

## 1 Lecture

- $\Gamma \vdash \diamond$  (“ $\Gamma$  is a well-formed environment”)
- $\Gamma \vdash A$  (“ $A$  is a well-formed type in environment  $\Gamma$ ”)
- $\Gamma \vdash e : A$  (“ $e$  is a well-formed term of type  $A$  in environment  $\Gamma$ ”)
- Basic setup

$$\overline{\emptyset \vdash \diamond} \quad (1)$$

$$\frac{\Gamma \vdash A \quad a \notin \text{dom}(\Gamma)}{\Gamma \cup \{(a, A)\} \vdash \diamond} \quad (2)$$

$$\frac{\Gamma \vdash \diamond}{\Gamma \vdash x : \Gamma(x)} \quad (3)$$

$$\frac{\Gamma : \diamond \quad A \in \text{Basic}}{\Gamma \vdash A} \quad (4)$$

- Function type

$$\frac{\Gamma \vdash A \quad \Gamma \vdash B}{\Gamma \vdash A \rightarrow B} \quad (5)$$

$$\frac{\Gamma \cup \{(x, A)\} \vdash e : B}{\Gamma \vdash (\lambda x : A. e) : A \rightarrow B} \quad (6)$$

$$\frac{\Gamma \vdash e : A \rightarrow B \quad \Gamma \vdash p : A}{\Gamma \vdash e(p) : B} \quad (7)$$

- Product type

$$\frac{\Gamma \vdash A_1 \quad \Gamma \vdash A_2}{\Gamma \vdash A_1 \times A_2} \quad (8)$$

$$\frac{\Gamma \vdash e_1 : A_1 \quad \Gamma \vdash e_2 : A_2}{\Gamma \vdash (e_1, e_2) : A_1 \times A_2} \quad (9)$$

$$\frac{\Gamma \vdash e : A_1 \times A_2}{\Gamma \vdash \mathbf{first} \ e : A_1} \quad (10)$$

$$\frac{\Gamma \vdash e : A_1 \times A_2}{\Gamma \vdash \mathbf{second} \ e : A_2} \quad (11)$$

- Union type

$$\frac{\Gamma \vdash A_1 \quad \Gamma \vdash A_2}{\Gamma \vdash A_1 + A_2} \quad (12)$$

$$\frac{\Gamma \vdash e : A_1 \quad \Gamma \vdash A_2}{\Gamma \vdash \mathbf{inLeft}_{A_2} e : A_1 + A_2} \quad (13)$$

$$\frac{\Gamma \vdash A_1 \quad \Gamma \vdash e : A_2}{\Gamma \vdash \mathbf{inRight}_{A_1} e : A_1 + A_2} \quad (14)$$

$$\frac{\Gamma \vdash e : A_1 + A_2}{\Gamma \vdash \mathbf{isLeft} e : \mathit{Boolean}} \quad (15)$$

$$\frac{\Gamma \vdash e : A_1 + A_2}{\Gamma \vdash \mathbf{isRight} e : \mathit{Boolean}} \quad (16)$$

$$\frac{\Gamma \vdash e : A_1 + A_2}{\Gamma \vdash \mathbf{asLeft} e : A_1} \quad (17)$$

$$\frac{\Gamma \vdash e : A_1 + A_2}{\Gamma \vdash \mathbf{asRight} e : A_2} \quad (18)$$

- Record type

$$\frac{\Gamma \vdash A_1 \quad \dots \quad \Gamma \vdash A_n}{\Gamma \vdash \mathbf{Record}(l_1 : A_1, \dots, l_n : A_n)} \quad (19)$$

$$\frac{\Gamma \vdash e_1 : A_1 \quad \dots \quad \Gamma \vdash e_n : A_n}{\Gamma \vdash \mathbf{record}(l_1=e_1, \dots, l_n=e_n) : \mathbf{Record}(l_1 : A_1, \dots, l_n : A_n)} \quad (20)$$

$$\frac{\Gamma \vdash e : \mathbf{Record}(l_1 : A_1, \dots, l_j : A_j, \dots, l_n : A_n)}{e.l_j : A_j} \quad (21)$$

- Reference type

$$\frac{\Gamma \vdash A}{\Gamma \vdash \mathbf{Ref} A} \quad (22)$$

$$\frac{\Gamma \vdash e : A}{\Gamma \vdash \mathbf{ref} e : \mathbf{Ref} A} \quad (23)$$

$$\frac{\Gamma \vdash e : \mathbf{Ref} A}{\Gamma \vdash \mathbf{deref} e : A} \quad (24)$$

$$\frac{\Gamma \vdash e : \mathbf{Ref} A \quad \Gamma \vdash e' : A}{\Gamma \vdash e = e' : \diamond} \quad (25)$$

## 2 Seminar

Define an API that allows you to represent the above-defined types and their operations.