1 Lecture

1.1 Syntax

$$
\begin{aligned}
Expr ::= {} & Num \mid \\
& Bool \mid \\
& \triangle Expr \mid \\
& Expr \odot Expr \mid \\
& Expr \leq Expr \mid \\
& Expr \texttt{ nand } Expr \mid \\
& \texttt{if } Expr \texttt{ then } Expr \texttt{ else } Expr \mid \\
& VarName \\
Type ::= {} & \texttt{Number} \mid \texttt{Boolean} \\
Cmd ::= {} & Type\ VarName \mid \\
& VarName \texttt{ = } Expr \\
Program ::= {} & Cmd\texttt{;}\ Program \mid \\
& Cmd,
\end{aligned}
\tag{1}
$$

where Num is a predefined set of integer numbers (a.k.a. Z), $Bool$ is a predefined set of boolean values and $VarName$ is a predefined set of variable names.

1.2 Operational Semantics

$$\frac{}{(s, n) \Rightarrow n} \tag{2}$$

$$\frac{}{(s, b) \Rightarrow b} \tag{3}$$

$$\frac{(s, e) \Rightarrow n}{(s, \triangle e) \Rightarrow -n} \tag{4}$$

$$\frac{(s, e) \Rightarrow n \quad (s, e') \Rightarrow n'}{(s, e \odot e') \Rightarrow n + n'} \tag{5}$$

$$\vdots$$

$$\frac{}{(s, x) \Rightarrow s(x)} \tag{6}$$

$$\frac{}{(s, t\ x) \Longrightarrow s} \tag{7}$$

$$\frac{(s, e) \Rightarrow v}{(s, x \texttt{ = } e) \Longrightarrow s[x \mapsto v]} \tag{8}$$

$$\frac{(s, c) \Longrightarrow s' \quad (s', p) \Longrightarrow s''}{(s, c\texttt{;}\ p) \Longrightarrow s''} \tag{9}$$

1.3 Typing

Convention: $e, e', e'', \ldots \in Expr$, $b \in Bool$, $n \in Num$, $v \in VarName$, $p \in Program$ and $t \in \{Number, Boolean\}$.

$$\frac{}{\Gamma \vdash n : Number} \tag{10}$$

$$\frac{}{\Gamma \vdash b : Boolean} \tag{11}$$

$$\frac{\Gamma \vdash e : Number}{\Gamma \vdash \triangle e : Number} \tag{12}$$

$$\frac{\Gamma \vdash e : Number \quad \Gamma \vdash e' : Number}{\Gamma \vdash e \odot e' : Number} \tag{13}$$

$$\frac{\Gamma \vdash e : Number \quad \Gamma \vdash e' : Number}{e \leq e' : Boolean} \tag{14}$$

$$\frac{\Gamma \vdash e : Boolean \quad \Gamma \vdash e' : Boolean}{e \texttt{ nand } e' : Boolean} \tag{15}$$

$$\frac{\Gamma \vdash e : Boolean \quad \Gamma \vdash e' : t \quad \Gamma \vdash e'' : t}{\Gamma \vdash \texttt{if } e \texttt{ then } e' \texttt{ else } e'' : t} \tag{16}$$

$$\frac{}{\Gamma \vdash v : \Gamma(v)} \tag{17}$$

$$\frac{\Gamma \vdash v : t \quad \Gamma \vdash e : t}{\Gamma \vdash v \texttt{ = } e : \diamond} \tag{18}$$

$$\frac{v \notin dom(\Gamma)}{\Gamma \vdash t\ v : \diamond} \tag{19}$$

$$\frac{\Gamma \vdash v \texttt{ = } e : \diamond \quad \Gamma \vdash p : \diamond}{\Gamma \vdash v \texttt{ = } e\texttt{;}\ p : \diamond} \tag{20}$$

$$\frac{\Gamma \vdash t\ v : \diamond \quad \Gamma \cup \{(v, t)\} \vdash p : \diamond}{\Gamma \vdash t\ v\texttt{;}\ p : \diamond} \tag{21}$$