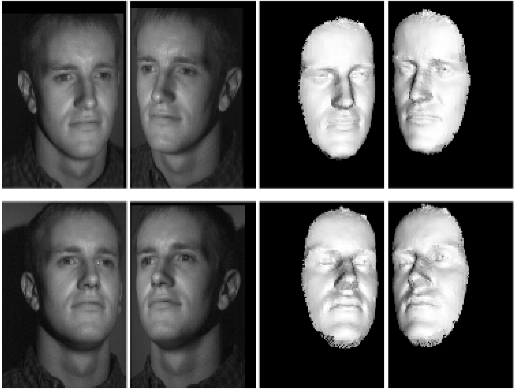
2D-3D registration using photo-consistency





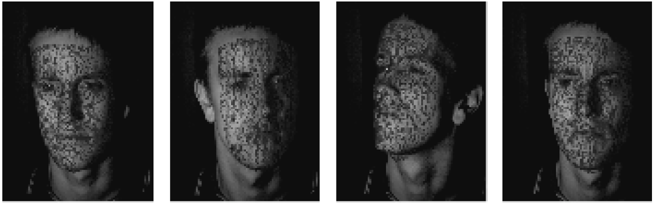
Clarkson et al., IEEE PAMI 2001

2D-3D registration using photo-consistency



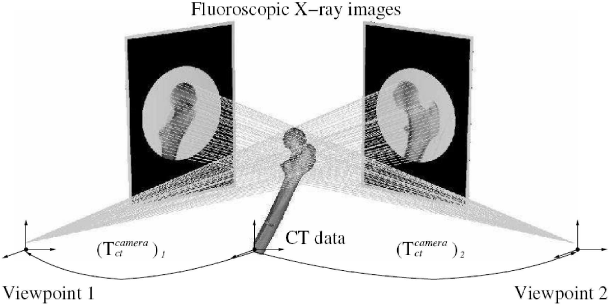
Clarkson et al., IEEE PAMI 2001

2D-3D registration using photo-consistency



Clarkson et al., IEEE PAMI 2001

Contour/Surface Projection



Registration conclusions

- Many different approaches
- Many different applications
- Very frequent in medical imaging
- ... but also video processing, 3D reconstruction...
- Trade-off between robustness, speed and generality
- A priori knowledge always usefull, sometimes essential