

Neuroinformatics — lab exercises manual

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Abstrakt

This is supportive text for labs of Neuroinformaticse course Kaunas 2014. Exercises for each task are provided in Matlab language.

We further recommend a book [?], which is nicely readable. Good summary is provided by [?]. Particular topics are covered by [?], [?] Some advance material cis cover by [?], [?].

Obsah

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0 Mathematical apparatus

We will introduce numerical apparatus, which will be applied during activity modelling of neurons.

0.1 Numerical solution of differential equations

As in other sectors dealing with dynamic systems, we will use differential equations throughout this course. Due to the fact that their exact analytical solutions can often be difficult or even impossible, we introduce in this exercise the approximate numerical solutions.

Euler's method

Let's consider first order

$$\frac{\delta x}{\delta t} = f(x, t), \quad (0.1)$$

Euler's first order metoda consist of discretization $\frac{\delta x}{\delta t} = \frac{\Delta x}{\Delta t}$. Lets' take:

$$\begin{aligned} \Delta t &= t_2 - t_1 \\ \Delta x &= x(t + \Delta t) - x(t), \end{aligned}$$

then we can express eq (0.1) as

$$\frac{\Delta x}{\Delta t} = f(x(t), t)$$

finally

$$x(t + \Delta t) = x(t) + \Delta t f(x(t), t)$$

We can approximate solution by taking into account the slope of the line at that point. However, if the slope is dependent on t, this will be a very rough approximation. In this case, we can refine the solutions by other parameters of the Taylor series according to the formula:

$$x(t + \Delta t) = x(t) + \Delta t \frac{\delta x}{\delta t} + \frac{1}{2} (\Delta t)^2 \frac{\delta^2 x}{\delta t^2} + O, \quad (0.2)$$

where O represents all members of the higher orders. In the second order therefore we model in addition the slope curvature. This approximation should be closer to analytical solutions.

Runge-Kutta metoda

In the methods of higher orders we can additionally refine the solution by estimation not in the point x and $x + \Delta x$, but in the middle of this period - the so-called "*midpoint method*". The resulting value should be more representative in a given interval and lead to a more accurate estimate. Moreover, if we do not have analytical expression of parameters of the Taylor expansion of higher orders, we can again estimate parameters of high orders using numerical methods.

The actual method of Runge-Kutta numerical integration is the 4th order, which combines estimation in the middle of the interval with a numerical estimate of higher order. It can therefore approximate solution of arbitrary functions. This leads to a higher computational cost, but - as we shall see - leads to very accurate estimates of the solution. Runge-Kutta method is implemented in Matlab function `ode45()`.

Exercise 0.1 Our task is numerical approximation of the following differential equation

$$\frac{dx}{dt} = t - x + 1, \quad (0.3)$$

initial conditions: $x(0) = 1$ using Euler's method (1 and 2 order) and Runge-Kutta methods.

The analytical solution of this equation has the form $x = t + e^{-t}$ ¹. Solve the equation in the interval $(0, 5)$, choose $\Delta t = 0.02$.

¹You can check e.g. at <http://www.wolframalpha.com/>, question: "solve differential equation x' = ..."

Task 0.1 (2 b) Plot the solution $x(t) = f(t)$ for $t = 0, \dots, 2s$ using Euler's method of first and second order ² (x_{Euler}) and plot into graph along with the analytical solution.

Task 0.2 (1 b) Solve the task using Runge-Kutta x_{Runge} and add the solution to the same graph.

Hints: function `ode45` has as first parameter a callback to the function, which describes the equation to be solved. In this case the callback can be easily defined using so called *function handle* and *anonymous function*³ as `ode_func = @(t,x,flag) 1-x+t;`

Task 0.3 (1.5 b) Plot the dependence of relative error of each numerical methods against analytical solution. For example (for Euler's method) $(x_{Euler} - x_{exact})/x_{Euler} = f(\Delta t)$

Task 0.4 (1.5 b) Plot absolute error of each numerical method taking into consideration size of integration step $\Delta t \in (0.001, 1)s$ in time $t = 1s$.

²when solving the second order Euler method you can proceed in two ways. 1) Derivating analytic function $f(x, t)$.
2) estimate value of $f'(x, t)$ using the slope $f(x, t)$ between points t and $t + \Delta t$

³see for example. http://www.mathworks.com/help/matlab/matlab_prog/creating-a-function-handle.html

1 Neuron models

In this part we will describe properties of action potential and behaviour of neurons.

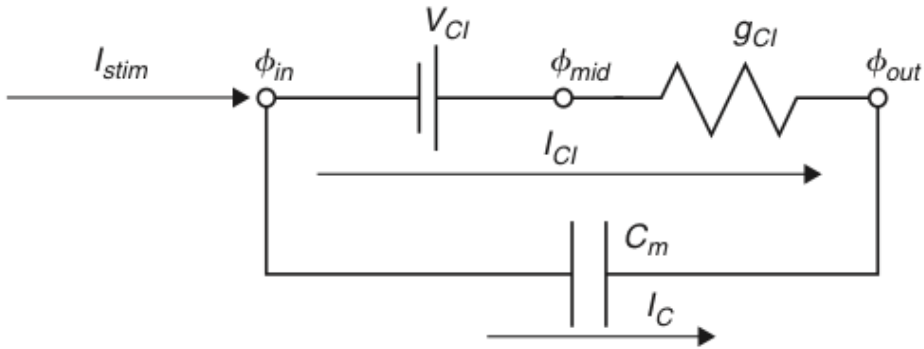
1.1 Model of membrane and synapse: simpler case

Taking into account that nerve impulses propagate through changes in electrical potential on the cell membrane, it seems natural to create the basic model as an equivalent electrical circuit. The simplest model, that we introduce in this exercise, is working only with one type of channel - leakage chloride channels (always opened) - and takes the form of an RC circuit, the resting battery, replacing the resting potential of the membrane (Nernst potential). As we shall see, the response to the input excitation is far away from the real action potential but serve us in understanding the basic concept.

Exercise 1.1 (RC model) Model membrane behavior using RC model - see figure 1. Input membrane current I_{stim} is rectangular signal $10pA$ lasting $20ms$. It is necessary to convert the stimulating current I_{Cl} and I_C to the electrode surface A^4 . The electrode is stimulating and at the same time recording the membrane current (expressed in cm^2). Hence $I'_{stim} = I_{stim}/A \approx 10^{-11} \cdot 10^6 \approx 10^{-5}$. The membrane parameters are the following:

- capacity: $C_m = 1 \mu F/cm^2$,
- conductance: $g_{Cl} = 0.3 ms/cm^2$,
- time constant : $\tau = C_m/g_{Cl}$,
- membrane surface : $A \approx 1 \cdot 10^{-6} cm^2$
- Nernst potential of Cl: $V_{Cl} = -68 mV$,
- initial conditions: $V(0) = -68 mV$, $I_{Cl}(0) = 0 \mu A/cm^2$, $I_C(0) = 0 \mu A/cm^2$.

Task 1.1 (2 b) Plot dependence $V(t)(= \phi_{in} - \phi_{out})$, $I_{Cl}(t)$, $I_C(t)$ for these time interval $(0, 40)ms$, $\Delta t = 0.01ms$



Obrázek 1: Membrane model with Cl leakage channel

Hence

$$I_C(t) = C_m \frac{dV}{dt}(t) \quad (1.1)$$

$$I_C(t) = \frac{I_{stim}(t)}{A} - I_{Cl}(t) \quad \rightarrow I_{stim} = A \cdot I_C(t) + A \cdot I_{Cl}(t) \quad (1.2)$$

$$I_{Cl}(t) = g_{Cl}(V(t) - V_{Cl}) \quad (1.3)$$

$$\tau = \frac{C_m}{g_{Cl}} \quad (1.4)$$

⁴This is current density when we should use the following convention g . However, we will stick to the terminology used during lectures I .

Combining the above equation we get

$$C_m \frac{dV}{dt} = \frac{I_{stim}(t)}{A} - g_{Cl}(V(t) - V_{Cl}) \quad (1.5)$$

$$\tau \frac{dV}{dt} = V_{Cl} - V(t) + \frac{I_{stim}(t)}{Ag_{Cl}} \quad (1.6)$$

$$(1.7)$$

Euler's method (forward)

$$\tau \frac{V(j) - V(j-1)}{dt} = V_{Cl} - V(j-1) + \frac{I_{stim}(j-1)}{Ag_{Cl}} \quad (1.8)$$

$$V(j) = V(j-1) + \frac{dt}{\tau} [V_{Cl} - V(j-1) + \frac{I_{stim}(j-1)}{Ag_{Cl}}] \quad (1.9)$$

Euler's method (backward)

$$\tau \frac{V(j) - V(j-1)}{dt} = V_{Cl} - V(j) + \frac{I_{stim}(j)}{Ag_{Cl}} \quad (1.10)$$

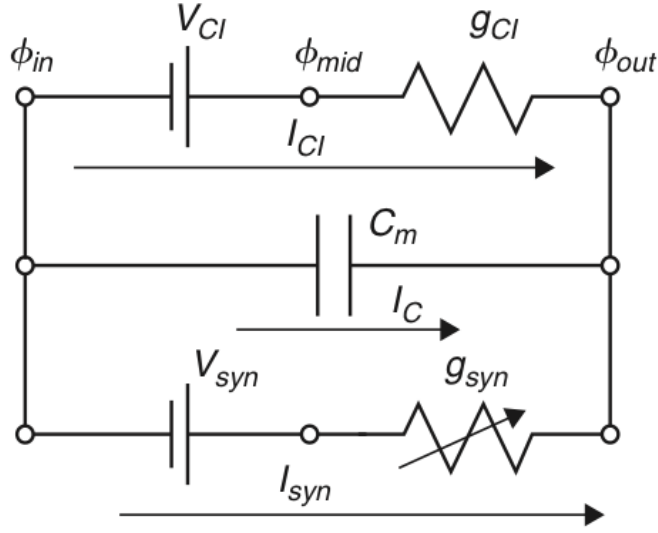
$$V(j)(1 + \frac{dt}{\tau}) = V(j-1) + dt(\frac{V_{Cl}}{\tau} + \frac{I_{stim}(j)g_{Cl}}{Ag_{Cl}C_m}) \quad (1.11)$$

$$V(j) = \frac{V(j-1) + dt(\frac{V_{Cl}}{\tau} + \frac{I_{stim}(j)}{AC_m})}{(1 + \frac{dt}{\tau})} \quad (1.12)$$

Exercise 1.2 (EPSP model) The task is analysis of a model depicted in figure 2. This model incorporates so called "excitatory postsynaptic potential"(EPSP) simulating behavior of membrane dendrite of the postsynaptic's neuron after receiving excitation. Synapse is modelled by changeable conductance g_{syn} . In other words: the model contains additionally channels, which react to release of the neurotransmitter (literally they are opening, hence their conductance is decreasing - see result of eq (1.13)) with time constant $\tau_{syn} = 1 \text{ mS}$. The parameters are the same like in the previous case. Stimulation current $I_{stim} = 0$, $V_{syn} = 10 \text{ mV}$. In time $t = 1 \text{ ms}$ the neurotransmitter is released, hence $g_{syn}(1 + \delta) = 1$. Initial conditions $V(1) = 0$, $I_{syn} = 0$, $g_{syn}(1) = 0$, $g_L = 1$.

$$\tau_{syn} \frac{dg_{syn}(t)}{dt} = -g_{syn}(t) + \delta(t - t_{pre} - t_{delay}) \quad (1.13)$$

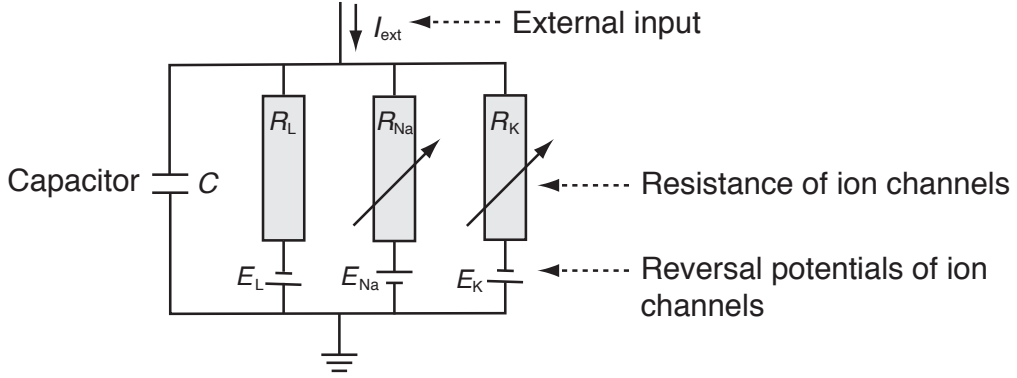
Task 1.2 (0 b) Plot dependence $V(t)$, $I_{Cl}(t)$, $I_C(t)$, $I_{syn}(t)$. Explain trends and compare them with the previous task 1.1.



Obrázek 2: EPSP: Synapse model with leakage Cl channel

1.2 Membrane and synapse modelling 2: Hodgkin-Huxley

Exercise 1.3 (Hodgkin-Huxley) The task is analysis of Hodgkin-Huxley(HH) model depicted in figure 3. Compared to the previous models, HH incorporates variables which are dependent both on time model and voltage, hence Na^+ a K^+ channels are controlled by voltage. The model is based on the following set of equations



Obrázek 3: Hodgkin-Huxley model. Each branch represents resting potential and resistivity of a particular channel: E_L, R_L - channels always opened ("Leakage channels"), E_{Na}, R_{Na} - sodium channels controlled by voltage, E_K, R_K - calcium channels controlled by voltage.

Current in ion channels can be described by Ohm's law:

$$I_{ion} = \hat{g}_{ion}(V - E_{ion}) \quad \left(= (V - E_{ion}) / \hat{R}_{ion} \right), \quad (1.14)$$

where \hat{g}_{ion} is the maximum conductance of ion channel

Next, we introduce auxiliary variables dependent on voltage and time $n(V, t)$, $m(V, t)$, $h(V, t)$, resulting in the following conductances:

$$\hat{g}_K(V, t) = g_K n^4 \quad (1.15)$$

$$g_{\hat{N}a}(V, t) = g_{Na} m^3 h \quad (1.16)$$

Combining eq.1.16-1.14 we get

$$C \frac{dV}{dt} = -g_K n^4 (V - E_K) - g_{Na} m^3 h (V - E_{Na}) - g_L (V - E_L) + I_{ext}(t) \quad (1.17)$$

Next, we define time constants

$$\tau_n(V) \frac{dn}{dt} = -[n - n_0(V)] \quad (1.18)$$

$$\tau_m(V) \frac{dm}{dt} = -[m - m_0(V)] \quad (1.19)$$

$$\tau_h(V) \frac{dh}{dt} = -[h - h_0(V)] \quad (1.20)$$

For each variable we get

$$\frac{dx}{dt} = -\frac{1}{\tau_x(V)} [x - x_0(V)], \quad x \in \{n, m, h\} \quad (1.21)$$

and after substituting Euler's numerical integration we get

$$x(t + \Delta t) = \left(1 - \frac{\Delta t}{\tau_x}\right) x(t) + \frac{\Delta t}{\tau_x} x_0. \quad (1.22)$$

Initial conditions hold:

$$x(0) = \frac{\alpha}{\alpha + \beta}, \quad \tau_x = \alpha\beta, \quad x \in \{n, m, h\} \quad (1.23)$$

$$\alpha_n = \frac{10 - V}{100 \left(e^{\frac{10-V}{10}} - 1\right)}, \quad \beta_n = 0.125e^{-\frac{V}{80}} \quad (1.24)$$

$$\alpha_m = \frac{25 - V}{10 \left(e^{\frac{25-V}{10}} - 1\right)}, \quad \beta_m = 4e^{-\frac{V}{18}} \quad (1.25)$$

$$\alpha_h = 0.07e^{\frac{V}{20}}, \quad \beta_h = \frac{1}{e^{\frac{30-V}{10}} + 1} \quad (1.26)$$

Task 1.3 (0 b) Download model implementation file `cv3_HH.m` from course website. Visualize time output of the model (membrane voltage V), which is excited by the constant current $I_{ext} = 60 \mu A/cm^2$ (default settings). Implementation works from time -30s to get steady system. Plot output for time 0 – 100ms.

Task 1.4 (1 b) Expand visualization by adding time trend of conductance g for each channel type (describe different time series using `legend`)

Task 1.5 (2 b) Visualize dependence of firing frequency on external constant current I_{ext} - so called *activation or transfer function*. Set the input current I_{ext} in the range 0 – 15 $\mu A/cm^2$. You should compute firing frequency after reaching the steady state (hint: use `findpeaks`, `find`, `diff`). Which shape does the activation function resemble? For which values of the current I_{ext} will the shape of the activation function change significantly (perform a discussion)?

Task 1.6 (2 b) Adding noise to the input current will change behaviour of the model. Visualize membrane voltage V and activation function as a response to input current with added noise. Use $I_{ext} = 30 \mu A/cm^2$ and white noise (`randn`) with the standard deviation 60 and zero mean, for plotting activation function use $I_{ext} = \langle 0, 15 \rangle \mu A/cm^2$, noise with standard deviation 30. How did the resulting output change? Try out for different noise levels.

Task 1.7 (bonus) (1 b) Plot ISI histogram for the input current $I_{ext} = 30 \mu A/cm^2$ and noise with standard deviation 60. Compare to the case with zero noise and discuss the results.

1.3 Spike train modelling

We showed in the previous tasks, how does noise on the input affect temporal and frequency characteristics of a neuron. Noise signal on neuron input is much more realistic than constant current - mainly due to the fact that neuron input is formed by a summary signal from many synapses of pre-synaptic neurons, connected to its dendrites. In this section, we will simulate such signal and use it as input to a neuron model.

You have learned in the lectures, that interspike intervals (ISI) of typical cortical neurons exhibit log-normal distribution. Before we start modelling spiketrains, let us verify this on a model.

Exercise 1.4 (LIF neuron model and Poisson spiketrain) The complex models from previous exercises, such as Hodgkin-Huxley, are able to model the action potential shape based on physiological modelling of ion channels' behavior. If we want to model more complex neural networks, even simpler models (such as Wilson model) are overly complex for such application. When modelling neural networks, the accurate shape of action potential is not as important (we will see it can be simply modelled using α functions) as the spike timing. For these purposes, the *Leaky integrate and fire* neuron is often used. The basics of this model includes simple integrator, which is reset to resting potential when a preset threshold is exceeded.

Task 1.8 (1 b) Download LIF model from the course website - `cv4_lif.m`. Show model output for constant input current (default setting). Plot output voltage, along with spike times (= reset times) over time. Compute firing rate and display an ISI histogram.

Task 1.9 (2 b) Modify previous task by applying input noise with mean 12 and standard deviation 100, instead of constant current (function `randn`). Simulate for times in the range $0 - 10^5$ and integration step $dt = 0.01$. Set bin number to 60 when computing histograms. Convert histograms to probability density function estimates ($\sum pdf = 1$).

Task 1.10 (3 b) Fit lognormal distribution to the data from the previous task. The lognormal distribution is given by:

$$pdf^{lognormal}(x; \mu, \sigma) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{(\log(x)-\mu)^2}{2\sigma^2}} \quad (1.27)$$

One possible solution would be to fit the analytical pdf to ISI histogram from the previous task. This, however, would be completely wrong, as one point in the histogram does not mean one, but many data points (think about this for a while!).

Thus, we will use the function `lognfit`, which estimates lognormal distribution parameters using maximum likelihood estimates. Based on these estimates, calculate the lognormal pdf and display along with the histogram. (You will probably have to adjust the bin number to get a good representation)

2 Real data analysis, modelling of neuronal populations

In this block, we will play with analyzing of real μEEG (*micro-EEG*) data, recorded from brains of Parkinson's disease patients. We will compare the data to our simulations from the previous block and we will analyze correlation with other biosignal - the electrooculogram (EOG).

Further in this section we will simulate more complicated network of artificial neurons and (so called *spiking network*) and analyze its behavior under varying conditions.

2.1 Spiketrain modelling II

Exercise 2.1 (Lognorm spiketrain) We showed in the last exercise how does LIF neuron model respond to a noisy input. In this task, we will show that real signal, produced as a combination of presynaptic potentials, really resembles gaussian noise in many ways.

You know from the lectures, that the *Poisson spiketrain*, - a spiketrain where the number of events per unit time follows poisson distribution - has exponential distribution of interspike intervals (ISIs). Further, the physiological constraints prevent neurons from firing earlier than after the *refractory period* - a fact we have to incorporate in our implementation.

In this example, we will generate a set of spiketrains using exponential distribution, taking refractory period into account, and mix them to create an artificial *muEEG* signal.

Task 2.1 (1 b) Generate 100 000 ISI intervals from the exponential distribution with mean $E[X] = 50ms$ (hint: use `expnrnd` function). Display ISI histogram.

Remove all ISI values lower than the refractory period of 1ms. Display ISI histogram and compare to the one before refractory period filtering. How many spikes were removed? Compute mean firing rate and coefficient of variation before and after filtering and compare.

Convert the resulting ISI intervals to a spiketrain (logical vector with ones at spike positions hint: `cumsum`). Use time step of $dt = 0.1ms$. Check for errors by plotting the result

Task 2.2 (1 b) To convert binary spiketrain to time course, we will use convolution with α -function, which will play the role of spike template.

Generate time course of the α -function according to:

$$I(t) = c \cdot t \cdot e^{-\frac{t}{t_{peak}}} \quad (2.1)$$

$$c = \frac{g_{peak}}{t_{peak}} e^{-1} \quad (2.2)$$

where $g_{peak} = 0.5$, $t_{peak} = 1ms$. Compute the function values for times in the interval $t \in \langle 0, 30 \rangle$, the time step should be again $dt = 0.1ms$. Plot the output. After assessing the result, you may reduce the time range if appropriate.

Convolve the alpha function with Poisson spiketrain from the previous task (`conv`). Check the result again by plotting.

Task 2.3 (1 b) Divide the time course from previous task into 1000 signals of the same length (`reshape`). In this task, we will simulate mixing of presynaptic potentials (represented by the simulated neurons), at dendrites. The resulting summary signal will be given by a linear combination of the individual spiketrains. Coefficients will be given by weights w , which will be modelled as random from exponential distribution with $\lambda = 1$.

In order to have better control of the amplitude of the resulting signal, we will normalize the weights so that their sum equals a constant k . Set the value $k = 100$.

Combine the 1000 spiketrains into one signal, using a linear combination with weights w . You can represent the linear combination as matrix multiplication (elegant + much faster in Matlab).

Compare the resulting signal to gaussian noise (time and distribution). What are the key differences?

Task 2.4 (1 b) Apply the signal from previous task to the input of LIF neuron from Exercise and observe the output. What changes do you observe when you change the normalizing constant $k = 150$. What does the ISI histogram look like, now? Investigate and comment behavior of the LIF neuron.

2.1.1 Artificial and real signal - comparison

We have now generated artificial signal as a combination of Poisson spiketrains, convoluted with alpha function. In these tasks, we will compare the simulated signal to real μEEG , recorded from basal ganglia of a Parkinson's disease patient during deep brain surgery. We will show, that this simple method of data generation is not far from reality.

Task 2.5 (1 b) Use artificial signal generated in the previous task. It is a time course with sampling rate $f_s = 10kHz$. Download μEEG data from the course website. The .mat file contains variable `realmEEG` with 1s recording of activity around Thalamus ($f_s = 24kHz$). Display both signals in parallel plots (with correct time coordinates). Investigate both signals.

Task 2.6 (1 b) Z porovnání pouhým okem máme hrubou představu, zda se signály podobají. Nyní je na čase vyzkoušet některé kvantitativní nástroje. Vyberte a implementujte vhodný způsob porovnání obou signálů (filtrovaného a reálného). Pro inspiraci můžete použít např. frekvenční spektrum, rozložení amplitud apod.