

1 Lecture

1.1 Syntax

$$\begin{aligned}
 \text{Expr} ::= & \text{Num} \mid \\
 & \text{Bool} \mid \\
 & \Delta \text{Expr} \mid \\
 & \text{Expr} \odot \text{Expr} \mid \\
 & \text{Expr} \leq \text{Expr} \mid \\
 & \text{Expr} \text{ nand } \text{Expr} \mid \\
 & \text{if } \text{Expr} \text{ then } \text{Expr} \text{ else } \text{Expr} \mid \\
 & \text{VarName} \\
 \text{Type} ::= & \text{Number} \mid \text{Boolean} \\
 \text{Cmd} ::= & \text{Type } \text{VarName} \mid \\
 & \text{VarName} = \text{Expr} \\
 \text{Program} ::= & \text{Cmd}; \text{Program} \mid \\
 & \text{Cmd},
 \end{aligned} \tag{1}$$

where Num is a predefined set of integer numbers (a.k.a. Z), Bool is a predefined set of boolean values and VarName is a predefined set of variable names.

1.2 Operational Semantics

$$\overline{(s, n) \Rightarrow n} \tag{2}$$

$$\overline{(s, b) \Rightarrow b} \tag{3}$$

$$\frac{(s, e) \Rightarrow n}{(s, \Delta e) \Rightarrow -n} \tag{4}$$

$$\frac{(s, e) \Rightarrow n \quad (s, e') \Rightarrow n'}{(s, e \odot e') \Rightarrow n + n'} \tag{5}$$

⋮

$$\overline{(s, x) \Rightarrow s(x)} \tag{6}$$

$$\overline{(s, t \ x) \Longrightarrow s} \tag{7}$$

$$\frac{(s, e) \Rightarrow v}{(s, x = e) \Longrightarrow s[x \mapsto v]} \tag{8}$$

$$\frac{(s, c) \Longrightarrow s' \quad (s', p) \Longrightarrow s''}{(s, c; p) \Longrightarrow s''} \tag{9}$$

1.3 Typing

Convention: $e, e', e'', \dots \in Expr$, $b \in Bool$, $n \in Num$, $v \in VarName$, $p \in Program$ and $t \in \{Number, Boolean\}$.

$$\frac{}{\Gamma \vdash n : Number} \quad (10)$$

$$\frac{}{\Gamma \vdash b : Boolean} \quad (11)$$

$$\frac{\Gamma \vdash e : Number}{\Gamma \vdash \Delta e : Number} \quad (12)$$

$$\frac{\Gamma \vdash e : Number \quad \Gamma \vdash e' : Number}{\Gamma \vdash e \odot e' : Number} \quad (13)$$

$$\frac{\Gamma \vdash e : Number \quad \Gamma \vdash e' : Number}{e \leq e' : Boolean} \quad (14)$$

$$\frac{\Gamma \vdash e : Boolean \quad \Gamma \vdash e' : Boolean}{e \mathbf{nand} e' : Boolean} \quad (15)$$

$$\frac{\Gamma \vdash e : Boolean \quad \Gamma \vdash e' : t \quad \Gamma \vdash e'' : t}{\Gamma \vdash \mathbf{if} \ e \ \mathbf{then} \ e' \ \mathbf{else} \ e'' : t} \quad (16)$$

$$\frac{}{\Gamma \vdash v : \Gamma(v)} \quad (17)$$

$$\frac{\Gamma \vdash v : t \quad \Gamma \vdash e : t}{\Gamma \vdash v = e : \diamond} \quad (18)$$

$$\frac{v \notin dom(\Gamma)}{\Gamma \vdash t v : \diamond} \quad (19)$$

$$\frac{\Gamma \vdash v = e : \diamond \quad \Gamma \vdash p : \diamond}{\Gamma \vdash v = e; p : \diamond} \quad (20)$$

$$\frac{\Gamma \vdash t v : \diamond \quad \Gamma \cup \{(v, t)\} \vdash p : \diamond}{\Gamma \vdash t v; p : \diamond} \quad (21)$$