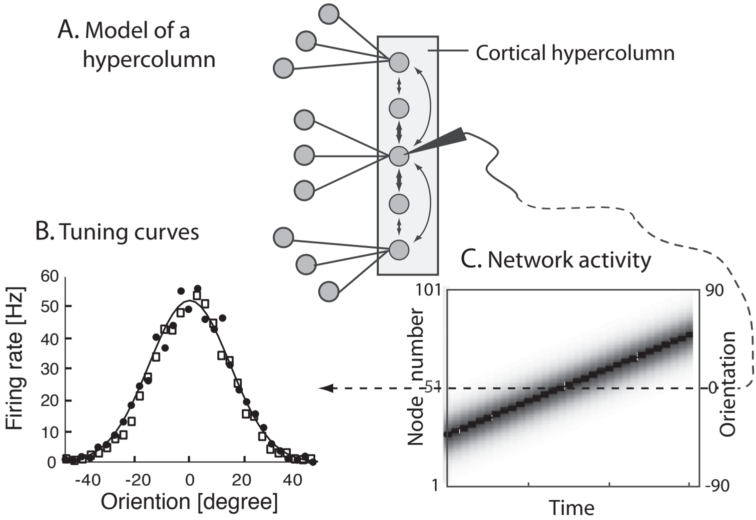
Neuroinformatics

April 25, 2013

Lecture 10: Cortical self-organized maps (SOM)

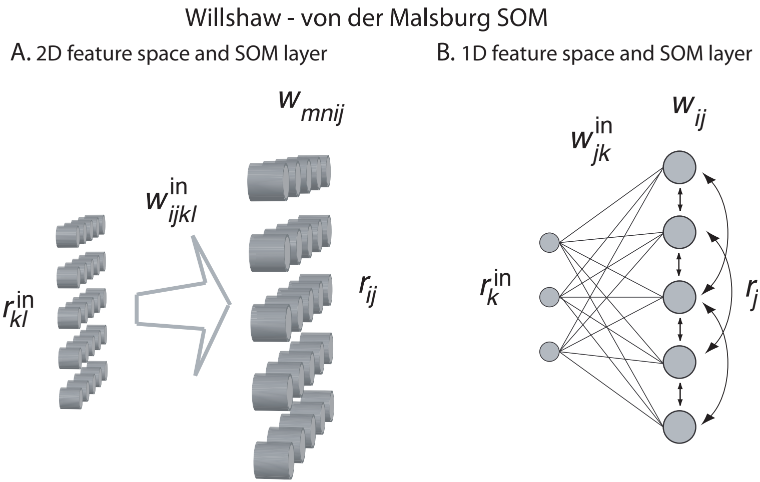
Motivation for SOM and DNF - Tuning Curves



Self-organizing maps (SOMs)

- The development of SOM as a neural model is motivated by the topographical nature of cortical maps.
- Visual, tactile, and acoustic inputs are mapped in a topographical manner. Visual: retinotopy (position in visual field), orientation, spatial frequency, direction, ocular dominance, etc. Tactile: somatotopy (position on skin,thumb and SMS) Acoustic: tonotopy (frequency)
- Self-organizing maps (SOM) is based on competitive learning, where output neurons compete with each other to be activated (Kohonen, 1982)
- The output neuron that activates is called the winner-takes-all neuron
- Lateral inhibition is one way to implement competition for map formation (von der Malsburg 1973)
- In SOM, neurons are placed on a lattice, on which a meaningful coordinate system for different features is created (feature map).
- The lattice thus forms a topographic map where the spatial location on the lattice is indicative of the input features.

SOM -von der Malsburg 1973



Network equations

Update rule of (recurrent) cortical network:

$$\tau \frac{\mathrm{d}u_i(t)}{\mathrm{d}t} = -u_i(t) + \frac{1}{N}\sum_j w_{ij} r_j(t) + \frac{1}{M}\sum_k w_{ik}^{\mathrm{in}} r_k^{\mathrm{in}}(t)$$

Activation function: $r_j(t) = \frac{1}{1+e^{\beta(u_j(t)-\alpha)}}$.

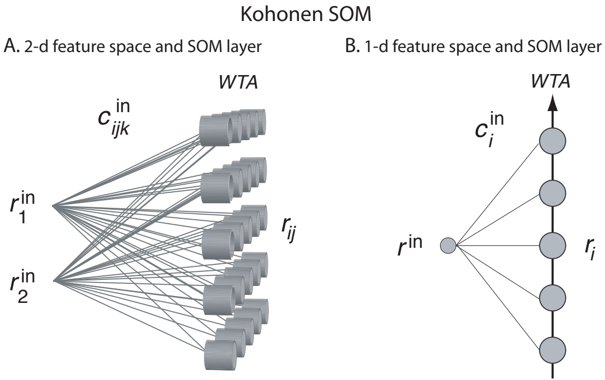
Lateral weight matrix: $w_{ij} \propto r_i r_j$

$$= A_{\mathrm{w}} \left(\mathrm{e}^{-((i-j)*\Delta x)^2/2\sigma^2} - C \right)$$

Input weight matrix: $w_{ij}^{\mathrm{in}} \propto r_i r_j^{\mathrm{in}}$

Kohonen - Shortcut

- Willshaw-von der Malsburg model: input neurons arranged in 2D lattice, output in 2D lattice. Lateral excitation/inhibition (Mexican hat) gives rise to soft competition. Normalized Hebbian learning. Biological motivation.
- Kohonen model: input of any dimension, output neurons in 1D, 2D, or 3D lattice. Relaxed winner-takes-all (neighborhood). Competetive learning rule. Computational motivation.



Kohonen model

- cortical sheet activation, σ_r^2 width of activated area, activation fce resembels tuning curves, radial-basis networks

$$r_{ij} = \exp(-\sum_k (c_{ijk} - r_k^{in})^2 / 2\sigma_r^2)$$

- strength connection around the winning node r_{ij}^*, WTA rule - winner takes all

$$\Delta c_{ijk} = \epsilon r_{ij}^* (r_{in} - c_{ijk})$$

- ML approach (Matlab implementation): $w^i(q) = w^i(q-1) + \alpha(p(q) - w^i(q))$, i are lying in neighborhood $N(i)_d = \{j, d_{ij} < d\}$

SOM Algorithm

1. Randomly initialize weight vectors w_i
2. Randomly sample input vector x
3. Find Best Matching Unit (BMU)

$$i(x) = \arg\min_j ||x - w_j||$$

4. Update weight vectors, where $h(j, i(x))$ is neighborhood function of BMU

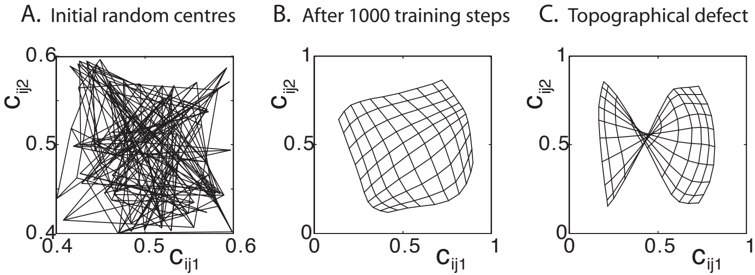
$$w_j = w_j + \epsilon h(j, i(x))(x - w_j)$$

5. Repeat steps 2-4

som.m

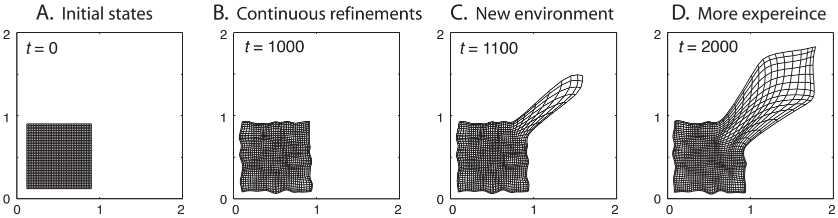
```
%% Two dimensional self-organizing feature map al la Kohonen
 clear; nn=10; lambda=0.2; sig=2; sig2=1/(2*sig^2);
 [X,Y]=meshgrid(1:nn,1:nn); ntrial=0;

 % Initial centres of prefered features:
 c1=0.5-.1*(2*rand(nn)-1);
 c2=0.5-.1*(2*rand(nn)-1);

%% training session
 while(true)
    if(mod(ntrial,100)==0) % Plot grid of feature centres
        clf; hold on; axis square; axis([0 1 0 1]);
        plot(c1,c2,'k'); plot(c1',c2','k');
        tstring=[int2str(ntrial) ' examples']; title(tstring);
        waitforbuttonpress;
    end
    r_in=[rand;rand];
    r=exp(-(c1-r_in(1)).^2-(c2-r_in(2)).^2);
    [rmax,x_winner]=max(max(r)); [rmax,y_winner]=max(max(r'));
    r=exp(-((X-x_winner).^2+(Y-y_winner).^2)*sig2);
    c1=c1+lambda*r.*(r_in(1)-c1);
    c2=c2+lambda*r.*(r_in(2)-c2);
    ntrial=ntrial+1;
 end
```

SOM simulation



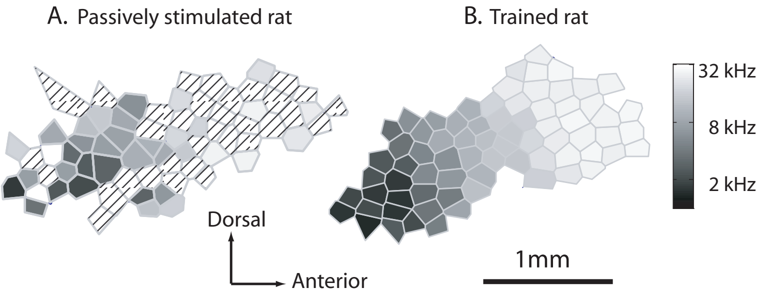
Another example

- Simulating development processes
- SOM can represent new domains, representation less fine-grained compared to initial domain
- Early in life exposed to broad feature space (learning languages)



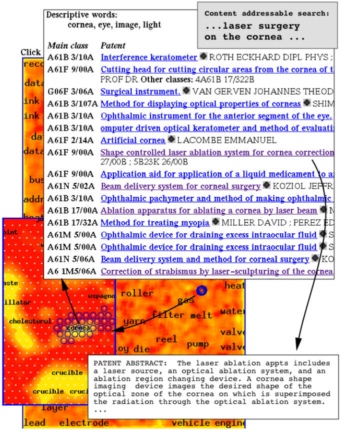
Representational plasticity - Zhou and Merzenich, PNAS 2007

- rat pups raised in noisy environment $\leftarrow$ severely impaired tonotopicity (tones representations) in primary auditory cortex - A1
- no recovery after stimulation with sounds of different frequencies
- stimulation by discrimination task with food reward $\leftarrow$ rats were able to recover tonotopic maps
- traditionally SOM maps are driven by data: bottom - up approach
- top-down processing explains those experimental results (reinforcement learning)



WEBSOM - Self-Organizing Maps for Internet Exploration

- find information on laser surgery on the cornea of eye, `http://websom.hut.fi`
- best matching locations marked with circles
- sparse feature vector, each raw representing single document, each term relative frequency of predefined entries (e.g. 50 000 words)



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Further Readings

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