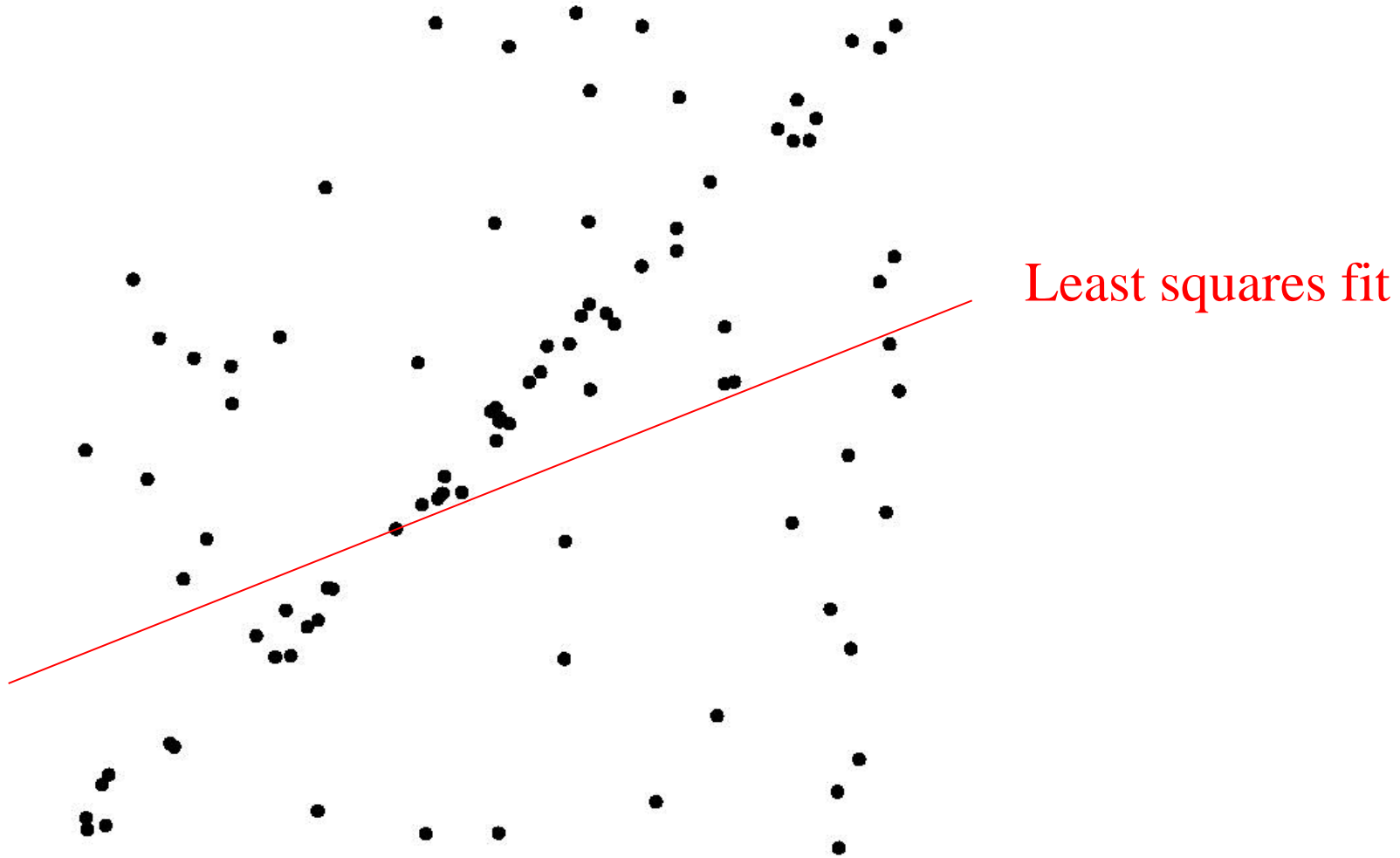


RANSAC

Robust model estimation from data
contaminated by outliers

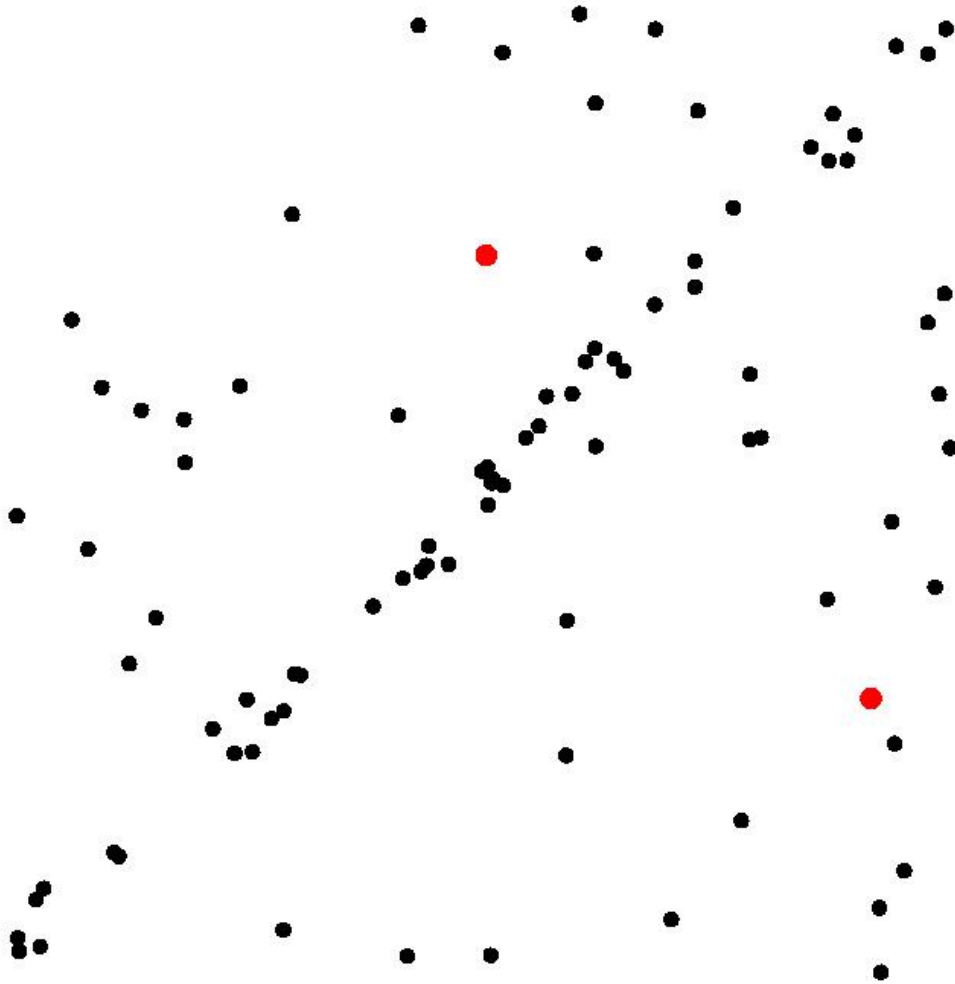
Ondřej Chum

Fitting a Line



RANSAC

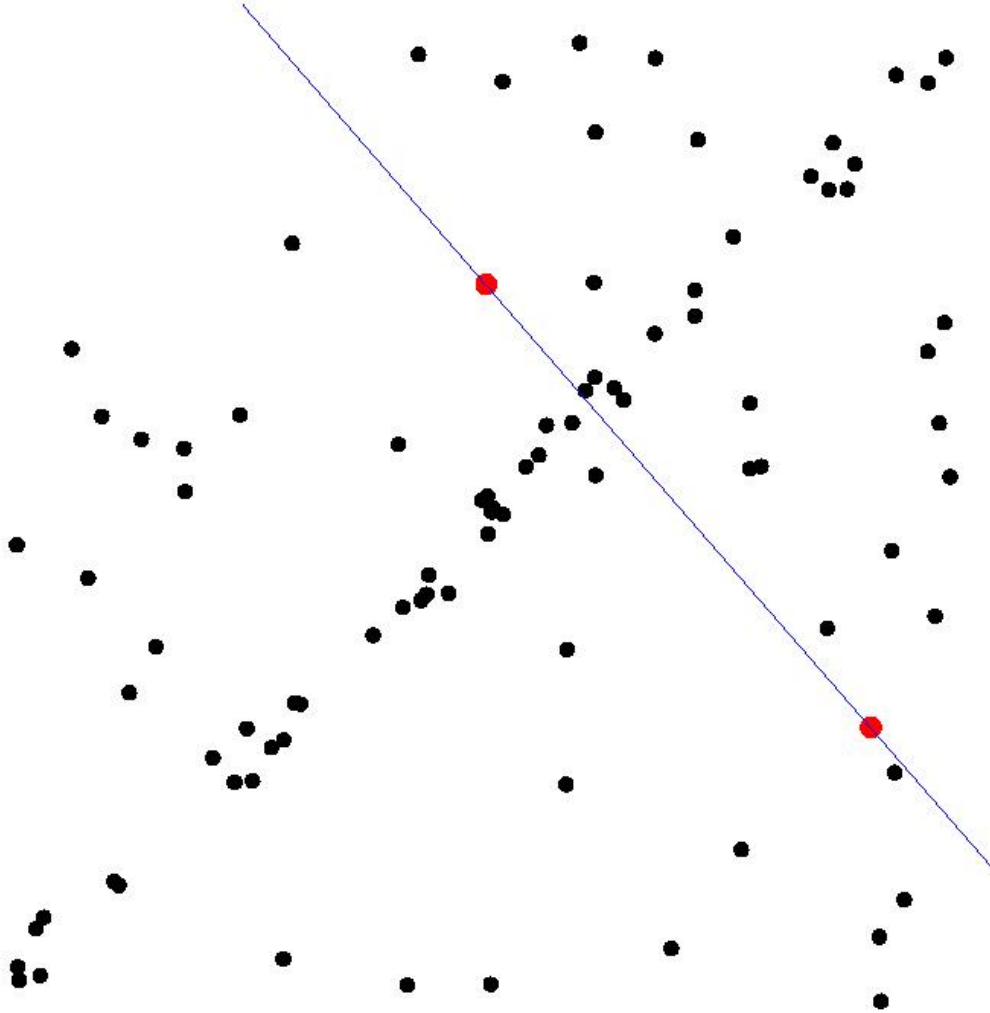
- **Select sample of m points at random**



RANSAC

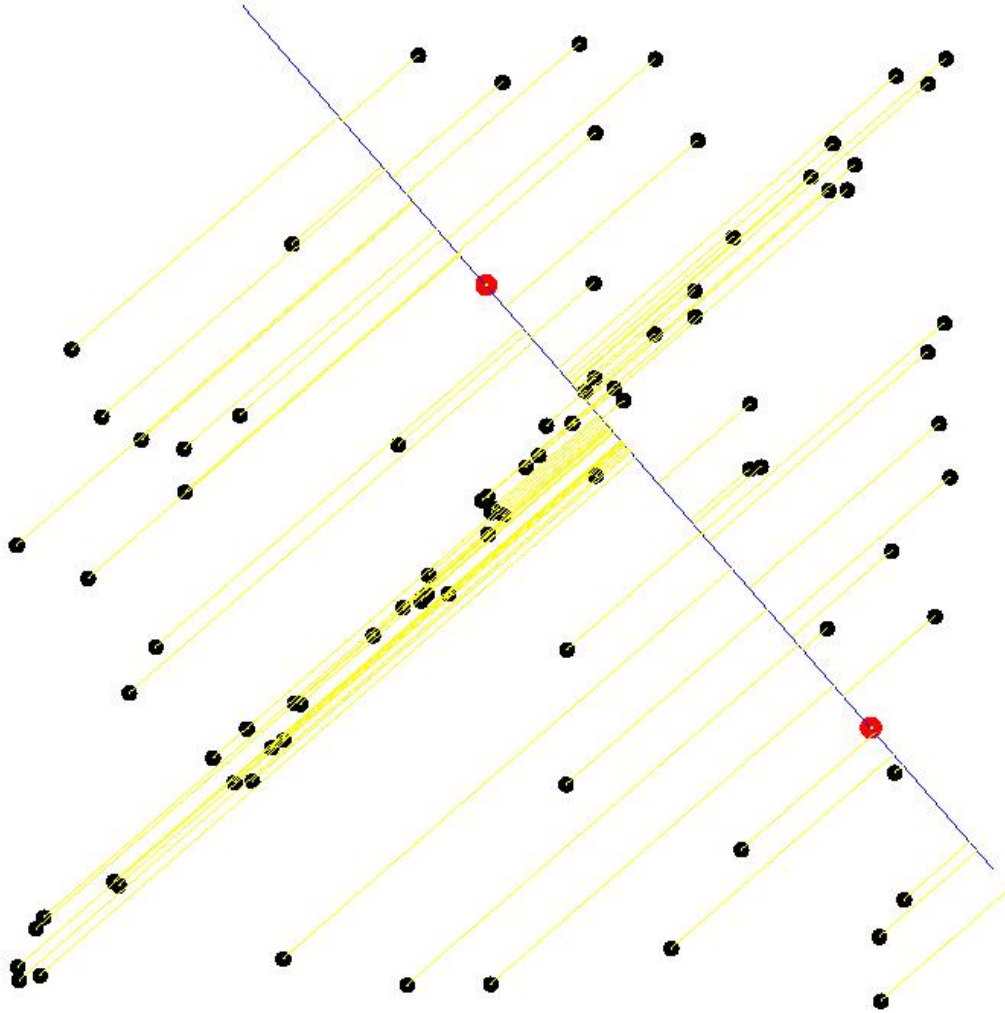
- Select sample of m points at random

- **Calculate model parameters that fit the data in the sample**



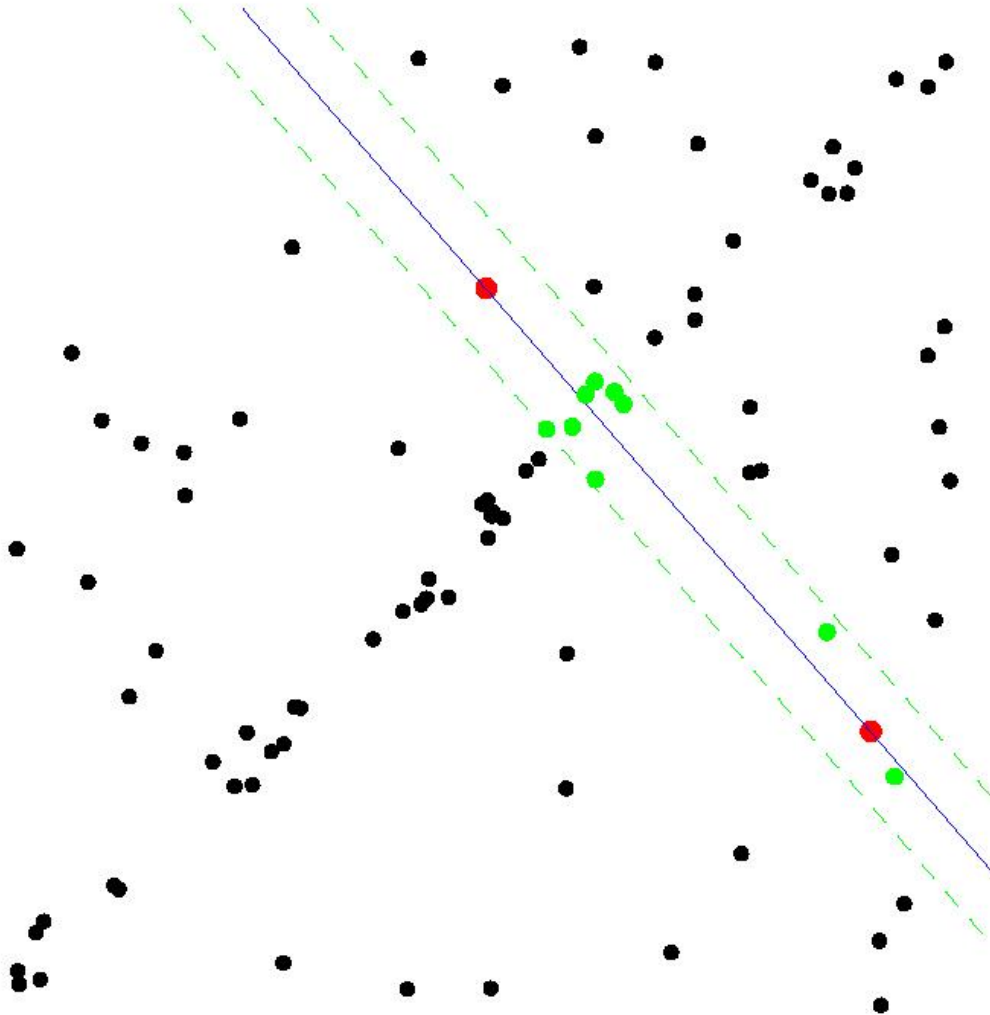
RANSAC

- Select sample of m points at random
- Calculate model parameters that fit the data in the sample
- **Calculate error function for each data point**

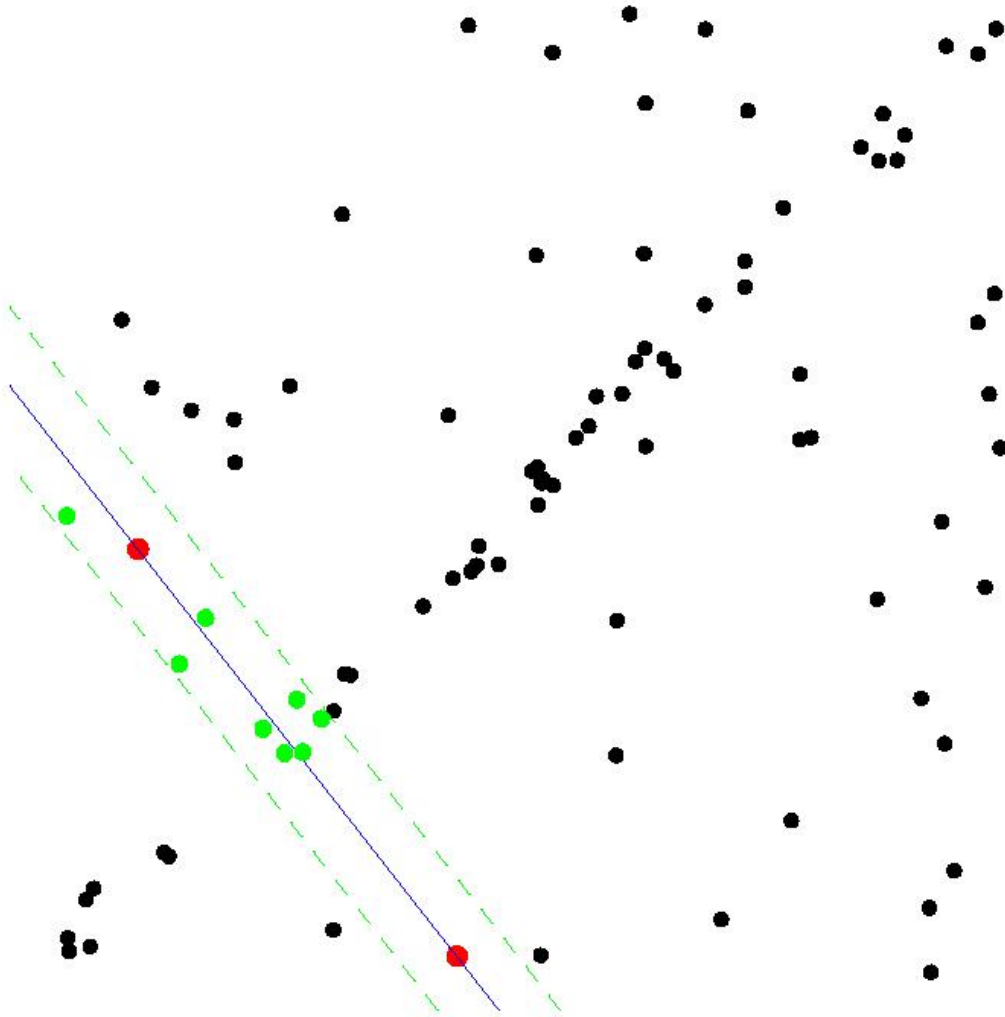


RANSAC

- Select sample of m points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- **Select data that support current hypothesis**

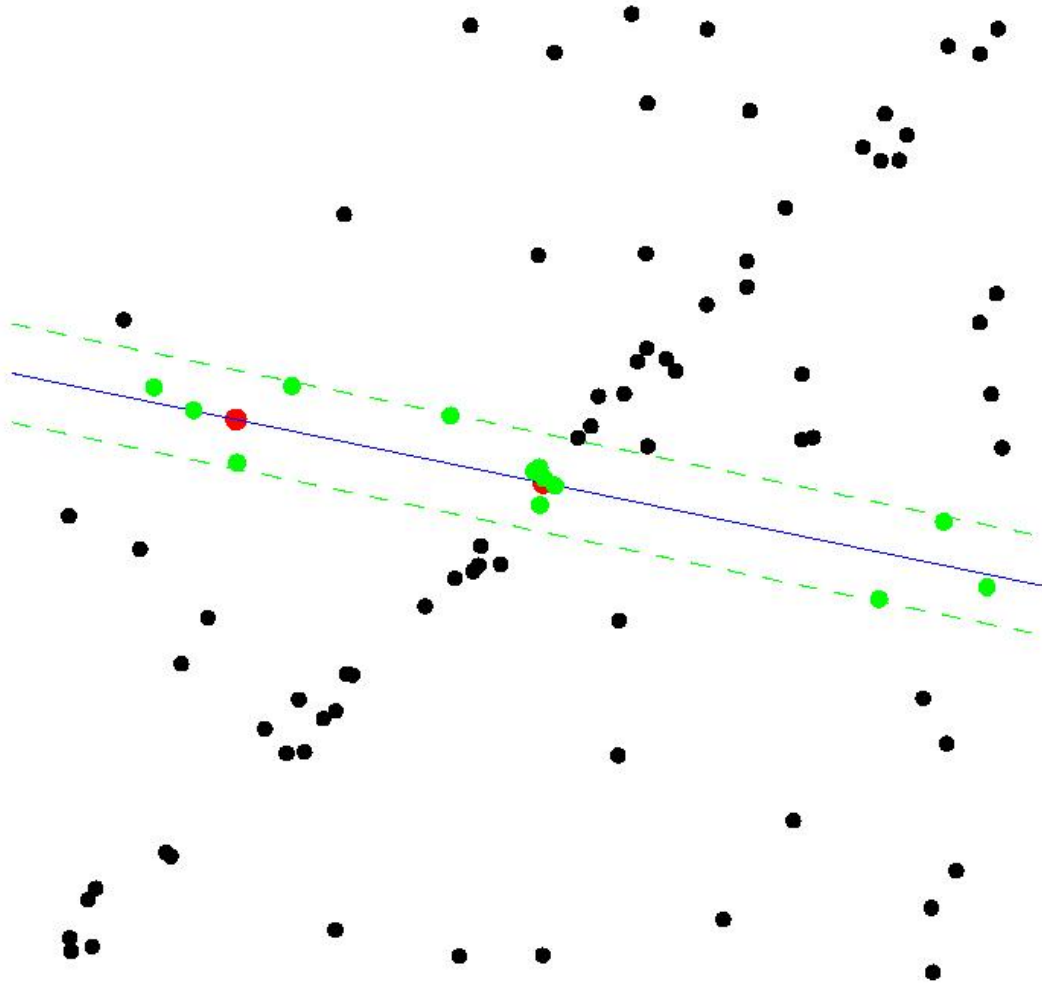


RANSAC



- Select sample of m points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- Select data that support current hypothesis
- **Repeat sampling**

RANSAC



- Select sample of m points at random
- Calculate model parameters that fit the data in the sample
- Calculate error function for each data point
- Select data that support current hypothesis
- **Repeat sampling**

How Many Samples?

On average

N ... number of point

I ... number of inliers

m ... size of the sample

$$P(\text{good}) = \frac{\binom{I}{m}}{\binom{N}{m}} = \prod_{j=0}^{m-1} \frac{I - j}{N - j}$$

mean time before the success

$$E(k) = 1 / P(\text{good})$$

How Many Samples?

With confidence p

How large k ?

... to hit at least one pair of points on the line l with probability larger than p (0.95)

Equivalently

... the probability of not hitting any pair of points on l is $\leq 1 - p$

How Many Samples?

With confidence p

N ... number of point

I ... number of inliers

m ... size of the sample

$$P(\text{good}) = \frac{\binom{I}{m}}{\binom{N}{m}} = \prod_{j=0}^{m-1} \frac{I - j}{N - j}$$

$$P(\text{bad}) = 1 - P(\text{good})$$

$$P(\text{bad } k \text{ times}) = (1 - P(\text{good}))^k$$

How Many Samples?

With confidence p

$$P(\text{bad } k \text{ times}) = (1 - P(\text{good}))^k \leq 1 - p$$

$$k \log (1 - P(\text{good})) \leq \log(1 - p)$$

$$k \geq \log(1 - p) / \log (1 - P(\text{good}))$$

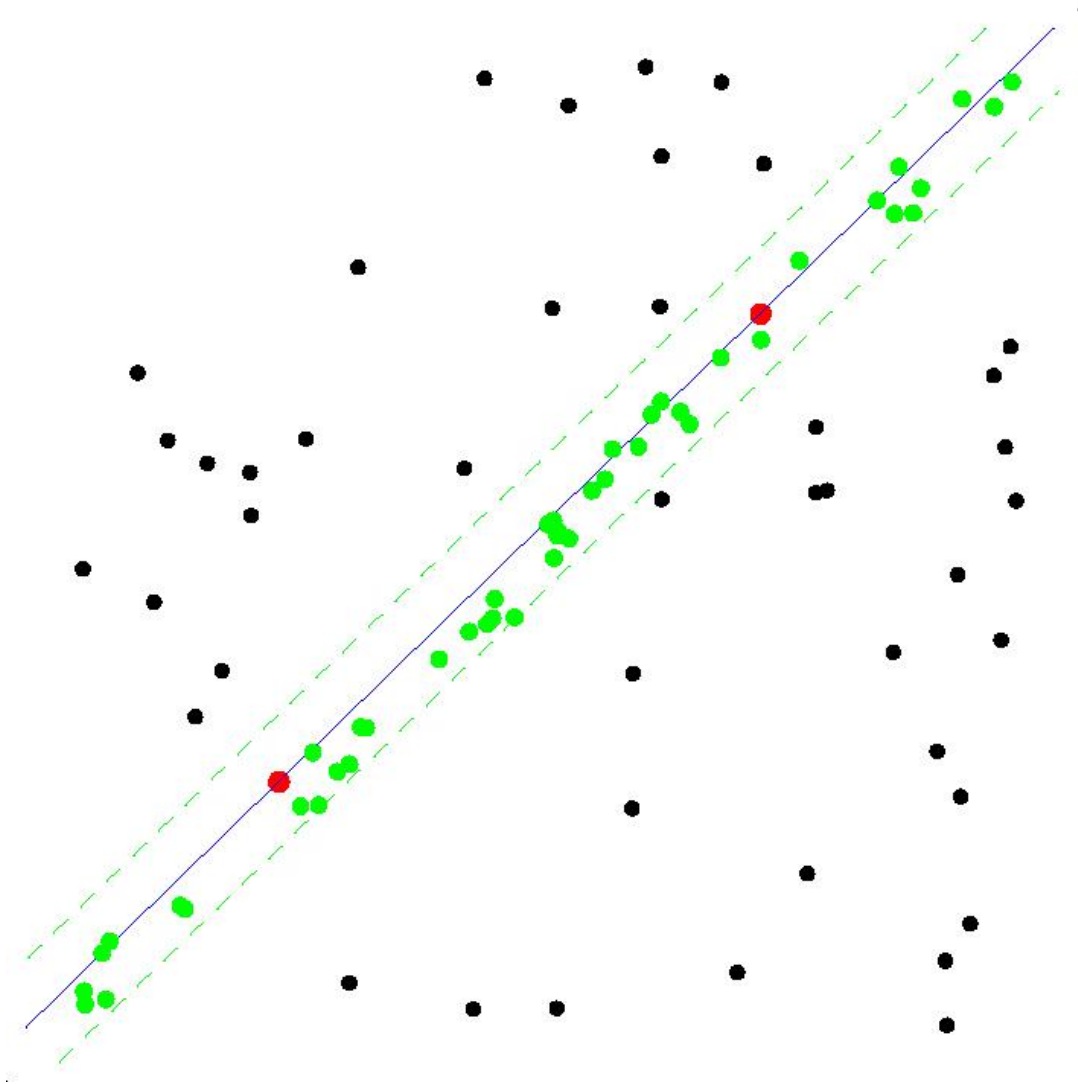
How Many Samples

I / N [%]

Size of the sample m

	15%	20%	30%	40%	50%	70%
2	132	73	32	17	10	4
4	5916	1871	368	116	46	11
7	$1.75 \cdot 10^6$	$2.34 \cdot 10^5$	$1.37 \cdot 10^4$	1827	382	35
8	$1.17 \cdot 10^7$	$1.17 \cdot 10^6$	$4.57 \cdot 10^4$	4570	765	50
12	$2.31 \cdot 10^{10}$	$7.31 \cdot 10^8$	$5.64 \cdot 10^6$	$1.79 \cdot 10^5$	$1.23 \cdot 10^4$	215
18	$2.08 \cdot 10^{15}$	$1.14 \cdot 10^{13}$	$7.73 \cdot 10^9$	$4.36 \cdot 10^7$	$7.85 \cdot 10^5$	1838
30	∞	∞	$1.35 \cdot 10^{16}$	$2.60 \cdot 10^{12}$	$3.22 \cdot 10^9$	$1.33 \cdot 10^5$
40	∞	∞	∞	$2.70 \cdot 10^{16}$	$3.29 \cdot 10^{12}$	$4.71 \cdot 10^6$

RANSAC



$$k = \frac{\log(1 - p)}{\log\left(1 - \frac{I}{N} \frac{I-1}{N-1}\right)}$$

k ... number of samples drawn

N ... number of data points

I ... time to compute a single model

p ... confidence in the solution (.95)

RANSAC [Fischler, Bolles '81]

In: $U = \{x_i\}$ set of **data points**, $|U| = N$

$f(S) : S \rightarrow p$ function f computes **model parameters** p given a sample S from U

$\rho(p, x)$ the **cost function** for a single data point x

Out: p^* p^* , parameters of the model maximizing the cost function

$k := 0$

Repeat until $P\{\text{better solution exists}\} < \eta$ (a function of C^* and no. of steps k)

$k := k + 1$

I. Hypothesis

(1) select randomly set $S_k \subset U$, **sample size** $|S_k| = m$

(2) compute parameters $p_k = f(S_k)$

II. Verification

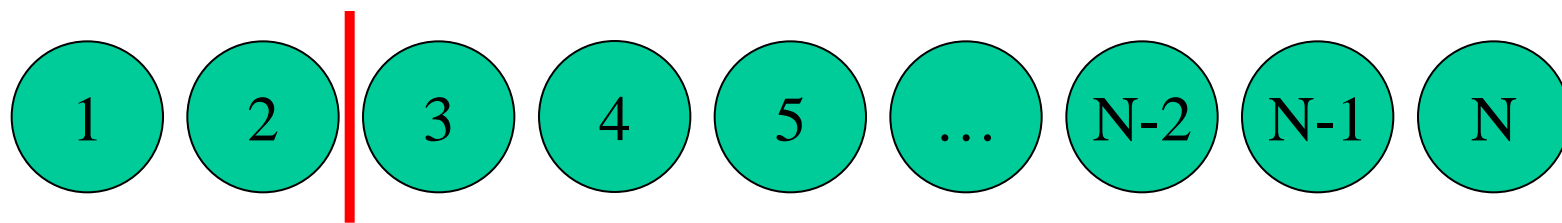
(3) compute cost $C_k = \sum_{x \in U} \rho(p_k, x)$

(4) if $C^* < C_k$ then $C^* := C_k$, $p^* := p_k$

end

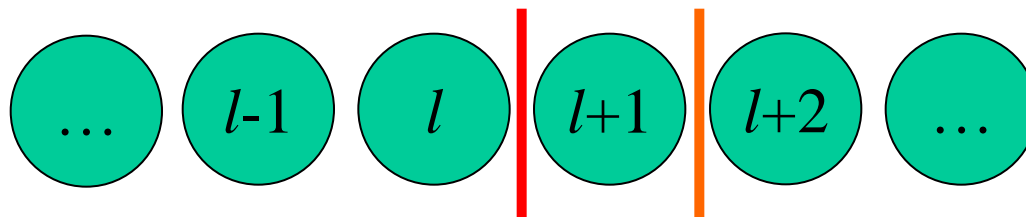
PROSAC – PROgressive SAmples Consensus

- Not all correspondences are created equally
- Some are better than others
- Sample from the best candidates first



Sample from here

PROSAC Samples



Draw T_l samples from $(1 \dots l)$

Draw T_{l+1} samples from $(1 \dots l+1)$

Samples from $(1 \dots l)$ that are not from $(1 \dots l+1)$ contain

$l+1$

Draw $T_{l+1} - T_l$ samples of size $m-1$ and add

$l+1$