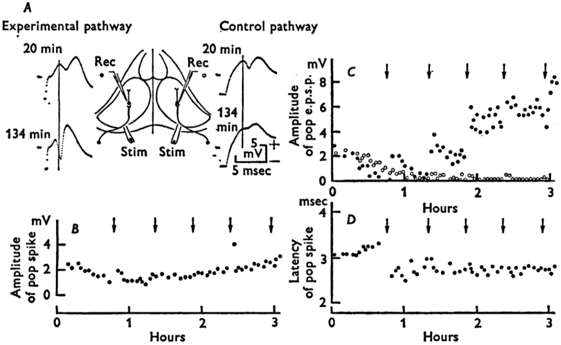
Fundamentals of Computational Neuroscience 2e

March 14, 2013

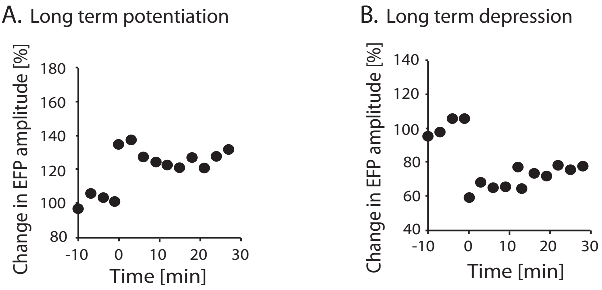
Lecture 6: Synaptic plasticity and Hebb's rule

Original LTP by Bliss and Lomo, 1973

- Long-lasting changes of synaptic response characteristics
- High frequency-stimulus is applied (plasticity-induced tetanus) $\rightarrow$ long-term potentiation(to strengthen, make more potent) (LTP) average amplitude of EPSP increased
- Long frequency stimulus $\rightarrow$ long-term depression (LTD)

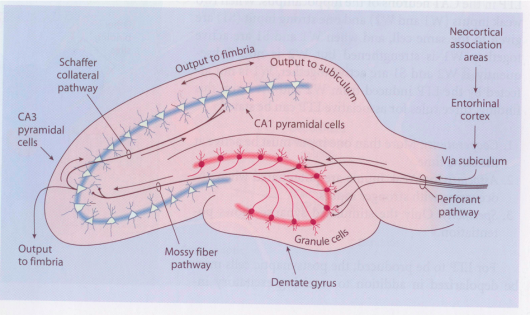


Classical LTP and LTD



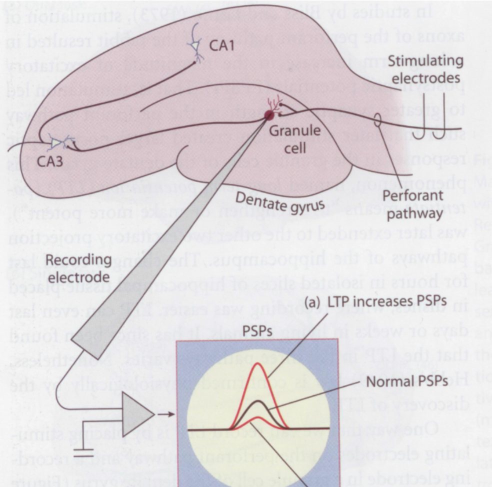
Hippocampus

- Hippocampus: centre of memory storage, The dentate gyrus is thought to contribute to the formation of new memories. It is notable as being one of a select few brainstructures currently known to have high rates of neurogenesis in adult rats
- Neurons must be plastic
- Experiment: isolated slices of hippocampal tissue placed in dishes



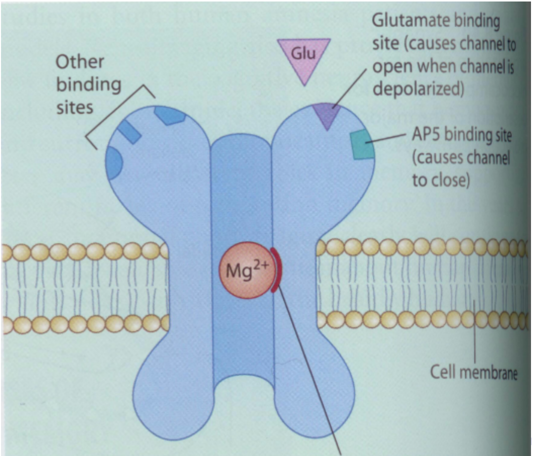
LTP experiment

- EXPERIMENTAL confirmation of Hebb's rule (1949)
- i) single pulse is presented ii) stimulation with burst of pulses: 100 pulses/sec ii) After LTP induced, single pulse stimulation
- Postsynaptic cells must be depolarized to LTP be produced AND receiving excitatory input - see Associative learning slide.



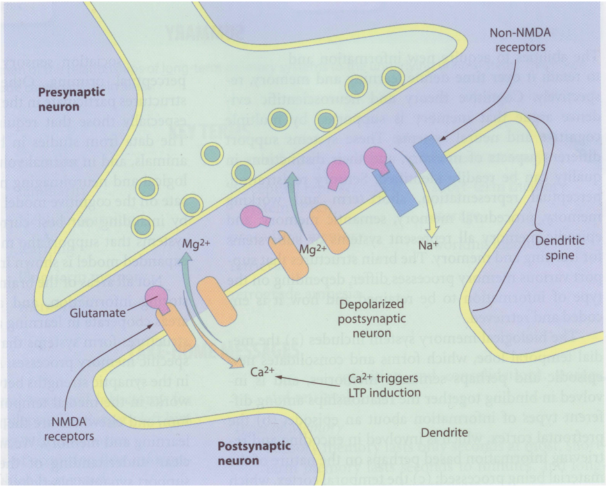
NMDA receptors

- N-methyl-D-aspartarte receptor lacated on dendrytic spines of postsynaptic neurons showing LTP
- i) NMDA receptors are blocked by Mg^{2+} ii) Channel unblocking after glutamate binding (glucamate is major excitatory transmitter in hippocampus) AND membrane depolarized (NMDA are voltage gated) $\rightarrow$ Mg^{2+} ejection, Ca^{2+} influx

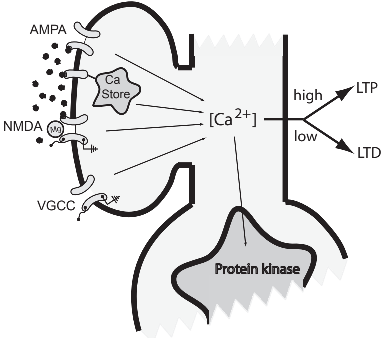


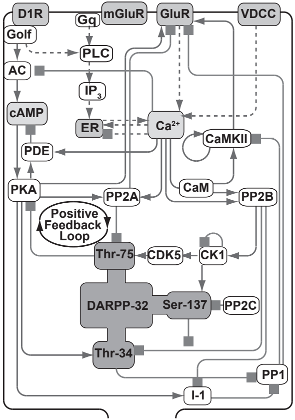
Ca^{2+} role

- Ca^{2+} changes enzyme activities that influence synaptic strength
- LTP raises sensitivity of non-NMDA glucamate receptors prompting release of more glucamate



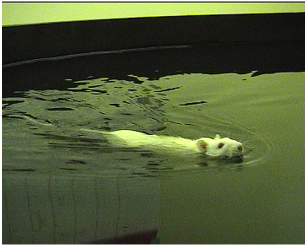
The calcium hypothesis and modeling chemical pathways

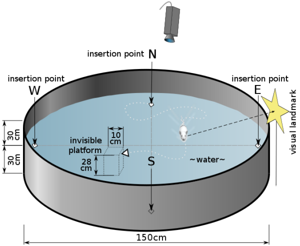




Morris Water Maze - spatial memory

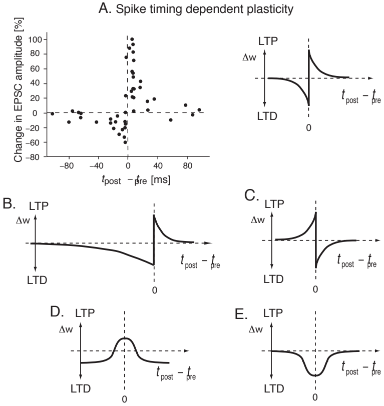
- i) mice training ii) Chemical blocking of LTP by AP5 impair spatial learning, keep control group iii) AP5-treated mice significantly impaired
- i) slices of the hippocampus were taken from both groups ii) LTP was easily induced in controls, but could not be induced in the brains of APV-treated rats
- Alzheimer's disease $\rightarrow$ cognitive decline seen in individuals with AD may result from impaired LTP ??



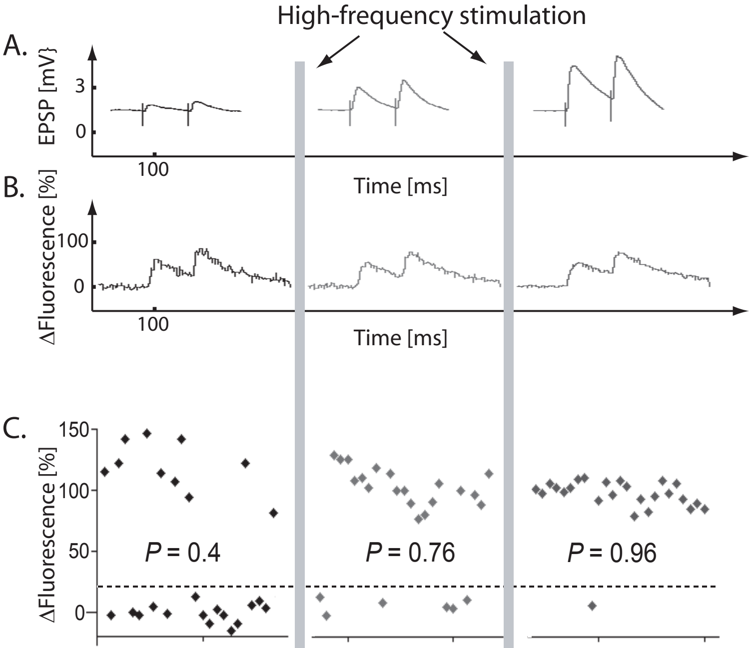


Spike timing dependent plasticity (STDP)

- Bi-Poo experiments: voltage clamp for hippocamal cells in vitro, $\rightarrow$ Excitatory PostSynaptic Current (EPSP) $\rightarrow$ critical time window $\Delta t = 40$ms
- critical window width is much larger, asymmetrical and symmetrical (for bursting neurons) form of Hebbian plasticity, inverse correlation in Purkinje cells (inhibitory) in the cerebellum

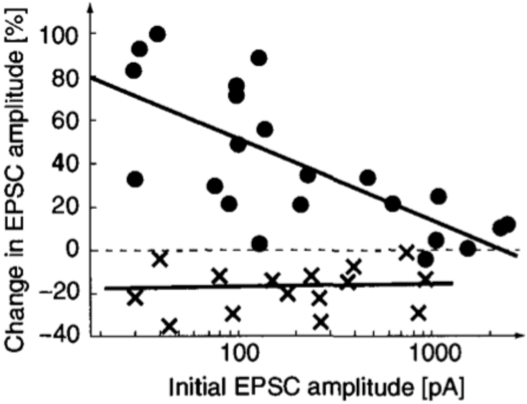


Synaptic neurotransmitter release probability



Initial weight dependence

- Bi-Poo experiments: synaptic efficiencies of LTD are proportional to the INITIAL synaptic strength, $\frac{\delta A}{A}$
- LTP: changes of EPSC are largest small initial EPSC amplitudes



Mathematical formulation of Hebbian plasticity - spiking models

$$w_{ij}(t+\Delta t) = w_{ij}(t) + \Delta w_{ij}(t_i^{\mathrm{f}}, t_j^{\mathrm{f}}, \Delta t; w_{ij}).$$

$$\Delta w_{ij}^{\pm} = \epsilon^{\pm}(w) K \pm (t^{\mathrm{post}} - t^{\mathrm{pre}})$$

Spike Timing Dependent Plasticity (SPDP)¿ (i) Exponential plasticity curve, (ii) Repeated spike pairings induced w UNBOUNDED growth $\rightarrow$ a weight dependent learning rate $\epsilon^{\pm}$

$$\Delta w_{ij}^{\pm} = \epsilon^{\pm}(w) \mathrm{e}^{\mp \frac{t^{\mathrm{post}} - t^{\mathrm{pre}}}{\tau^{\pm}}} \ \Theta(\pm[t^{\mathrm{post}} - t^{\mathrm{pre}}]).$$

Additive rule with hard (absorbing) boundaries:

$$\epsilon^{\pm} = \begin{cases} a^{\pm} & \text{for } w_{ij}^{\min} \le w_{ij} \le w_{ij}^{\max} \\ 0 & \text{otherwise} \end{cases},$$

Multiplicative rule (soft boundaries):

$$\begin{aligned} \epsilon^{+} &= a^{+}(w^{\max} - w_{ij}) \\ \epsilon^{-} &= a^{-}(w_{ij} - w^{\min}). \end{aligned} \quad (1)$$

Hebbian learning in rate (population) models

no spike timings! $\rightarrow$ plasticity depends on correlation of pre and post synaptic spikes!

General: $\Delta w_{ij} = \epsilon(t, w)[f_{\text{post}}(r_i) f_{\text{pre}}(r_j) - f(r_i, r_j, w)]$

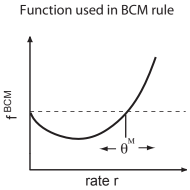
Mnemonic equation (Caianiello): f(w) is weight decay
$\Delta w_{ij} = \epsilon(w)[r_i r_j - f(w)]$

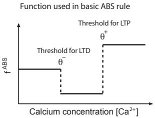
Basic Hebb:, f_{post} linear, f_{pre} linear: $\Delta w_{ij} = \epsilon r_i r_j$

$\langle r \rangle$ is average over many trials with different stimuli, if f_{post}, f_{pre} is $\langle r \rangle$
Covariance rule (plasticity threshold): $\Delta w_{ij} = \epsilon(r_i - \langle r_i \rangle)(r_j - \langle r_j \rangle)$

BCM theory, $\theta^M = f(r_j)$**, post!:** $\Delta w_{ij} = \epsilon(f^{\text{BCM}}(r_i; \theta^M)(r_j) - f(w))$

ABS rule: $\Delta w_{ij} = \epsilon(f_{\text{ABS}}(r_i; \theta^-, \theta^+)\text{sign}(r_j - \theta^{\text{pre}}))$

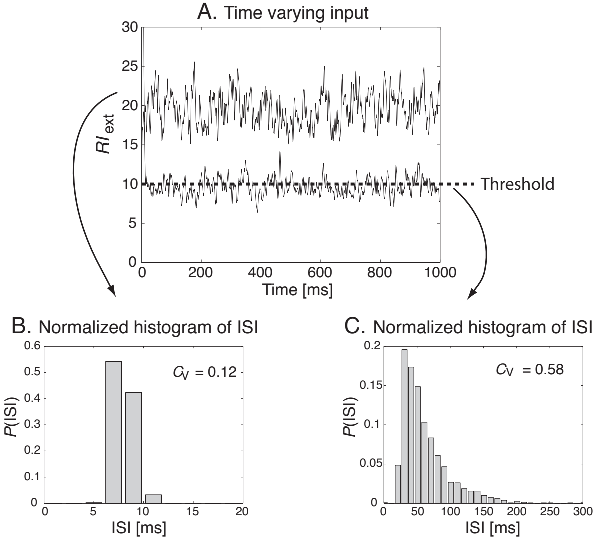




The LIF-neuron noise simulation I

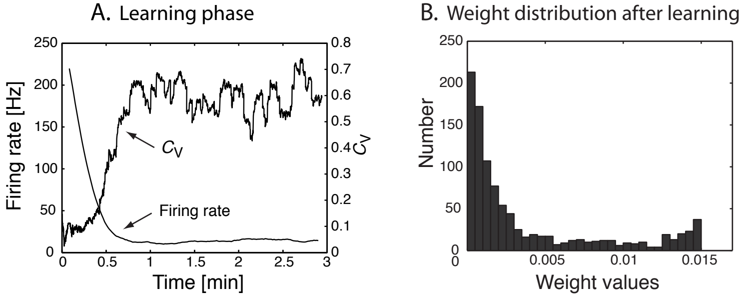
- real neuron with 5000 presynaptic neuron
- 10 % simulation $\rightarrow$ 500 Poisson-distributed spike trains (**??**) with refractory corrections
- mean firing rate = 20 Hz, after correction 19.3 Hz, refractory constant 2 ms.
- each presynaptic spike $\rightarrow$ EPSP in form of α function (**??**)
- $\omega = 0.5 \rightarrow$ regular firing, $C_V = 0.12$, average rate 118 Hz.
- $\omega = 0.25 \rightarrow$ irregular firing, $C_V = 0.58$, average rate 16 Hz. The $C_V >$ lower bound found in experiments

The LIF-neuron noise simulation II



Synaptic scaling and weight distributions

- IF neuron with 1000 excitatory synapses driven by presynaptic Poisson spike trains with average firing rate of 20 Hz, $\Delta w_{ij}^{\pm} = \epsilon^{\pm}(w)K\pm(t^{\text{post}} - t^{\text{pre}})$ applying additive rule and asymmetrical Gaussian plasticity windows
- (i) weights set to large values (ii) large frequency firing (see lec4) (iii) apply additive STDP rule with marginally stronger LTD than LTP
- increased CV, firing rate reduction, weight BINOMICAL distribution after 5 mins

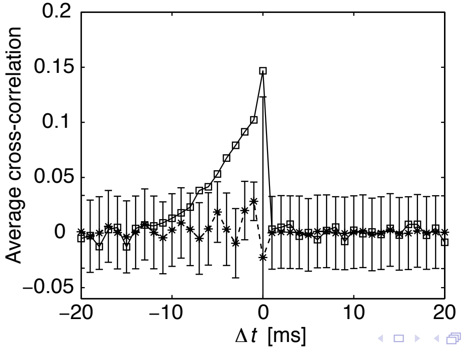


after Song, Miller and Abbott 2000

Cross-correlation function

- $s(\Delta t)$, $s = 1$ if a spike occurs in Δt
- star line:$C(n) = 0$ for regular IF firing 270 Hz, $w = 0.015$, LTP occurs as much as LTD
- square line:after Hebb's learning, IF firing 18 Hz, some presynaptic spikes elicits post-synaptic spikes
- $C < 0$, if presynaptic spikes reduce postsynaptic (anti-correlation) and vice-versa

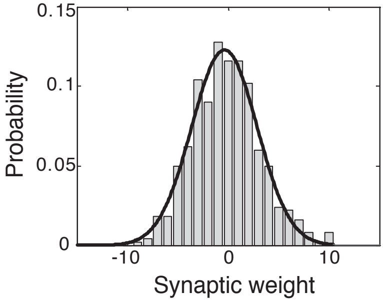
$$C(n) = \langle s^{pre}(t) s^{post}(t + n\delta t)\rangle - \langle s^{pre} s^{post}\rangle$$



Hebbian rate rules on random pattern - weight distribution ??

- Central limit theorem: sum of random variables approaches Gaussian distribution with ZERO mean and $\frac{\sigma}{N_p}$ variance (after 1000 runs).
- After learning N_p patterns, where $\epsilon = \frac{1}{N_p} \rightarrow$ the width of distribution does not change with the number of training patterns
- Rates are exponential distributed as in real case ($\langle r \rangle log(x)$) -

$$w_{ij} = \frac{1}{\sqrt{N_p}} \sum_{\mu} (r_i^{\mu} - \langle r_i \rangle)(r_j^{\mu} - \langle r_j \rangle)$$



Matlab code

- 500 hundred presynaptic nodes, 1 postsynaptic node, 1000 patterns
- matrix notation: $\Delta w = ba^{'}$. a firing rate presynaptic and b postynaptic.
- covariance Hebb's rule: w=(rPost-ar)*(rPre-ar)'

```
%% Weight distribution of Hebbian synapses in rate model
 clear; clf; %clear workspace and figure
 nn=500; npat=1000; %number of nodes and patterns
%% Random pattern; firing rates are exponential distributed
 ar=40; %average firing rate of pattern
 rPre =-ar.*log(rand(nn,npat)); %exponential distr. pre rates
 rPost=-ar.*log(rand(1,npat));  %exponential distr. post rate
%% Weight matrix
 w=(rPost-ar)*(rPre-ar)'; %Hebbian covariance rule
 w=w/sqrt(npat); %standard scaling to keep variance constant
%% Histogram plotting
 x=-10:1:10;
 [n,x]=hist(w/nn,x); %calculate histogram
 n=n/sum(n); %normalizaton to get probability distribution
 h=bar(x,n); set(h,'facecolor','none');
%% Fit normal ditribution to data
 a0=[0 5];
 a=lsqcurvefit('normal',a0,x,n);
 n2=normal(a,-15:0.1:15);
 hold on; plot(-15:0.1:15,n2,'r')
```