Two-player Games

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Game-tree Search / Adversarial Search

- planning – only the searching player acts in the environment
- there could be others:
  - Nature – stochastic environment (MDP, POMDP, …)
  - other agents – rational opponents

- Game Theory
  - mathematical framework that describes optimal behavior of rational self-interested agents
  - A4M36MAS (Multi-agent Systems)

Game-tree Search / Adversarial Search

- What are the basic games categories?
  - perfect / imperfect information
  - deterministic / stochastic
  - zero-sum / general-sum
  - finite / infinite
  - two-player / n-player
  - …

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- What is the goal?

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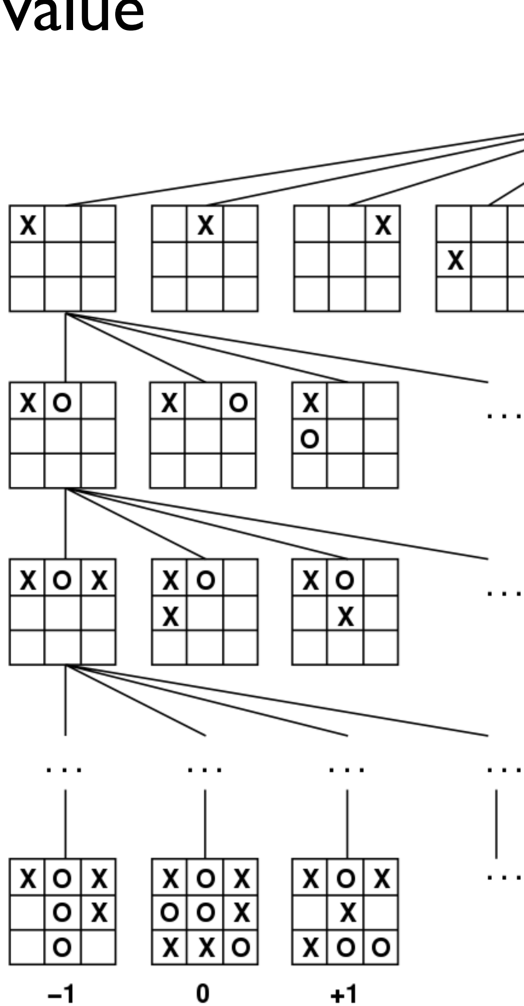
- What is the goal?
  - finding an optimal **strategy** (i.e., selecting an action to play in each possible situation)

Game-tree Search / Adversarial Search

- Players are rational – each player wants to maximize her/his utility value



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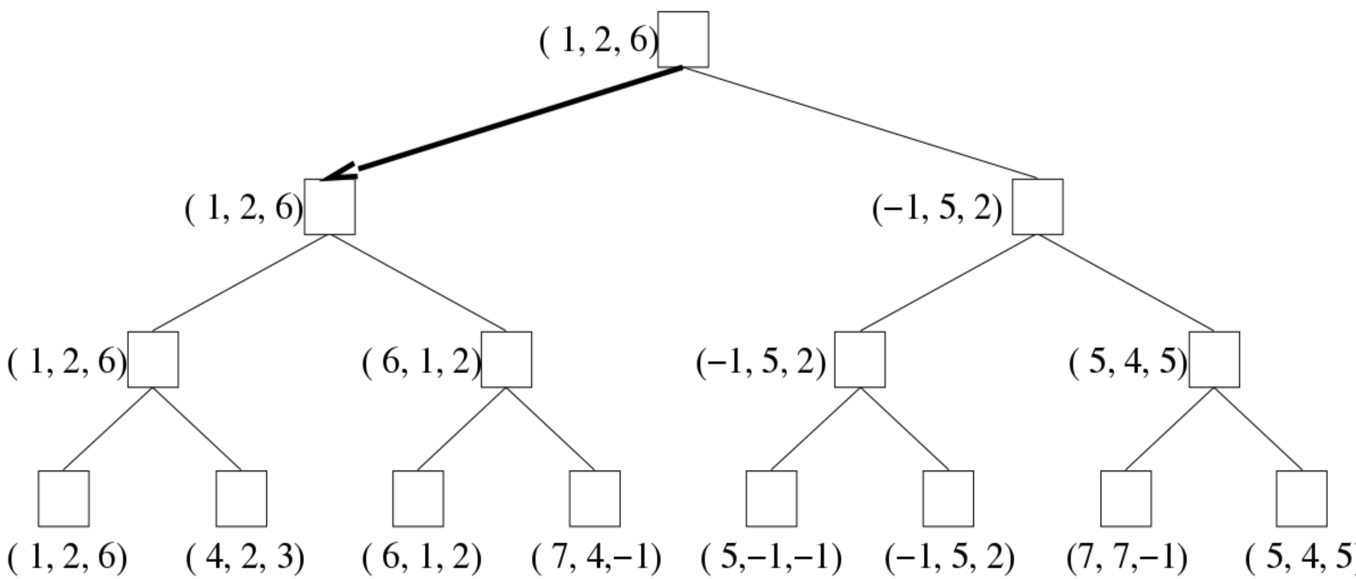

Minimax

- **function** minimax(node, Player)
- **if** (node is a terminal node) **return** utility value of node
- **if** (Player = MaxPlayer)
- **for each** child of node
- v ← max(v, minimax(child, switch(Player)))
- **return** v
- **else**
- **for each** child of node
- v ← min(v, minimax(child, switch(Player)))
- **return** v

Minimax in Real Games

- search space in games is typically **very large**
  - exponential in branching factor b^d
    - e.g., 35 in chess, up to 360 in Go, up to 45000 in Arimaa
- we have to limit the depth of the search
- we need an evaluation function

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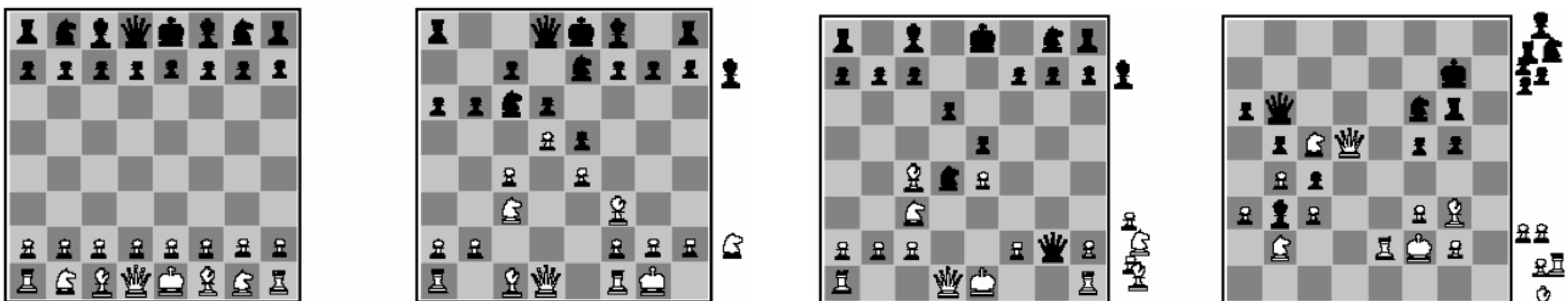

(a) White to move
Fairly even

(b) Black to move
White slightly better

(c) White to move
Black winning

(d) Black to move
White about to lose

Minimax

- **function** minimax(node, depth, Player)
- **if** (depth = 0 or node is a terminal node) **return** evaluation value of node
- **if** (Player = MaxPlayer)
- **for each** child of node
- v ← max(v, minimax(child, depth-1, switch(Player)))
- **return** v
- **else**
- **for each** child of node
- v ← min(v, minimax(child, depth-1, switch(Player)))
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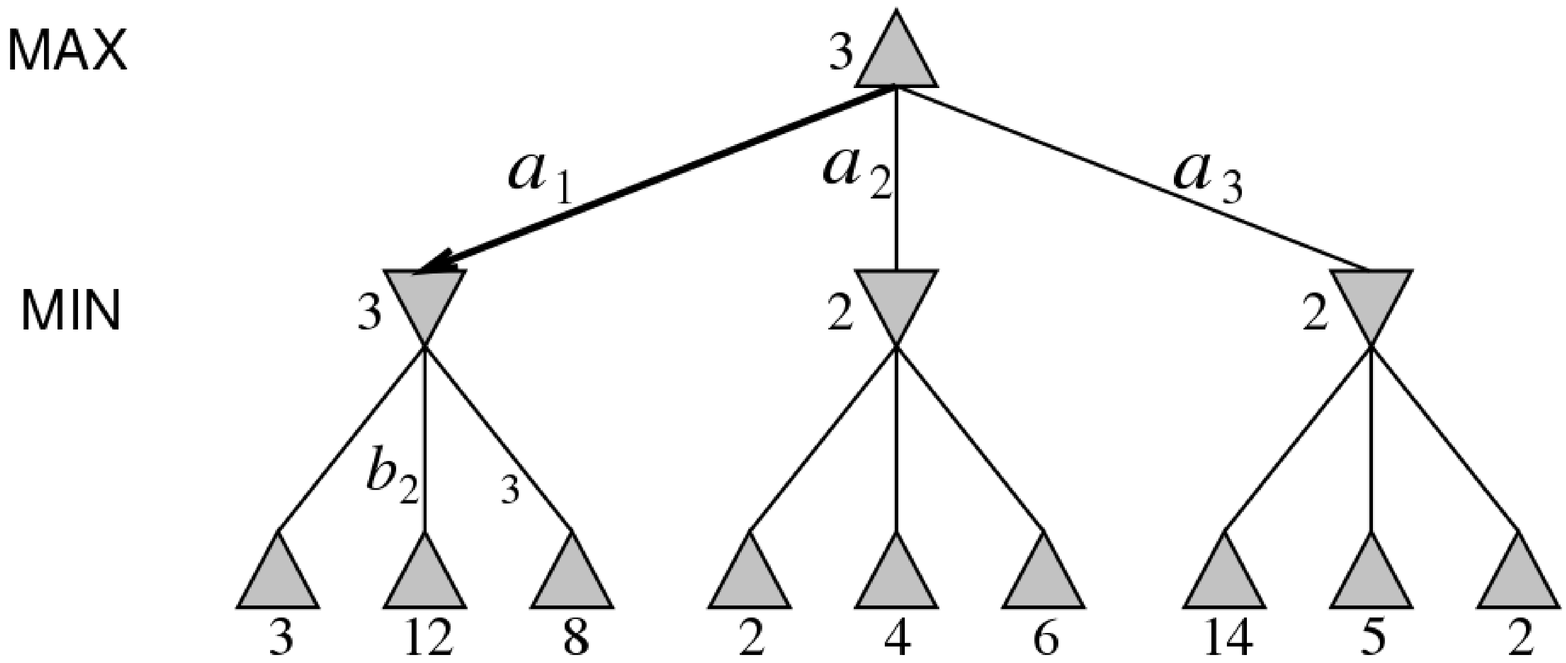
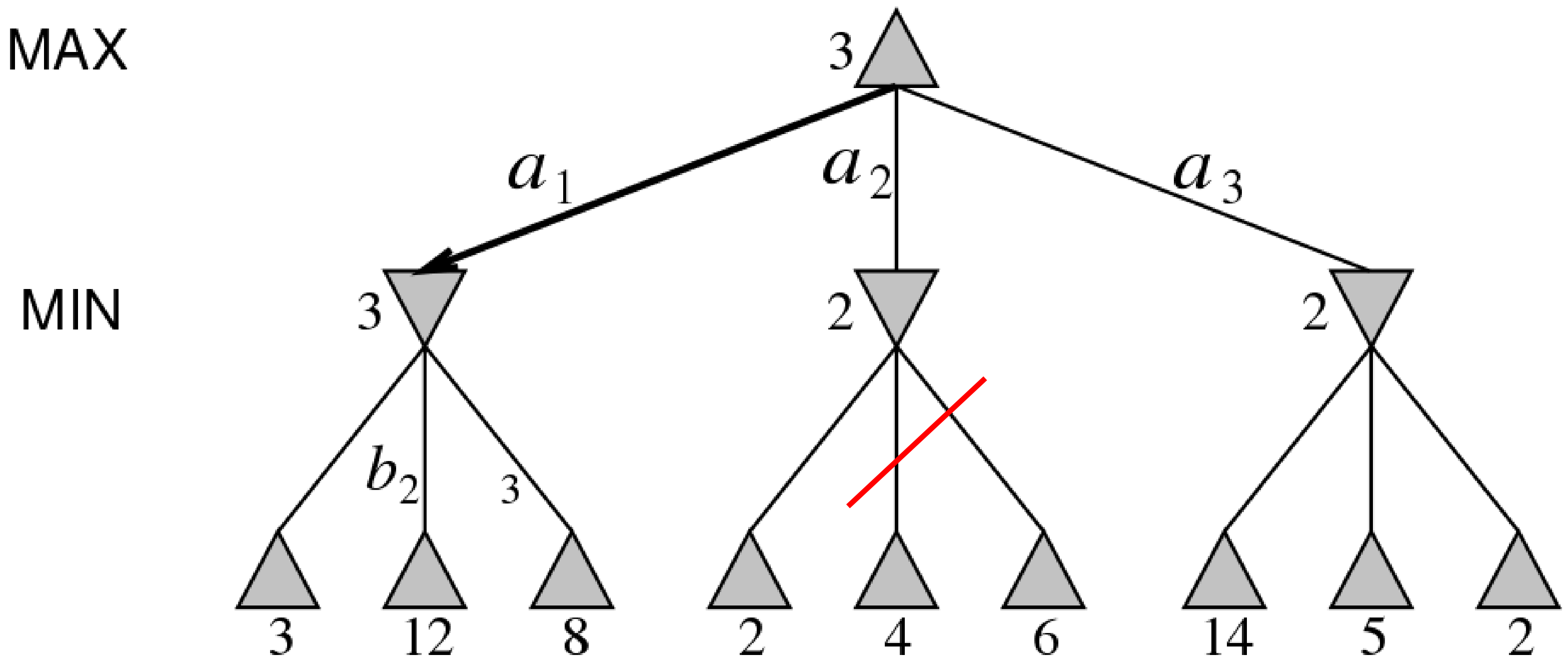
Minimax in Real Games - Problems

- good evaluation function
- depth?
  - horizon problem
  - iterative deepening
  - not always searching deeper improve the results
- caching the results (transposition tables)
- …

Alpha-Beta Pruning



Alpha-Beta Pruning



Alpha-Beta Pruning

- **function** alphabeta(node, depth, α, β, Player)
- **if** (depth = 0 or node is a terminal node) **return** evaluation value of node
- **if** (Player = MaxPlayer)
- **for each** child of node
- v ← max(v, alphabeta(child, depth-1, α, β, switch(Player)))
- α := max(α,v)
- **if** (β≤α) break
- **return** v
- **else**
- **for each** child of node
- v ← min(v, alphabeta(child, depth-1, α, β, switch(Player)))
- β := min(β, v)
- **if** (β≤α) break
- **return** v

Negamax

- **function** negamax(node, depth, α, β, Player)
- **if** (depth = 0 or node is a terminal node) **return** evaluation value of node
- **if** (Player = MaxPlayer)
- **for each** child of node
- v ← max(v, -negamax(child, depth-1, -β, -α, switch(Player)))
- α := max(α,v)
- **if** (β≤α) break
- **return** v
- **else**
- **for each** child of node
- v ← min(v, alphabeta(child, depth-1, α, β, switch(Player)))
- β := min(β, v)
- **if** (β≤α) break
- **return** v

Aspiration Search

- $[\alpha, \beta]$ interval – window
- alphabeta initialization $[-\infty, +\infty]$
- what if we use $[\alpha_0, \beta_0]$
  - x = alphabeta(node, depth, α_0, β_0,player)
  - $\alpha_0 \leq x \leq \beta_0$ - we found a solution
  - $x \leq \alpha_0$ - failing low (run again with $[-\infty, x]$)
  - $x \geq \beta_0$ - failing high (run again with $[x, +\infty]$)

Scout – Idea

- assume we are in a MAX node
- we are about to search a child 'c'
- we already have obtained a lower bound 'α'

- Is it worth searching the branch 'c'?

- we need to have some test ...

Scout –Test

- what we really need at that moment is a bound (not the precise value)
- Remember Aspiration Search?
  - $x \leq \alpha_0$ - failing low (we know, that solution is $\leq x$)
  - $x \geq \beta_0$ - failing high (we know, that solution is $\geq x$)
- What if we use a null-window $[\alpha, \alpha+1]$ (or $[\alpha,\alpha]$)?
  - we obtain a bound …

NegaScout

function negascout(node, depth, α, β, Player)

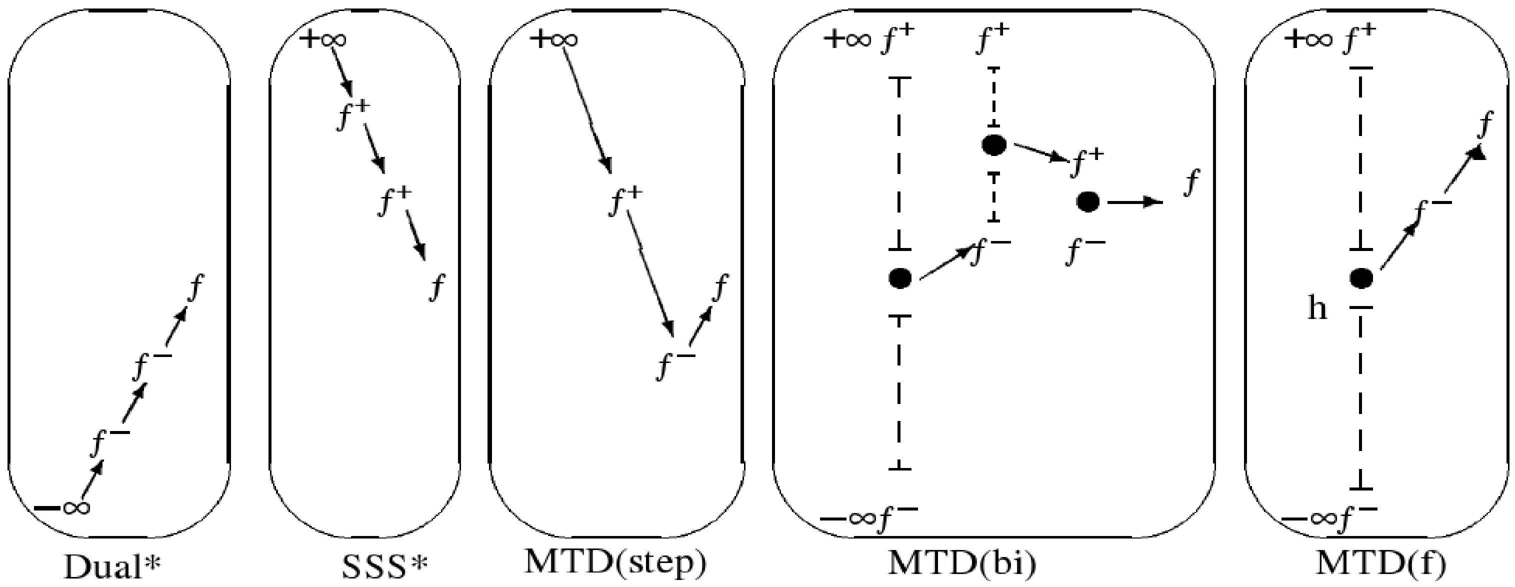
- **if** ((depth = 0) or (node is a terminal node)) **return** eval(node)
- b := β
- **for each** child of node
- v := max(v, -negascout(child, depth-1, -b, -α, switch(Player))))
- **if** ((α < v < β) and (child is not the first child))
- v := max(v, -negascout(child, depth-1, -β, -α, switch(Player))))
- α := max(α, v)
- **if** (β≤α) break
- b := α + 1
- **return** v

NegaScout

- also termed Principal Variation Search (PVS)
- dominates alpha-beta
  - never evaluates more different nodes than alpha-beta
  - can evaluate some nodes more than once
- depends on the move ordering
- can benefit from transposition tables
- generally 10-20% faster compared to alpha-beta

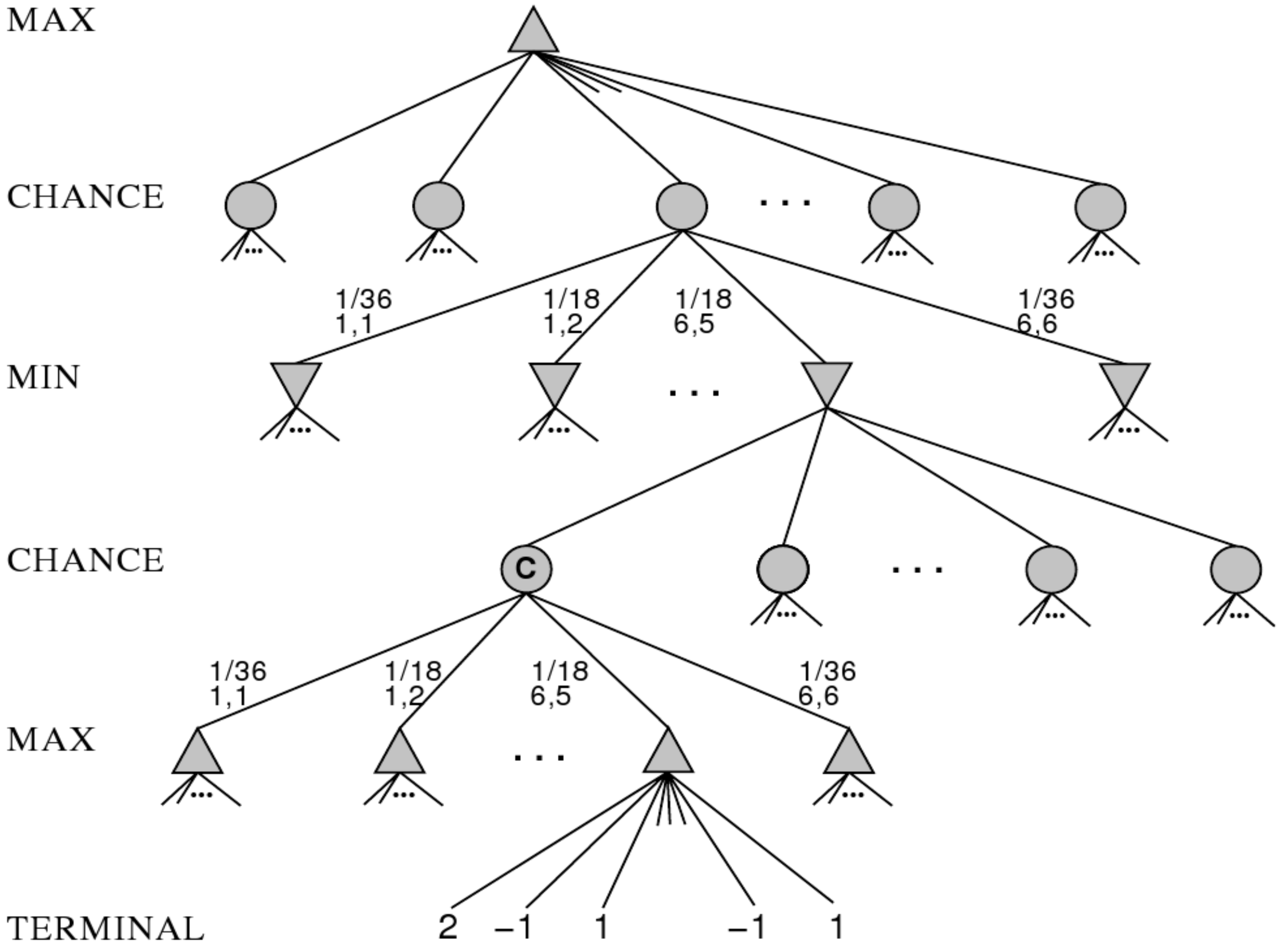
MTD

- Memory-enhanced Test Driver

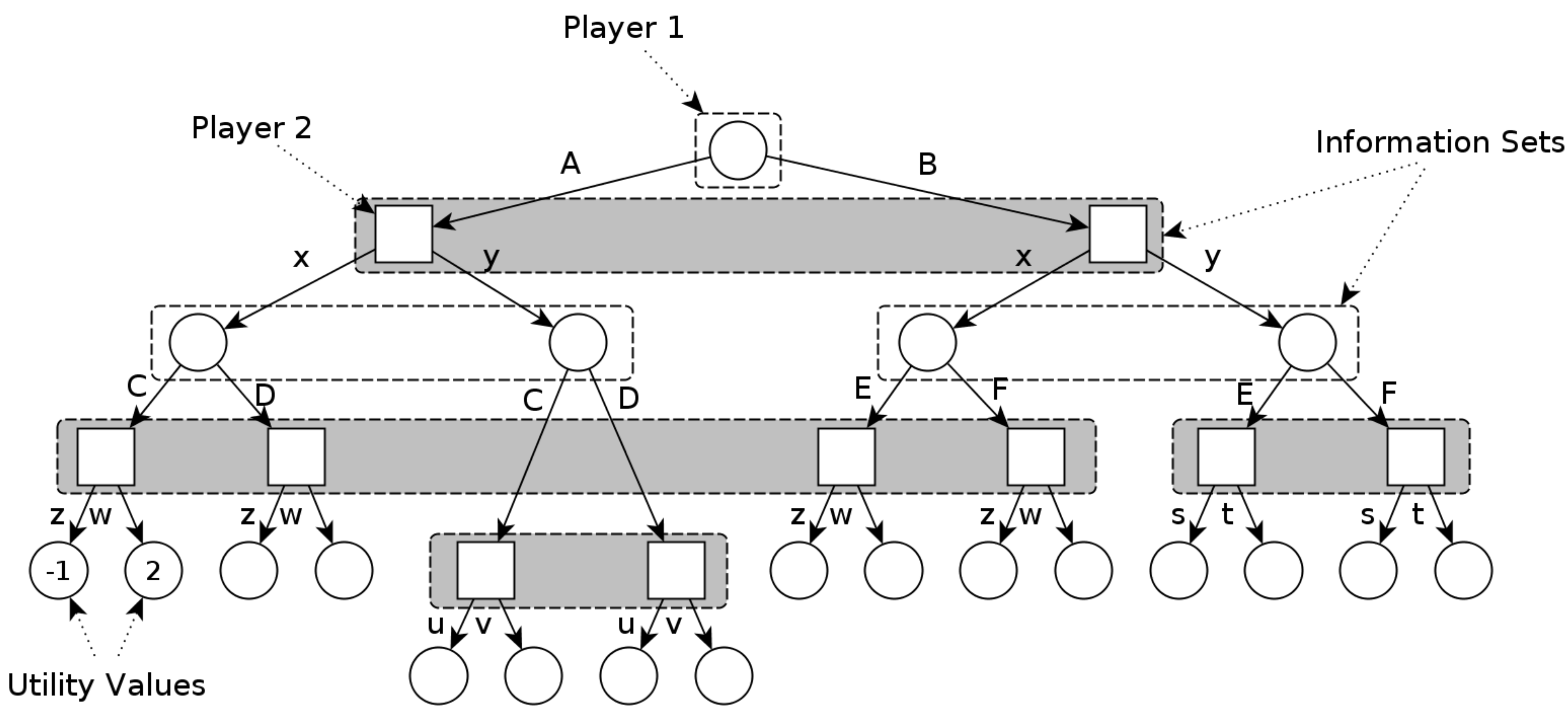


- Best-first fixed-depth minimax algorithms. Plaat et. al. , In *Artificial Intelligence,* Volume 87, Issues 1-2, November 1996, Pages 255-293

Other Games - Chance nodes



Other Games – Imperfect Information



Other Algorithms

- Sequence-form linear program
- Counterfactual Regret Minimization
- Monte-carlo Tree Search
- Double-oracle methods
- …

Game Theory in ATG

- generic algorithms
- sequential games
  - with simultaneous moves
  - with imperfect information (Poker, Security Games)
- complex strategies, abstractions
- more general types of ‘solutions’
- …