

# Operational Semantics

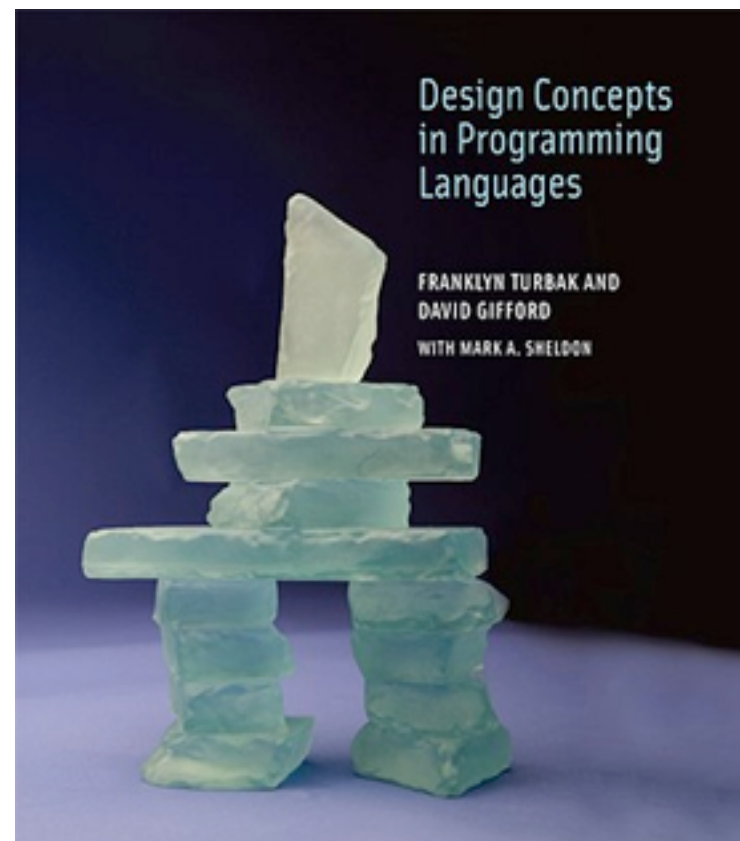
A4M36TPJ, 2013/2014

# Overview

- Introduction
- Small-step Operational Semantics (SOS)
- Big-step Operational Semantics (BOS)

# References

- Franklyn Turbak, David Gifford, Mark A. Sheldon:  
*Design Concepts in Programming Languages*,  
The MIT Press, 2008, ISBN: 978-0262201759



- <https://edux.feld.cvut.cz/courses/A4M36TPJ>

# Programming Languages

1. Syntax — the *form*...
  2. Semantics — the *meaning*...
  3. Pragmatics — the *implementation*...
- ...of programming languages

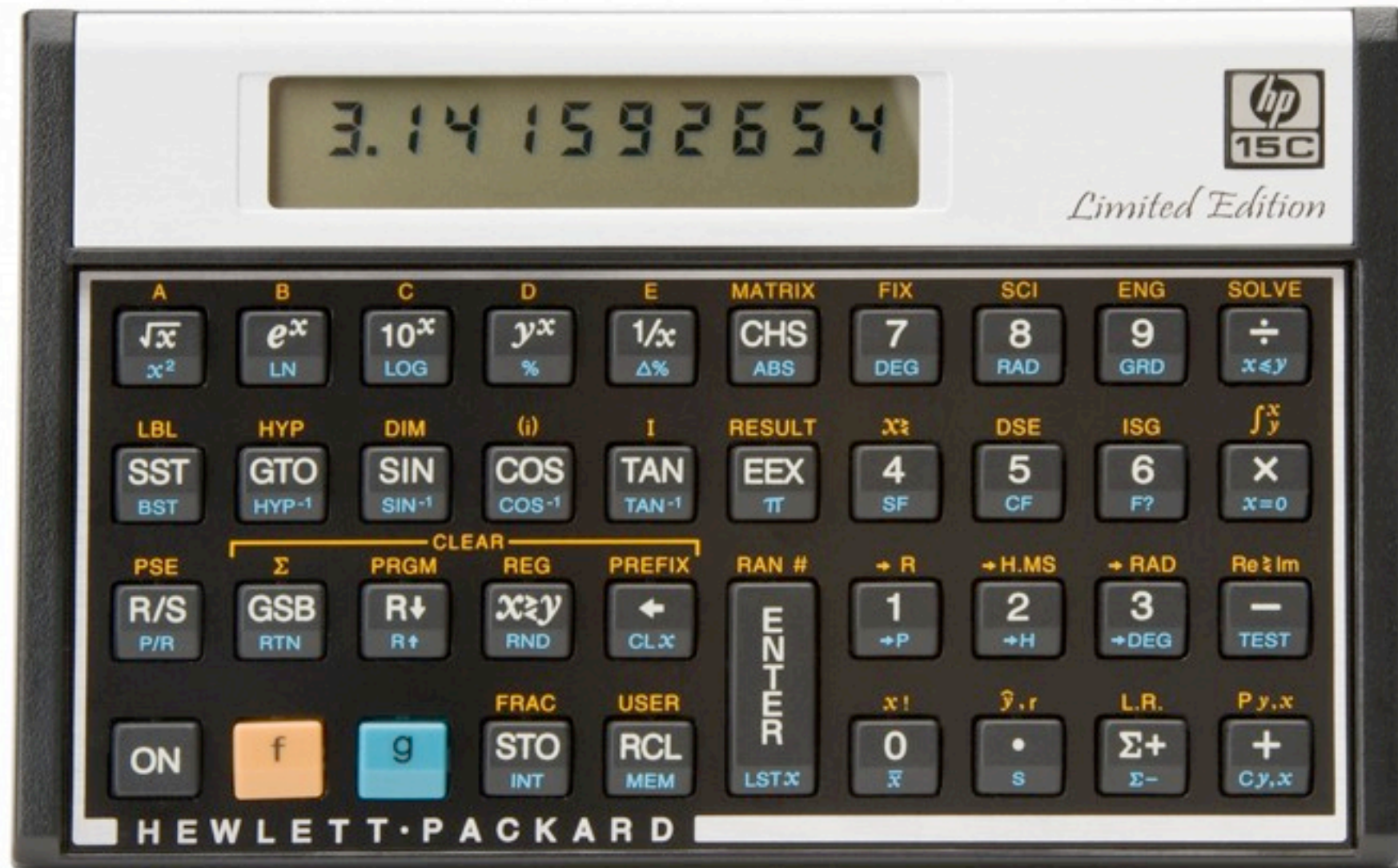
# Abstract Machine

1. Code component
2. State component

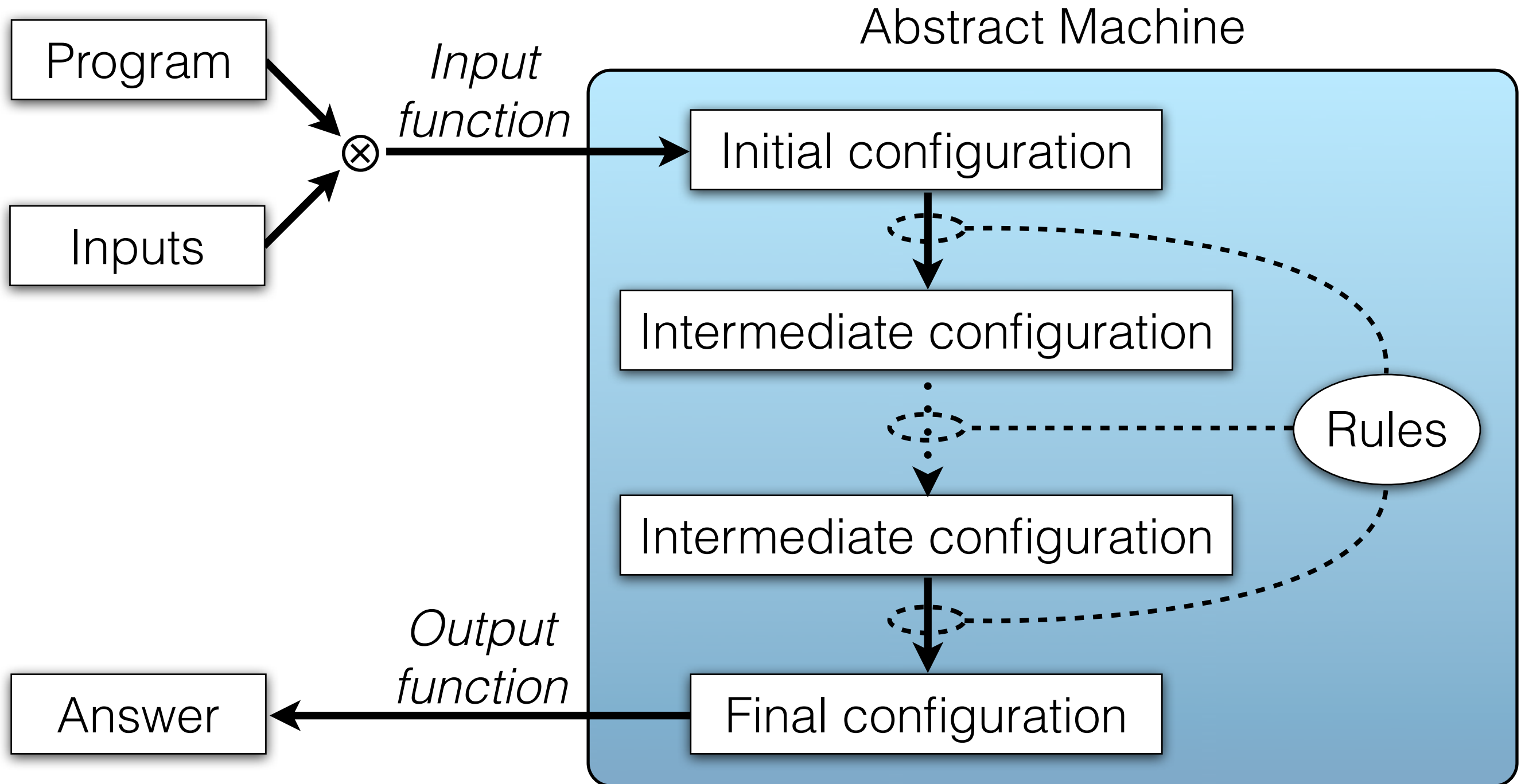
# Operational Execution

- Program  $\otimes$  Inputs  $\rightarrow$  Input function
- Initial configuration
- (*Intermediate configurations...*)
- Final configuration
- Output function  $\rightarrow$  Answer

# Configuration



# The OS “game board”





# Example Language - Syntax

$$\begin{aligned} \textit{Expr} ::= & \textit{Num} \mid \\ & \Delta \textit{Expr} \mid \\ & \textit{Expr} \odot \textit{Expr} \end{aligned}$$

Num is a predefined set of integer numbers (a.k.a.  $\mathbb{Z}$ ).

# Expressions

$$\begin{aligned} Expr = \{ \dots, 0, 1, 2, \dots, \Delta 0, \Delta 1, \Delta 2, \dots, 0 \odot 0, 0 \odot 1, 0 \odot 2, \dots, (\Delta 0) \odot 0, \\ (\Delta 0) \odot 1, (\Delta 0) \odot 2, 0 \odot (\Delta 0), 0 \odot (\Delta 1), 0 \odot (\Delta 2), \Delta(0 \odot 0), \Delta(0 \odot 1), \\ \Delta(0 \odot 2), \dots \} \end{aligned}$$

# Small-step Operational Semantics

$n, n' \in Num$

$$\frac{\overline{\Delta n \Rightarrow -n}}{n \odot n' \Rightarrow n + n'}$$

$e, e_1, e_2, e' \in Expr$

$$\frac{e \Rightarrow e'}{\Delta e \Rightarrow \Delta e'}$$
$$\frac{e_1 \Rightarrow e'}{e_1 \odot e_2 \Rightarrow e' \odot e_2}$$
$$\frac{e_2 \Rightarrow e'}{e_1 \odot e_2 \Rightarrow e_1 \odot e'}$$