# Homework assignment

Use the template source file (`hw1.wl` in your repository), implement the semantics described bellow.

Grammar rules:

$$\begin{aligned} Expr ::= & \; Num \mid \\ & \texttt{X} \mid \\ & Expr \oplus Expr \mid \\ & Expr \odot Expr \mid \\ & Expr' \mid \\ & Expr^n \end{aligned} \tag{1}$$

Convention: $n, m \in Num$, $e, f, g \in Expr$ and $\cdot, +$ are standard operators on numbers.

$$\frac{}{n' \Rightarrow 0} \tag{2}$$

$$\frac{}{\texttt{X}' \Rightarrow 1} \tag{3}$$

$$\frac{}{(e \oplus f)' \Rightarrow e' \oplus f'} \tag{4}$$

$$\frac{}{(e \odot f)' \Rightarrow (e \odot f') \oplus (e' \odot f)} \tag{5}$$

$$\frac{e \Rightarrow g}{e \oplus f \Rightarrow g \oplus f} \tag{6}$$

$$\frac{f \Rightarrow g}{e \oplus f \Rightarrow e \oplus g} \tag{7}$$

$$\frac{e \Rightarrow g}{e \odot f \Rightarrow g \odot f} \tag{8}$$

$$\frac{f \Rightarrow g}{e \odot f \Rightarrow e \odot g} \tag{9}$$

$$\frac{}{(Expr^n)' \Rightarrow Expr' \odot (n \odot Expr^{n-1})} \tag{10}$$

Neutral element rules:

$$\frac{}{0 \oplus e \Rightarrow e} \tag{11}$$

$$\frac{}{e \oplus 0 \Rightarrow e} \tag{12}$$

$$\frac{}{1 \odot e \Rightarrow e} \tag{13}$$

$$\frac{}{e \odot 1 \Rightarrow e} \tag{14}$$

Absorbing element rules:

$$\frac{}{0 \odot e \Rightarrow 0} \tag{15}$$

$$\frac{}{e \odot 0 \Rightarrow 0} \tag{16}$$

Evaluation rules:

$$\frac{}{n \oplus m \Rightarrow n + m} \tag{17}$$

$$\frac{}{n \odot m \Rightarrow n \cdot m} \tag{18}$$

$$\frac{}{e^0 \Rightarrow 1} \tag{19}$$

$$\frac{}{0^n \Rightarrow 0} \quad \text{if } n > 0 \tag{20}$$

# Naming convention

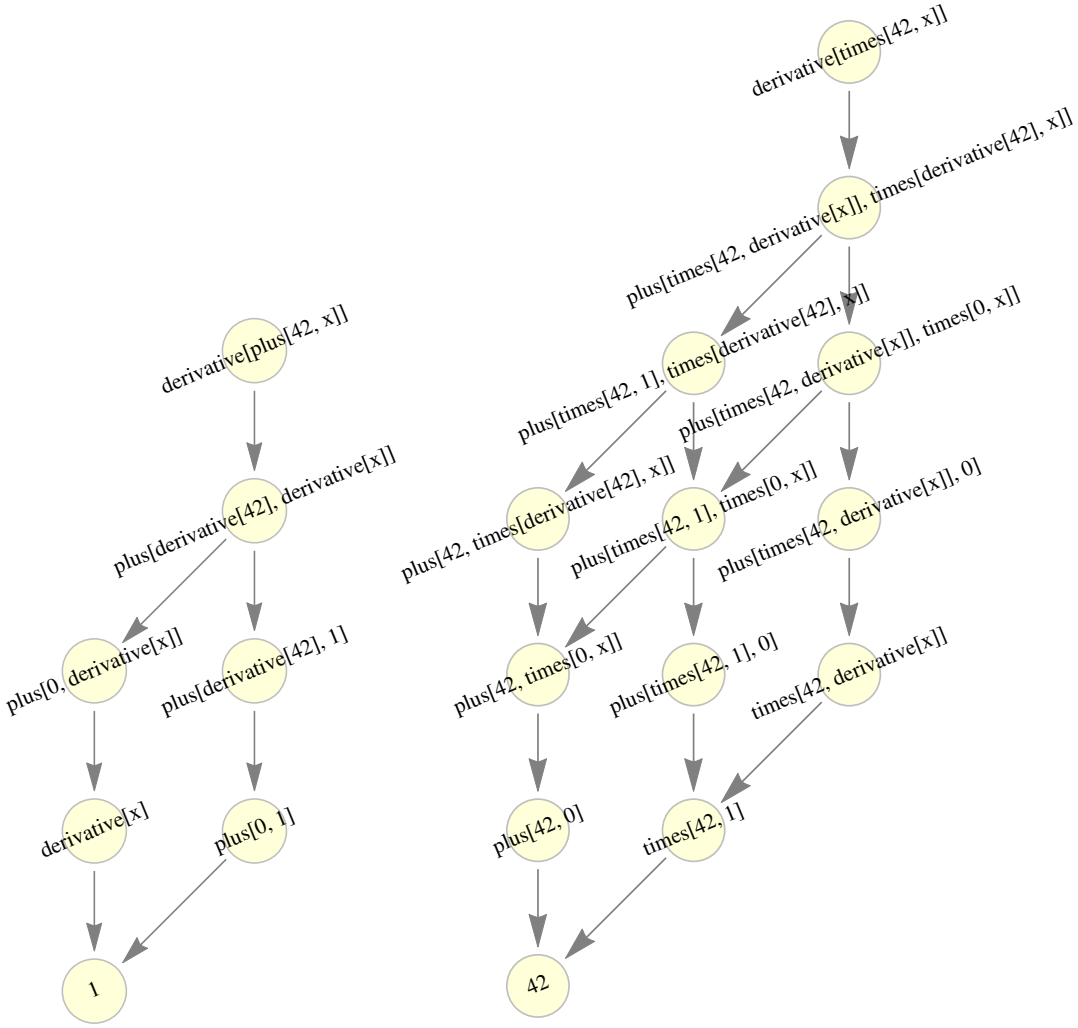
Use the following names and symbols for expressions (take care about the case sensitivity in Wolfram *Mathematica*):

- $Num$ – `_Integer`
- `X` – `x`
- $Expr \oplus Expr$ – `plus`
- $Expr \odot Expr$ – `times`
- $Expr^n$ – `pow`
- $Expr'$ – `derivative`

Function for expression rewrite (one application of SOS rules) is called `oneStepRewrite`.

## Examples

- $x'$ – `derivative[x]`
- $(42 + x)'$ – `derivative[plus[42,x]]`
- $(42 \cdot x)'$ – `derivative[times[42,x]]`
- $(x^{42})'$ – `derivative[pow[x,42]]`
- `oneStepRewrite[derivative[x]] = {1}`
- `oneStepRewrite[derivative[plus[42,x]]] = {plus[derivative[42], derivative[x]]}`




(a) `derivative[plus[42, x]]` (b) `derivative[times[42, x]]`

Figure 1: Example expressions evaluation, each arrow represents one call of `oneStepRewrite` function.