

# Combinatorial Optimization

## Lab No. 5

### Applications of the Shortest Path Problem

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March 21, 2017

#### Abstract

In this lab, we present an interesting application of shortest paths and we remind ourselves, that SPT problem is generally  $\mathcal{NP}$ -hard.

## 1 Approximation of piecewise-linear functions

Approximation of piecewise linear functions is lesser-known, however quite an interesting application of the shortest path problem [1]. As an input a set of  $n$  points is given, i.e.  $\{(x_i, f(x_i))\}$ .

$x = [0, 1.26, 2.51, 3.77, 5.03, 6.28]$   
 $f = [0.01, 1.16, 0.70, -0.34, -0.80, 0.2100]$

The goal is to select the smallest possible number of points on condition that approximation error is reasonable (see Figure 1).

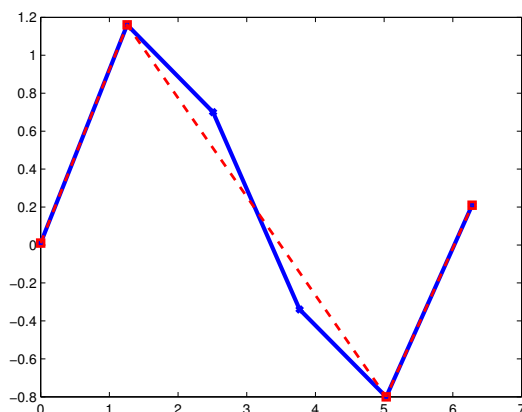


Figure 1: The input function (blue) and the approximated function (red).

This problem can be solved using the shortest path problem applied to graph  $G$  where each node  $v_i$  corresponds to a data point  $(x_i, f(x_i))$  and each edge  $e = (i, j)$  (such that  $i < j$ ) has weight  $c_{i,j}$  which corresponds to a penalty if the part of the function from  $(x_i, f(x_i))$  to  $(x_j, f(x_j))$  is approximated by a line segment. The penalty consists of the approximation error (weighted by  $\beta$ ) and the number of required samples (the 1 term). An example of such graph can be seen in Figure 1 and penalty  $c_{i,j}$  is calculated as follows.

$$c_{i,j} = 1 + \beta \left[ \sum_{k=i}^j (f(x_k) - f'(x_k))^2 \right] \quad (1)$$

$f'(x_k)$  is a precomputed value of the approximating function.

$$f'(x) = f(x_i) + (x - x_i) \cdot \frac{f(x_j) - f(x_i)}{x_j - x_i} \quad (2)$$

In a similar way the shortest path problem can be applied to some vector pictures which are composed of points  $(x_i, y_i)$ . The difference is that weights  $c_{i,j}$  are computed as a sum of squared distances from points  $(x_k, y_k)$  to the line segment defined by  $(x_i, y_i)$  and  $(x_j, y_j)$  points where  $i < k < j$ . An example of two-dimensional approximation, where the frontiers of the Czech Republic are input data, is depicted in Figure 2.

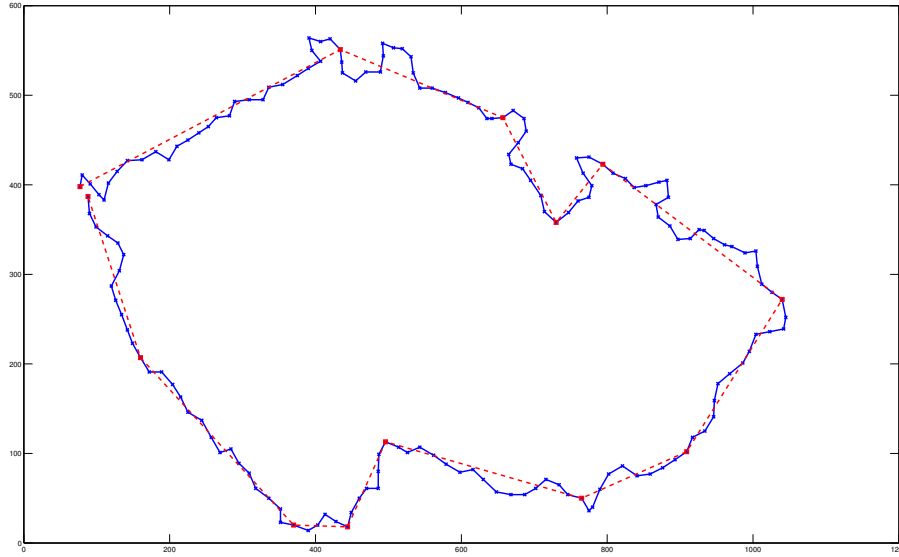


Figure 2: Entered points (blue) and selected points (red).

## 2 Shortest paths with negative cycles

In this example, we demonstrate that Bellman-Ford's algorithm fails to produce correct answer on the following instance, whereas an ILP formulation of SPT yields correct solution.

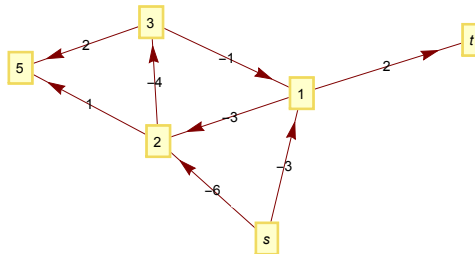


Figure 3: A graph with a negative cycle.

Codes are available at CourseWare.

## References

- [1] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, *Network Flows: Theory, Algorithms, and Applications*. Prentice Hall; United States Ed edition, 1993.