## 1 Lecture

### 1.1 Foundations

Given a set of characters $A$, a sequence is a function from $\{1, \ldots, \mathrm{n}\}$ to $A$ (e.g. $\langle a, b, c\rangle=\{(1, a),(2, b),(3, c)\})$. A set of all sequences over $A$ is denoted $A^{*}$.

### 1.2 Featherweight Java

- Notable features: Turing completeness, evaluation order, inheritance.
- Key theorems:
- FJ programs preserve their types.
- The only FJ programs that may get stuck are the ones with an invalid downcast in them.
- FJ programs not containing invalid downcasts are sound.


## 2 Seminar

1. Implement method Boolean. and (Boolean that).
2. Consider function value: Expression $\rightarrow$ Bool defined as

$$
\begin{align*}
\text { value }(\text { new } \operatorname{True}()) & =\text { true }  \tag{1}\\
\text { value }(\text { new } \operatorname{False}()) & =\text { false } . \tag{2}
\end{align*}
$$

Prove that

$$
\begin{equation*}
\frac{\operatorname{value}(e)=v \quad \operatorname{value}\left(e^{\prime}\right)=v^{\prime}}{\exists\left(e^{\prime \prime} \in \text { Expression }: e . \text { and }\left(e^{\prime}\right) \Rightarrow^{*} e^{\prime \prime} \wedge \operatorname{value}\left(e^{\prime \prime}\right) \Leftrightarrow v \wedge v^{\prime}\right)} . \tag{3}
\end{equation*}
$$

3. Implement method Number.minus (Number that) $(0-x$ is defined as 0$)$.
4. Prove correctness of your implementation of Number.minus. Hint: define function value : Expression $\rightarrow N$ and choose carefully what proposition needs to be proved.
5. Use lists to represent numbers in binary format. Implement the following operations: halving, increment and addition.
