We assume the existence of the set of all locations loc. Consequently, we may construct the set of all possible links:

$$link = loc \times loc \times N \times N$$

The first component of each element of $link$ represents the start location of the link, the second component represents the destination, the third represents departure time and the last component represents duration of the link. Your task is to implement function $conns : loc \times loc \times N \times \mathcal{P}(link) \to \mathcal{P}(link^*)$ that adheres to the following specification:

$$\begin{aligned} conns(s, d, dep, links) = \{&c \in link^* : connValid\ c, links \wedge connStart(c) = s \wedge \\ &connDest(c) = d \wedge connDep(c) \geq dep \wedge \\ &\neg \exists c' \in conns(s, d, dep, links) : obsoletes\ c', c\} \end{aligned}$$

Implementation

$$\begin{aligned} direct &: loc \times loc \times N \times \mathcal{P}(link) \to \mathcal{P}(link) \\ direct(s, d, dep, links) &= \{l \in links : start(l) = s \wedge dest(l) = d \wedge dep(l) \geq dep\} \end{aligned}$$

$$\begin{aligned} allConns &: loc \times loc \times N \times \mathcal{P}(link) \to \mathcal{P}(link^*) \\ allConns(s, d, dep, links) &= direct(s, d, dep, links) \cup \\ &\{\langle l \rangle \cdot c : l \in links \wedge start(l) = s \wedge dep(l) \geq dep \wedge \\ &c \in allConns(dest(l), d, arr(l), links)\} \end{aligned}$$

$$\begin{aligned} conns'(s, d, dep, links) = \{&c \in link^* : c \in allConns(s, d, dep, links) \wedge \\ &\neg \exists c' \in conns(s, d, dep, links) : obsoletes\ c', c\} \end{aligned}$$

Auxiliaries

$$\begin{aligned} connStart &: link^* \to loc \\ connStart(\langle l_1, \ldots, l_n \rangle) &= start(l_1) \\ connDest &: link^* \to loc \\ connDest(\langle l_1, \ldots, l_n \rangle) &= dest(l_n) \\ connDep &: link^* \to N \\ connDep(\langle l_1, \ldots, l_n \rangle) &= dep(l_1) \\ connArr &: link^* \to N \\ connArr(\langle l_1, \ldots, l_n \rangle) &= arr(l_n) \end{aligned}$$

$$\begin{aligned} connValid\ \langle l_1, \ldots, l_n \rangle, links \equiv\ &\forall i \in \{1, \ldots, n\} : l_i \in links \wedge \\ &\forall i \in \{1, \ldots, n-1\} : dest(l_i) = start(l_{i+1}) \wedge \\ &arr(l_i) \leq dep(l_{i+1}) \end{aligned}$$

$$obsoletes\ c, d \equiv connDep(c) > connDep(d) \wedge connArr(c) < connArr(d)$$

Accessors

$$\begin{aligned}
start &: link \to loc \\
start(l) &= \pi_1(l) \\
dest &: link \to loc \\
dest(l) &= \pi_2(l) \\
dep &: link \to N \\
dep(l) &= \pi_3(l) \\
arr &: link \to N \\
arr(l) &= \pi_3(l) + \pi_4(l)
\end{aligned}$$