

Automated Action Planning

Implicit Planning Task Structure: Landmark Heuristics

Carmel Domshlak



Finite Domain Representation (FDR) Language

Definition (FDR planning tasks)

An **FDR** planning task is a tuple $\Pi = \langle V, A, I, G \rangle$

- V is a finite set of state variables with **finite** domains $dom(v_i)$
- initial state I is a complete assignment to V
- goal G is a partial assignment to V
- A is a finite set of actions a specified via **pre**(a) and **eff**(a), both being partial assignments to V

In cost-sensitive planning, each action a is also associated with a **cost** $C(a)$

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What
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Landmarks

- A **landmark** is a formula that must be true at some point in **every** plan
- Landmarks can be (partially) **ordered** according to the order in which they must be achieved
- Some landmarks and orderings can be discovered automatically
- Most current approaches consider only landmarks that are **facts** or **disjunctions** of facts
(Some recent work on conjunctive landmarks)

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Action Landmarks

- An **action landmark** is an action which occurs in every valid plan
- Landmarks may imply actions landmarks (e.g., sole achievers)
- Action landmarks imply landmarks (e.g., preconditions and effects)
- Some action landmarks can be discovered automatically

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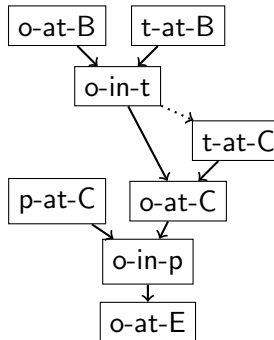
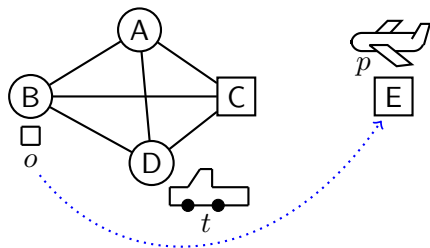
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Example Planning Problem - Logistics



Partial landmarks graph

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Sound Landmark Orderings

Sound landmark orderings are guaranteed to hold - they do not prune the solution space

- *Natural* ordering $A \rightarrow B$, iff A true some time before B
- *Necessary* ordering $A \rightarrow_n B$, iff A always true **one step** before B becomes true
- *Greedy-necessary* ordering $A \rightarrow_{gn} B$, iff A true **one step** before B becomes true for the **first time**

Note that $A \rightarrow_n B \implies A \rightarrow_{gn} B \implies A \rightarrow B$

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Landmark Complexity

- Everything is PSPACE-complete
- Deciding if a given fact is a landmark is PSPACE-complete
- Proof Sketch: it's the same as deciding if the problem without operators that achieve this fact is unsolvable
- Deciding if there is a natural / necessary / greedy-necessary between two landmarks is PSPACE-complete

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Landmark Discovery in Theory

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Theory

- A is a landmark $\iff \Pi'_A$ is unsolvable
where Π'_A is Π without the operators that achieve A
- The delete relaxation of Π'_A is unsolvable $\implies \Pi'_A$ is unsolvable (delete-relaxation landmarks)
- An abstraction of Π'_A is unsolvable $\implies \Pi'_A$ is unsolvable (abstraction landmarks)

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Landmark Discovery in Theory

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Theory

- A is a landmark $\iff \Pi'_A$ is unsolvable
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- The delete relaxation of Π'_A is unsolvable $\implies \Pi'_A$ is unsolvable (**delete-relaxation landmarks**)
- An abstraction of Π'_A is unsolvable $\implies \Pi'_A$ is unsolvable (**abstraction landmarks**)

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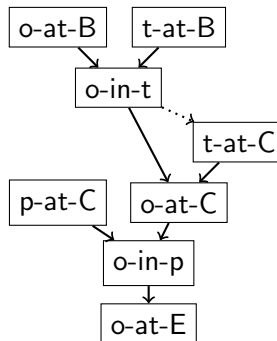
Landmark Discovery I

Delete Relaxation Landmarks

Find landmarks and orderings by **backchaining**

- Every goal is a landmark
- If B is landmark and **all actions that achieve B share A as precondition**, then
 - A is a landmark
 - $A \rightarrow_n B$

Useful restriction: consider only the case where B is achieved **for the first time** \rightsquigarrow find more landmarks (and $A \rightarrow_{gn} B$)



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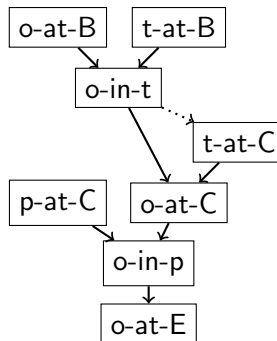
Landmark Discovery I

Delete Relaxation Landmarks

PSPACE-complete to find first achievers
 \rightsquigarrow **over-approximation** by building relaxed
planning graph for Π'_B

- This graph contains no actions that add B
- Any action applicable in this graph can possibly be executed before B first becomes true \rightsquigarrow **possible first achievers**

Additionally, if C not in the graph and C later proven to be a landmark, introduce $B \rightarrow C$



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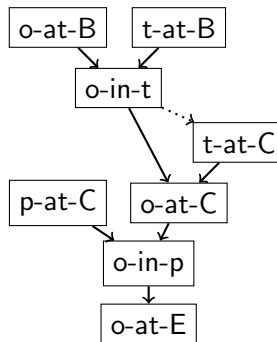
Landmark
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Landmark Discovery I

Delete Relaxation Landmarks

Disjunctive landmarks also possible,
e.g., $(o\text{-in-}p_1 \vee o\text{-in-}p_2)$:

- If B is landmark and all actions that (first) achieve B have A or C as precondition, then $A \vee C$ is a landmark
- Generalises to any number of disjuncts
- Large number of possible disjunctive landmarks \rightsquigarrow must be restricted



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Domain Transition Graphs (DTGs)

Find landmarks through DTGs (Richter et al. 2008)

The **domain transition graph of $v \in V$** (DTG_v) represents how the value of v can change.

Given: an FDR task $\langle V, A, s_0, G \rangle$

DTG_v is a directed graph with **nodes \mathcal{D}_v** that has **arc $\langle d, d' \rangle$** iff

- $d \neq d'$, and
- \exists action with $v \mapsto d'$ as effect, and either
 - $v \mapsto d$ as precondition, or
 - no precondition on v

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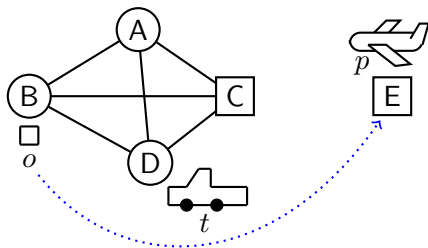
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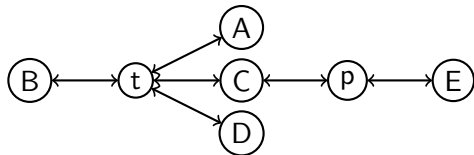
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DTG Example



DTG_{vo}:



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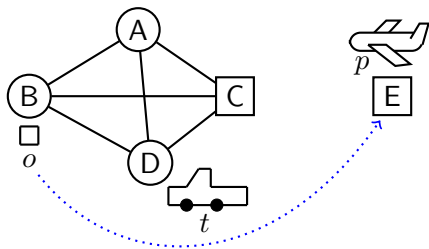
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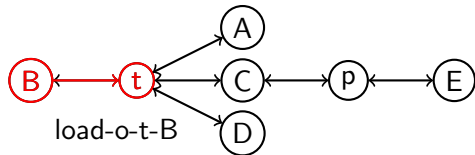
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DTG Example



DTG_{vo}:



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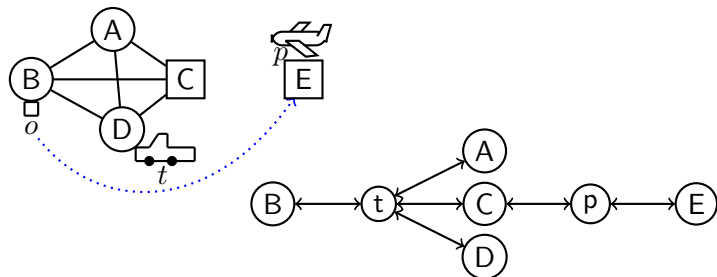
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Landmark Discovery II

Abstraction Landmarks



- Find landmarks through DTGs: if
 - $s_0(v) = d_0$,
 - $v \mapsto d$ landmark, and
 - **every path** from d_0 to d passes through d' ,then $v \mapsto d'$ landmark, and $(v \mapsto d') \rightarrow (v \mapsto d)$

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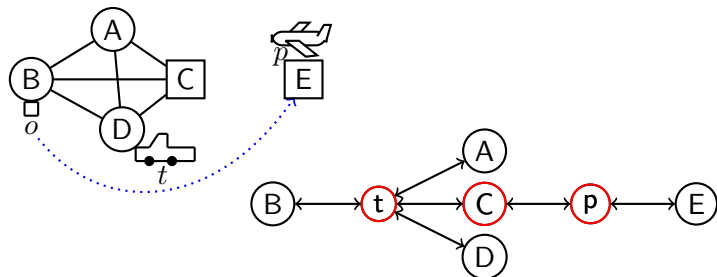
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Landmark Discovery II

Abstraction Landmarks



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Using Landmarks

- Some landmarks and orderings can be discovered efficiently
- So what can we do once we have these landmarks?
- We assume that landmarks and orderings are discovered in a pre-processing phase, and the same landmark graph is used throughout the planning phase

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Using Landmarks as Subgoals

- Landmarks can be used as subgoals for a base planner
- The first layer of landmarks that have not yet been achieved is passed as a disjunctive goal to a base planner

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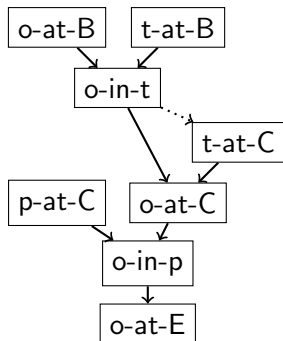
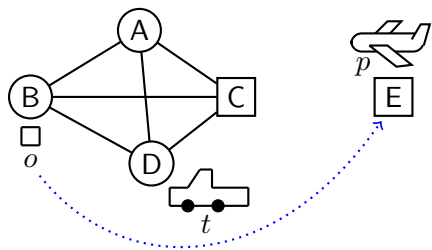
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Using Landmarks as Subgoals - Logistics Example



- Partial plan:
- Goal:

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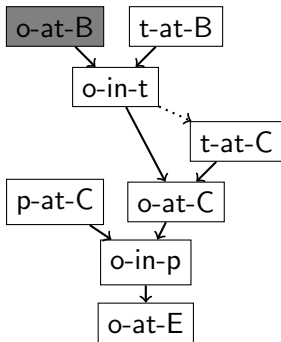
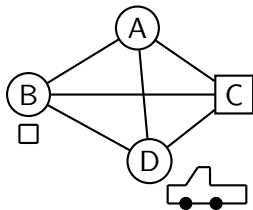
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Using Landmarks as Subgoals - Logistics Example



- Partial plan: \emptyset
- Goal: $t\text{-at-B} \vee p\text{-at-C}$

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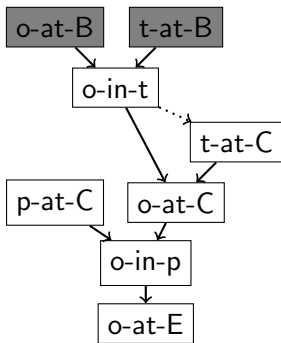
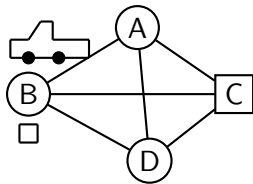
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Using Landmarks as Subgoals - Logistics Example



- Partial plan: Drive-t-B
- Goal: $o\text{-in-t} \vee p\text{-at-C}$

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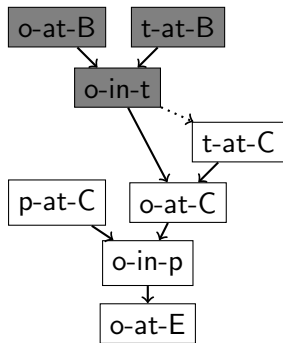
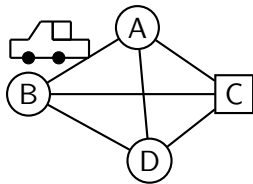
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Using Landmarks as Subgoals - Logistics Example



- Partial plan: Drive-t-B, Load-o-B
- Goal: $t\text{-at-C} \vee p\text{-at-C}$

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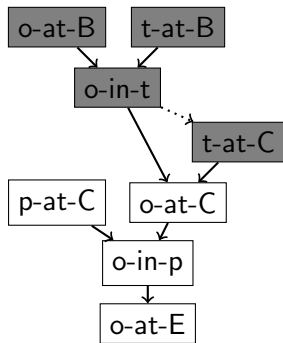
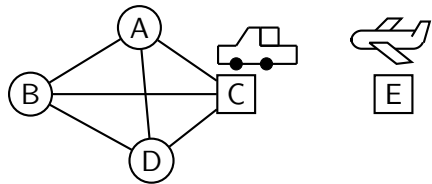
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Using Landmarks as Subgoals - Logistics Example



- Partial plan: Drive-t-B, Load-o-B, Drive-t-C
- Goal: o-at-C \vee p-at-C

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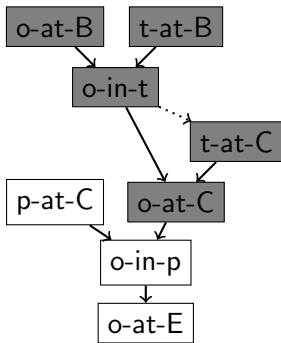
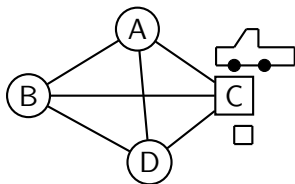
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Using Landmarks as Subgoals - Logistics Example



- Partial plan: Drive-t-B, Load-o-B, Drive-t-C, Unload-o-C
- Goal: p-at-C

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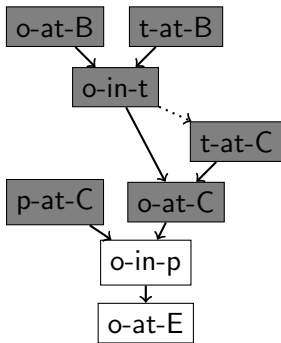
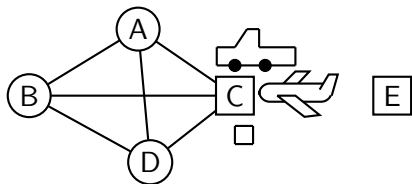
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Using Landmarks as Subgoals - Logistics Example



- Partial plan: Drive-t-B, Load-o-B, Drive-t-C, Unload-o-C, Fly-p-C
- Goal: o-in-p

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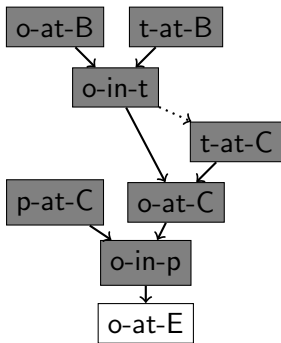
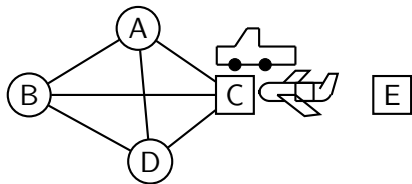
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Using Landmarks as Subgoals - Logistics Example



- Partial plan: Drive-t-B, Load-o-B, Drive-t-C, Unload-o-C, Fly-p-C, Load-o-p
- Goal: o-at-E

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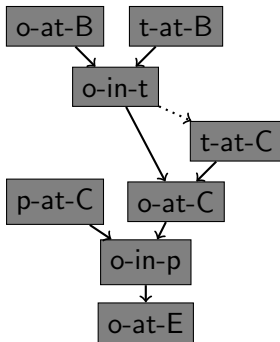
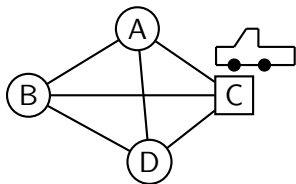
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Using Landmarks as Subgoals - Logistics Example



- Partial plan: Drive-t-B, Load-o-B, Drive-t-C, Unload-o-C, Fly-p-C, Load-o-p, Fly-p-E, Unload-o-E
- Goal: \emptyset

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Using Landmarks as Subgoals

- That was a good example
- Now let's see a bad one

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Using Landmarks as Subgoals - Sussman Example

- Consider the following blocks problem (“The Sussman Anomaly”)



- Goal: *on-A-B*, *on-B-C*

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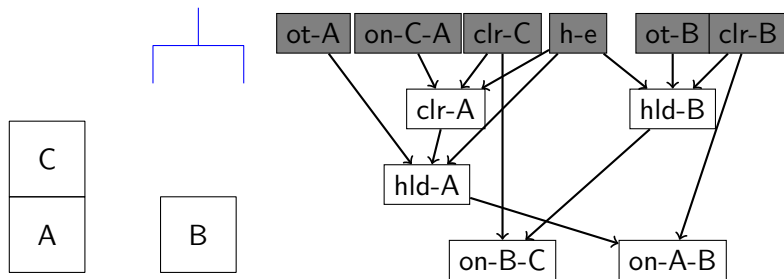
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Using Landmarks as Subgoals - Sussman Example



- Partial plan: \emptyset
- Goal: $\text{clear-A} \vee \text{holding-B}$

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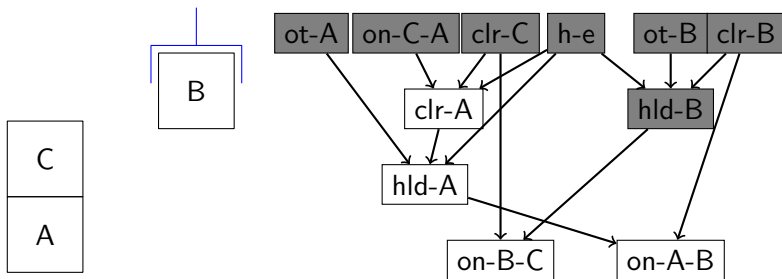
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Using Landmarks as Subgoals - Sussman Example



- Partial plan: Pickup-B
- Goal: clear-A \vee on-B-C

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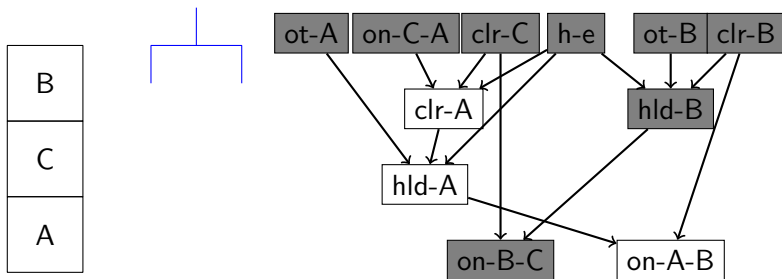
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Using Landmarks as Subgoals - Sussman Example



- Partial plan: Pickup-B, Stack-B-C
- Goal: clear-A

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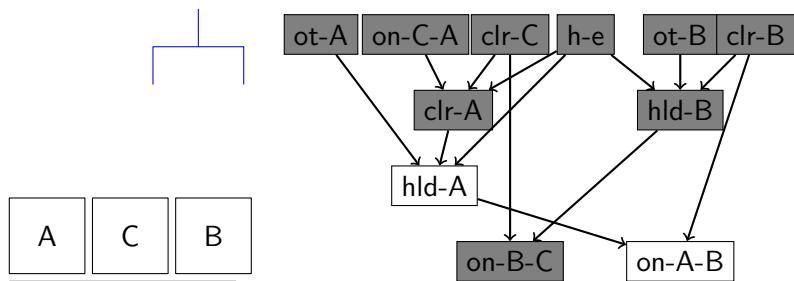
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Using Landmarks as Subgoals - Sussman Example



- Partial plan: Pickup-B, Stack-B-C, Unstack-B-C, Putdown-B, Unstack-C-A, Putdown-C
- Goal: holding-A

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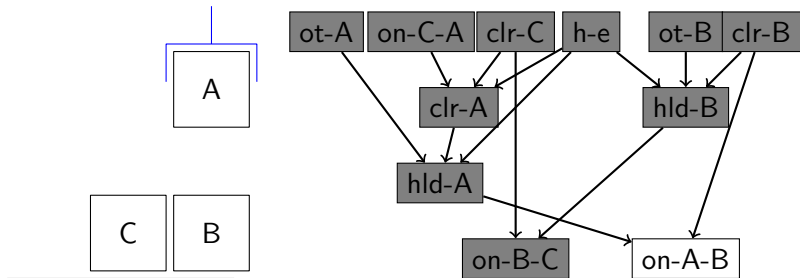
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Using Landmarks as Subgoals - Sussman Example



- Partial plan: Pickup-B, Stack-B-C, Unstack-B-C, Putdown-B, Unstack-C-A, Putdown-C, Pickup-A
- Goal: on-A-B

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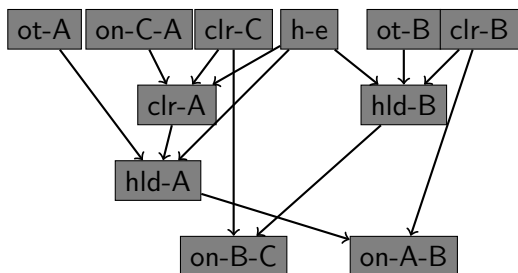
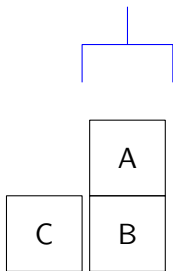
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Using Landmarks as Subgoals - Sussman Example



- Partial plan: Pickup-B, Stack-B-C, Unstack-B-C, Putdown-B, Unstack-C-A, Putdown-C, Pickup-A, Stack-A-B
- Goal: Still need to achieve on-B-C

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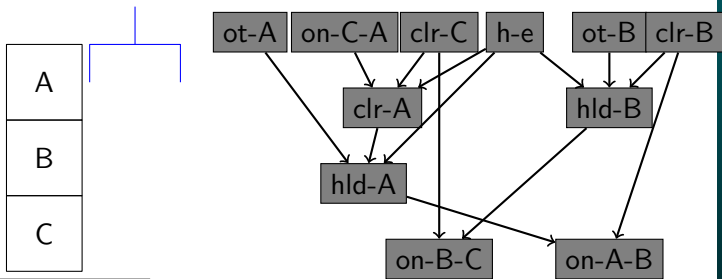
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Using Landmarks as Subgoals - Sussman Example



- Partial plan: Pickup-B, Stack-B-C, Unstack-B-C, Putdown-B, Unstack-C-A, Putdown-C, Pickup-A, Stack-A-B, Unstack-A-B, Putdown-A, Pickup-B, Stack-B-C, Pickup-A, Stack-A-B
- Goal: \emptyset

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Subgoals
Heuristic
Estimates
Admissible
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Estimates

Using Landmarks as Subgoals - Pros and Cons

- Pros:
 - Planning is very fast - the base planner needs to plan to a lesser depth
- Cons:
 - Can lead to much longer plans
 - Not complete in the presence of dead-ends

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Using Landmarks for Heuristic Estimates

- The number of landmarks that still need to be achieved is a heuristic estimate
- Used by **LAMA** (Richter, Helmert and Westphal 2008), winner of the IPC-2008 and IPC-2011 sequential satisficing track!

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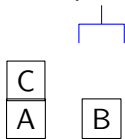
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Path-dependent Heuristics

- Suppose we are in state s . Did we achieve landmark A yet?
- Example: did we achieve holding(B)?



- There is no way to tell just by looking at s
- Achieved landmarks are a function of path, not state
- The number of landmarks that still need to be achieved is a **path-dependent** heuristic

The Landmark Heuristic

- The landmarks that still need to be achieved after reaching state s via path π are

$$L(s, \pi) = (L \setminus \text{Accepted}(s, \pi)) \cup \text{ReqAgain}(s, \pi)$$

- L is the set of all (discovered) landmarks
- $\text{Accepted}(s, \pi) \subset L$ is the set of *accepted* landmarks
- $\text{ReqAgain}(s, \pi) \subseteq \text{Accepted}(s, \pi)$ is the set of *required again* landmarks - landmarks that must be achieved again

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Accepted Landmarks

- In LAMA, a landmark A is first accepted by path π in state s if
 - all predecessors of A in the landmark graph have been accepted, and
 - A becomes true in s
- Once a landmark has been accepted, it remains accepted

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Required Again Landmarks

- A landmark A is required again by path π in state s if:
 - false-goal A is false in s and is a goal, or
 - open-prerequisite A is false in s and is a greedy-necessary predecessor of some landmark that is not accepted

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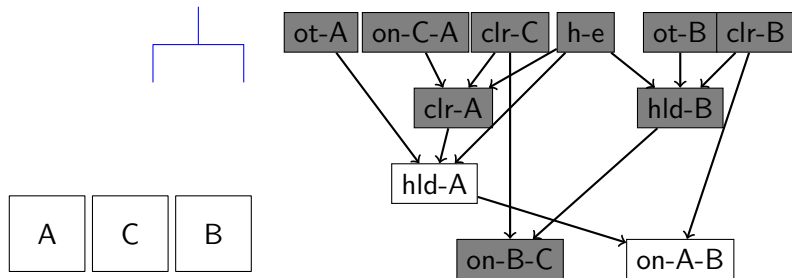
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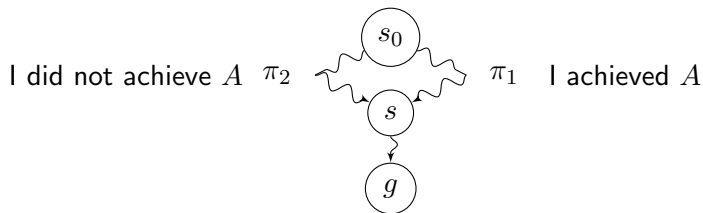
Accepted and Required Again Landmarks - Example

- In the Sussman anomaly, after performing: Pickup-B, Stack-B-C, Unstack-B-C, Putdown-B, Unstack-C-A, Putdown-C



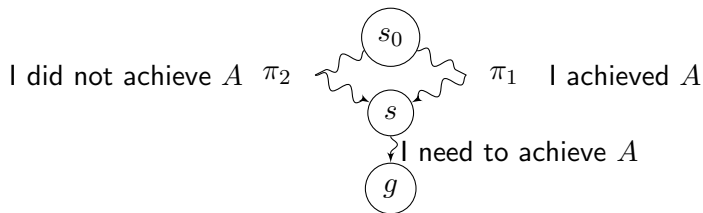
- on-B-C is a *false-goal*, and so it is required again

Multi-path Dependence



- Suppose state s was reached by paths π_1, π_2
- Suppose π_1 achieved landmark A and π_2 did not
- Conclusion:

Multi-path Dependence



- Suppose state s was reached by paths π_1, π_2
- Suppose π_1 achieved landmark A and π_2 did not
- Conclusion: A needs to be achieved after state s
- Proof: A is a landmark, therefore it needs to be true in **all** valid plans, including valid plans that start with π_2

Fusing Data from Multiple Paths

- Suppose \mathcal{P} is a set of paths from s_0 to a state s . Define

$$L(s, \mathcal{P}) = (L \setminus \text{Accepted}(s, \mathcal{P})) \cup \text{ReqAgain}(s, \mathcal{P})$$

where

- $\text{Accepted}(s, \mathcal{P}) = \bigcap_{\pi \in \mathcal{P}} \text{Accepted}(s, \pi)$
- $\text{ReqAgain}(s, \mathcal{P}) \subseteq \text{Accepted}(s, \mathcal{P})$ is specified as before by s and the various rules
- $L(s, \mathcal{P})$ is the set of landmarks that we know still needs to be achieved after reaching state s via the paths in \mathcal{P}

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Admissible Heuristic Estimates

- LAMA's heuristic: the number of landmarks that still need to be achieved (Richter, Helmert and Westphal 2008)
- LAMA's heuristic is inadmissible - a single action can achieve multiple landmarks
 - Example: *hand-empty* and *on-A-B* can both be achieved by *stack-A-B*
- Admissible heuristic: assign a cost to each landmark, sum over the **costs** of landmarks (Karpas and Domshlak, 2009)

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Conditions for Admissibility

- Each action shares its cost between all the landmarks it achieves

$$\forall a \in A : \sum_{A \in L(a|s, \mathcal{P})} cost(a, A) \leq C(a)$$

$cost(a, A)$: cost “assigned” by action a to A

$L(a|s, \mathcal{P})$: the set of landmarks achieved by a

- Each landmark is assigned at most the cheapest cost any action assigned it

$$\forall A \in L(s, \mathcal{P}) : cost(A) \leq \min_{a \in ach(A|s, \mathcal{P})} cost(a, A)$$

$cost(A)$: cost assigned to landmark A

$ach(A|s, \mathcal{P})$: the set of actions that can achieve A

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Admissible Cost Sharing

- Idea: the cost of a set of landmarks is no greater than the cost of any single action that achieves them
- Given that, the sum of costs of landmarks that still need to be achieved is an admissible heuristic, h_L

$$h_L(s, \pi) := \text{cost}(L(s, \pi)) = \sum_{A \in L(s, \pi)} \text{cost}(A)$$

- Proof: Homework 😊

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Cost Partitioning - how?

- How can we find such a partitioning?
- Easy answer - **uniform cost sharing** - each action shares its cost equally between the landmarks it achieves

$$\text{cost}(a, A) = \frac{C(a)}{|L(a|s, \pi)|}$$

$$\text{cost}(A) = \min_{a \in \text{ach}(A|s, \pi)} \text{cost}(a, A)$$

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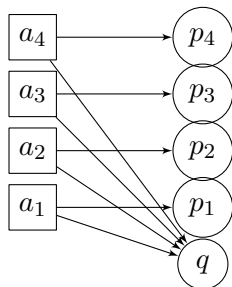
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Uniform Cost Sharing

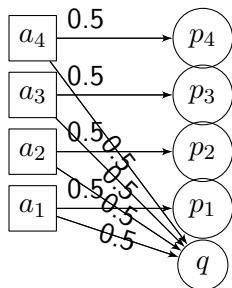
- Advantage: Easy and fast to compute
- Disadvantage: Can be much worse than the optimal cost partitioning



Uniform Cost Sharing

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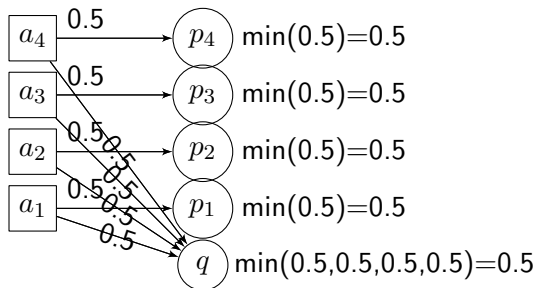
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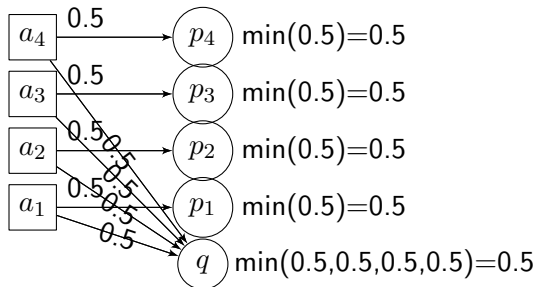


Uniform Cost Sharing

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Uniform cost sharing

$$h_L = 2.5$$

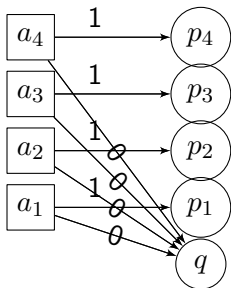


Uniform Cost Sharing

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Optimal cost sharing

uniform $h_L = 2.5$

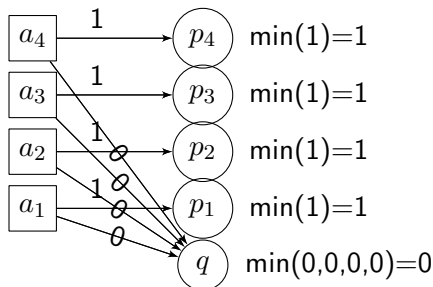


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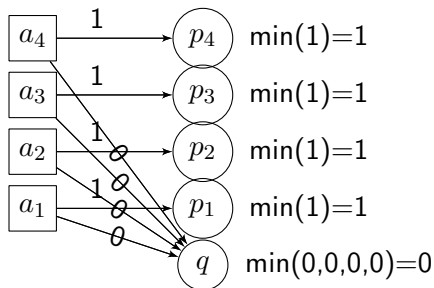
Uniform Cost Sharing

- Advantage: Easy and fast to compute
- Disadvantage: Can be much worse than the optimal cost partitioning

Optimal cost sharing

$$h_L = 4$$

$$\text{uniform } h_L = 2.5$$



Optimal Cost Sharing

- The good news: the optimal cost partitioning is poly-time to compute
 - The constraints for admissibility are linear, and can be used in a **Linear Program** (LP)
 - Objective: maximize the sum of landmark costs
 - The solution to the LP gives us the optimal cost partitioning
- The bad news: poly-time can still take a long time
- Sounds familiar?
 - 1 Indeed, but can be just a coincidence.

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 - 2 Not a coincidence: special case of action-cost partitioning for abstractions.

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How can we do better?

- So far:
 - Uniform cost sharing is easy to compute, but suboptimal
 - Optimal cost sharing takes a long time to compute
- Q: How can we get better heuristic estimates that don't take a long time to compute?
- A:

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- So far:
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 - Optimal cost sharing takes a long time to compute
- Q: How can we get better heuristic estimates that don't take a long time to compute?
- A: Exploit additional information - **action landmarks**

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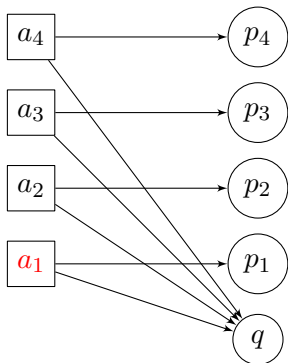
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Using Action Landmarks - by Example



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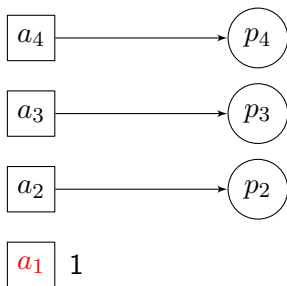
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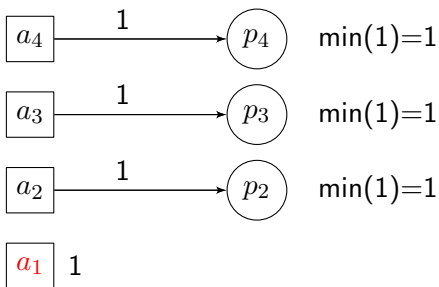
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Using Action Landmarks - by Example

Uniform Cost Sharing

$$h_{LA} = 4$$



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Summary

- Landmarks provide a way to utilize the implicit structure of a planning problem
- They can (and have been) used successfully for both satisficing and optimal planning
- The envelope of heuristic functions has been pushed!

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