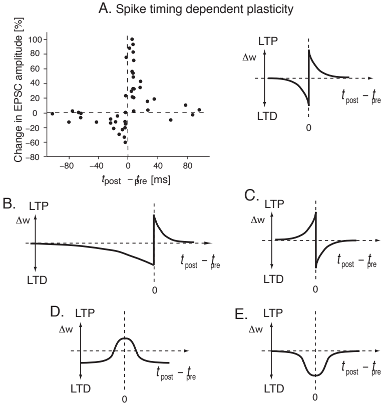
Fundamentals of Computational Neuroscience 2e

March 26, 2014

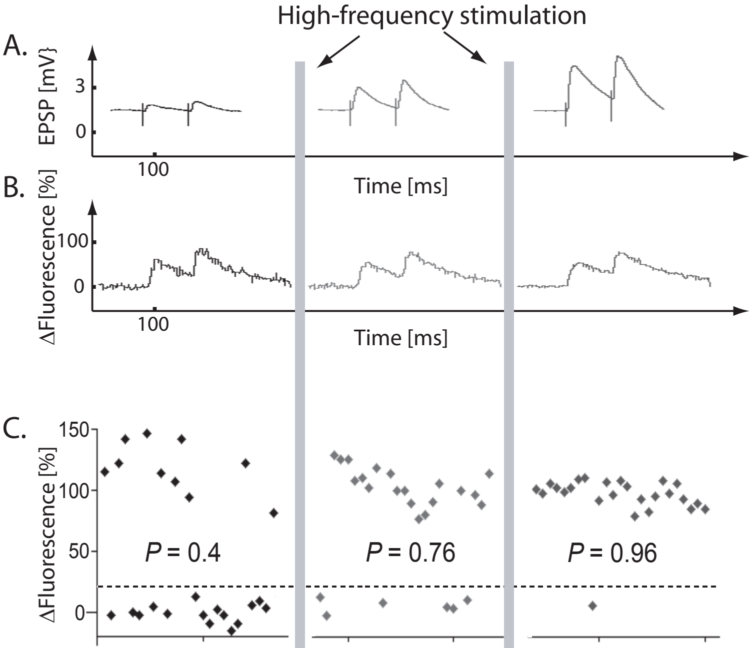
Lecture 6: Synaptic plasticity and Hebb's rule

Spike timing dependent plasticity (STDP)

- Bi-Poo experiments: voltage clamp for hippocamal cells in vitro, $\rightarrow$ Excitatory PostSynaptic Current (EPSP) $\rightarrow$ critical time window $\Delta t = 40$ms
- critical window width is much larger, asymmetrical and symmetrical (for bursting neurons) form of Hebbian plasticity, inverse correlation in Purkinje cells (inhibitory) in the cerebellum

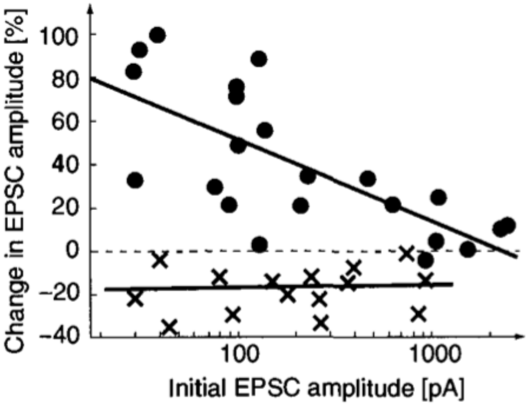


Synaptic neurotransmitter release probability



Initial weight dependence

- Bi-Poo experiments: synaptic efficiencies of LTD are proportional to the INITIAL synaptic strength, $\frac{\delta A}{A}$
- LTP: changes of EPSC are largest small initial EPSC amplitudes



Mathematical formulation of Hebbian plasticity - spiking models

$$w_{ij}(t+\Delta t) = w_{ij}(t) + \Delta w_{ij}(t_i^{\mathrm{f}}, t_j^{\mathrm{f}}, \Delta t; w_{ij}).$$

$$\Delta w_{ij}^{\pm} = \epsilon^{\pm}(w) K \pm (t^{\mathrm{post}} - t^{\mathrm{pre}})$$

Spike Timing Dependent Plasticity (SPDP)¿ (i) Exponential plasticity curve, (ii) Repeated spike pairings induced w UNBOUNDED growth $\rightarrow$ a weight dependent learning rate $\epsilon^{\pm}$

$$\Delta w_{ij}^{\pm} = \epsilon^{\pm}(w) \mathrm{e}^{\mp \frac{t^{\mathrm{post}} - t^{\mathrm{pre}}}{\tau^{\pm}}} \; \Theta(\pm[t^{\mathrm{post}} - t^{\mathrm{pre}}]).$$

Additive rule with hard (absorbing) boundaries:

$$\epsilon^{\pm} = \begin{cases} a^{\pm} & \text{for } \; w_{ij}^{\mathrm{min}} \leq w_{ij} \leq w_{ij}^{\mathrm{max}} \\ 0 & \text{otherwise} \end{cases},$$

Multiplicative rule (soft boundaries):

$$\begin{aligned} \epsilon^{+} &= a^{+}(w^{\mathrm{max}} - w_{ij}) \\ \epsilon^{-} &= a^{-}(w_{ij} - w^{\mathrm{min}}). \end{aligned} \quad (1)$$

Hebbian learning in rate (population) models

no spike timings! $\rightarrow$ plasticity depends on correlation of pre and post synaptic spikes!

General: $\Delta w_{ij} = \epsilon(t, w)[f_{\text{post}}(r_i) f_{\text{pre}}(r_j) - f(r_i, r_j, w)]$

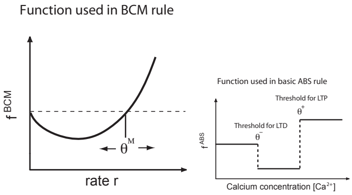
Mnemonic equation (Caianiello): f(w) is weight decay
$\Delta w_{ij} = \epsilon(w)[r_i r_j - f(w)]$

Basic Hebb:,f_{post} linear,f_{pre} linear: $\Delta w_{ij} = \epsilon r_i r_j$

$\langle r \rangle$ is average over many trials with different stimuli, if f_{post},f_{pre} is $\langle r \rangle$
Covariance rule (plasticity threshold): $\Delta w_{ij} = \epsilon(r_i - \langle r_i \rangle)(r_j - \langle r_j \rangle)$

BCM theory, $\theta^M = f(r_j)$, post!: $\Delta w_{ij} = \epsilon(f^{\text{BCM}}(r_i; \theta^M)(r_j) - f(w))$

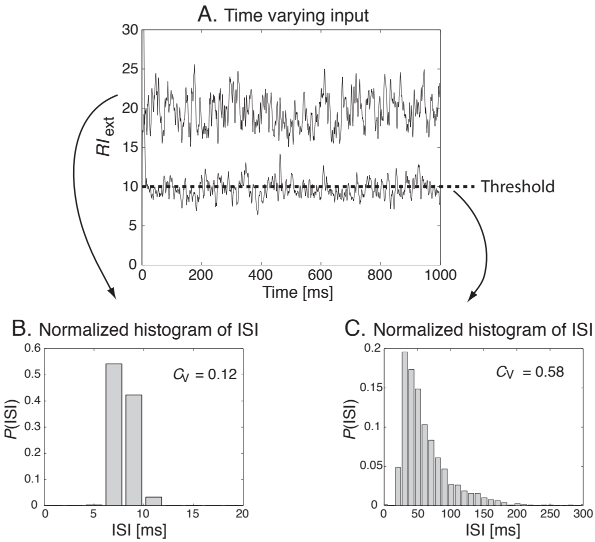
ABS rule: $\Delta w_{ij} = \epsilon(f_{\text{ABS}}(r_i; \theta^-, \theta^+)\text{sign}(r_j - \theta^{\text{pre}}))$



The LIF-neuron noise simulation I

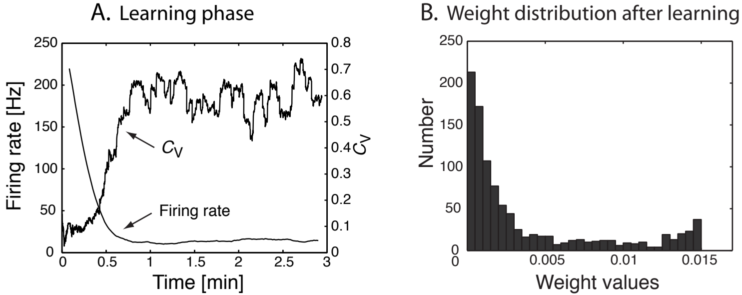
- real neuron with 5000 presynaptic neuron
- 10 % simulation $\rightarrow$ 500 Poisson-distributed spike trains (**??**) with refractory corrections
- mean firing rate = 20 Hz, after correction 19.3 Hz, refractory constant 2 ms.
- each presynaptic spike $\rightarrow$ EPSP in form of α function (**??**)
- $\omega = 0.5 \rightarrow$ regular firing, $C_V = 0.12$, average rate 118 Hz.
- $\omega = 0.25 \rightarrow$ irregular firing, $C_V = 0.58$, average rate 16 Hz. The $C_V >$ lower bound found in experiments

The LIF-neuron noise simulation II



Synaptic scaling and weight distributions

- IF neuron with 1000 excitatory synapses driven by presynaptic Poisson spike trains with average firing rate of 20 Hz, $\Delta w_{ij}^{\pm} = \epsilon^{\pm}(w)K\pm(t^{\text{post}} - t^{\text{pre}})$ applying additive rule and asymmetrical Gaussian plasticity windows
- (i) weights set to large values (ii) large frequency firing (see lec4) (iii) apply additive STDP rule with marginally stronger LTD than LTP
- increased CV, firing rate reduction, weight BINOMICAL distribution after 5 mins

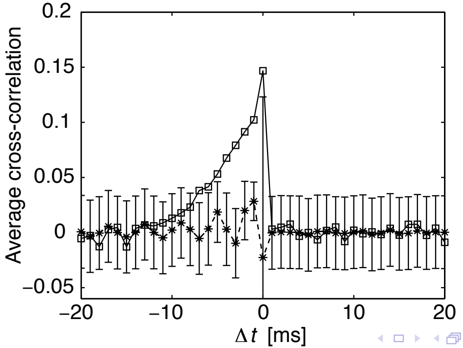


after Song, Miller and Abbott 2000

Cross-correlation function

- $s(\Delta t)$, $s = 1$ if a spike occurs in Δt
- star line:$C(n) = 0$ for regular IF firing 270 Hz, $w = 0.015$, LTP occurs as much as LTD
- square line:after Hebb's learning, IF firing 18 Hz, some presynaptic spikes elicits post-synaptic spikes
- $C < 0$, if presynaptic spikes reduce postsynaptic (anti-correlation) and vice-versa

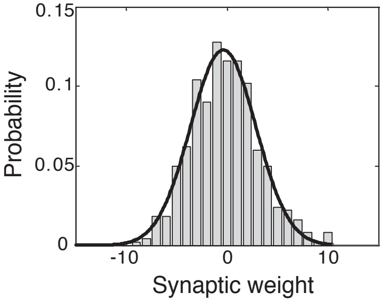
$$C(n) = \langle s^{pre}(t) s^{post}(t + n\delta t) \rangle - \langle s^{pre} s^{post} \rangle$$



Hebbian rate rules on random pattern - weight distribution ??

- Central limit theorem: sum of random variables approaches Gaussian distribution with ZERO mean and $\frac{\sigma}{N_p}$ variance (after 1000 runs).
- After learning N_p patterns, where $\epsilon = \frac{1}{N_p} \rightarrow$ the width of distribution does not change with the number of training patterns
- Rates are exponential distributed as in real case ($\langle r \rangle log(x)$) -

$$w_{ij} = \frac{1}{\sqrt{N_p}} \sum_{\mu} (r_i^{\mu} - \langle r_i \rangle)(r_j^{\mu} - \langle r_j \rangle)$$



Matlab code

- 500 hundred presynaptic nodes, 1 postsynaptic node, 1000 patterns
- matrix notation: $\Delta w = ba'$. a firing rate presynaptic and b postynaptic.
- covariance Hebb's rule: w=(rPost-ar)*(rPre-ar)'

```
%% Weight distribution of Hebbian synapses in rate model
clear; clf; %clear workspace and figure
nn=500; npat=1000; %number of nodes and patterns
%% Random pattern; firing rates are exponential distributed
ar=40; %average firing rate of pattern
rPre =-ar.*log(rand(nn,npat)); %exponential distr. pre rates
rPost=-ar.*log(rand(1,npat));  %exponential distr. post rate
%% Weight matrix
w=(rPost-ar)*(rPre-ar)'; %Hebbian covariance rule
w=w/sqrt(npat); %standard scaling to keep variance constant
%% Histogram plotting
x=-10:1:10;
[n,x]=hist(w/nn,x); %calculate histogram
n=n/sum(n); %normalizaton to get probability distribution
h=bar(x,n); set(h,'facecolor','none');
%% Fit normal ditribution to data
a0=[0 5];
a=lsqcurvefit('normal',a0,x,n);
n2=normal(a,-15:0.1:15);
hold on; plot(-15:0.1:15,n2,'r')
```