

Combinatorial Optimization

Lab No. 5

Applications of the Shortest Path Problem

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Abstract

In this lab, we present an interesting application of shortest paths and we remind ourselves, that SPT problem is generally \mathcal{NP} -hard.

1 Approximation of piecewise-linear functions

Approximation of piecewise linear functions is lesser-known, however quite an interesting application of the shortest path problem [1]. As an input a set of n points is given, i.e. $\{(x_i, f(x_i))\}$

$x = [0, 1.26, 2.51, 3.77, 5.03, 6.28]$
 $f = [0.01, 1.16, 0.70, -0.34, -0.80, 0.2100]$

The goal is to select the smallest possible number of points on condition that approximation error is reasonable (see Figure 1).

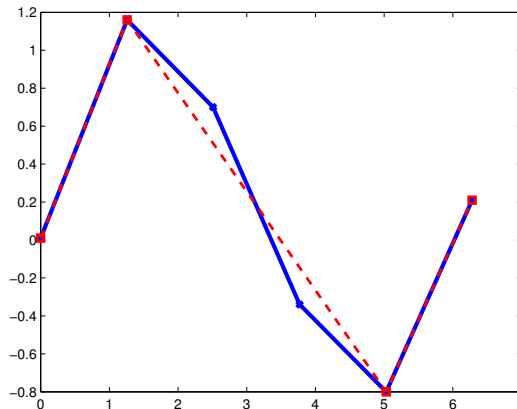


Figure 1: The input function (blue) and the approximated function (red).

This problem can be solved using the shortest path problem applied to graph G where each node v_i corresponds to a data point $(x_i, f(x_i))$ and each edge $e = (i, j)$ (such that $i < j$) has weight $c_{i,j}$ which corresponds to a penalty if the part of the function from $(x_i, f(x_i))$ to $(x_j, f(x_j))$ is approximated by a line segment. The penalty consists of the approximation error (weighted by β) and the number of required samples (weighted by α). An example of such graph can be seen in Figure 1 and penalty $c_{i,j}$ is calculated as follows.

$$c_{i,j} = \alpha + \beta \left[\sum_{k=i}^j (f(x_k) - f'(x_k))^2 \right] \quad (1)$$

$f'(x_k)$ is a precomputed value of the approximating function.

$$f'(x) = f(x_i) + (x - x_i) \cdot \frac{f(x_j) - f(x_i)}{x_j - x_i} \quad (2)$$

In a similar way the shortest path problem can be applied to some vector pictures which are composed of points (x_i, y_i) . The difference is that weights $c_{i,j}$ are computed as a sum of squared distances from points (x_k, y_k) to the line segment defined by (x_i, y_i) and (x_j, y_j) points where $i < k < j$. An example of two-dimensional approximation, where the frontiers of the Czech Republic are input data, is depicted in Figure 2.

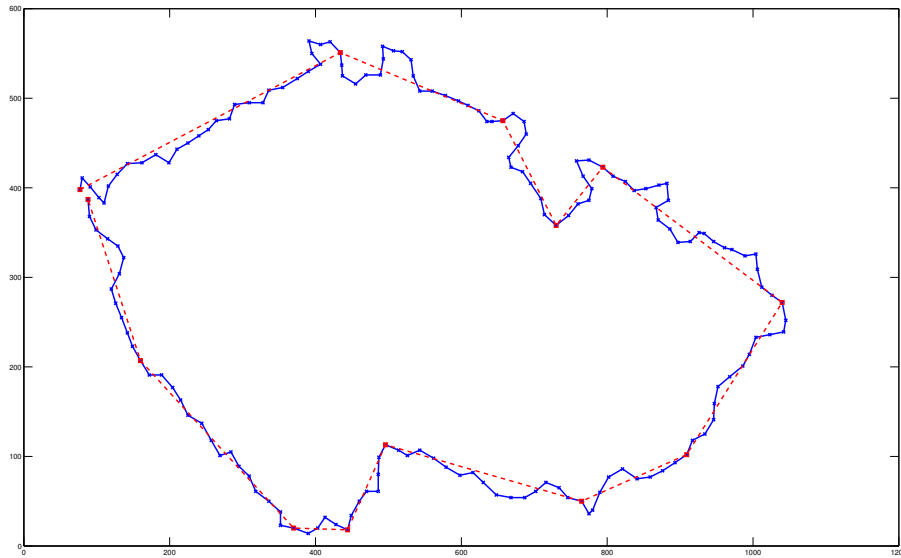


Figure 2: Entered points (blue) and selected points (red).

2 Shortest paths with negative cycles

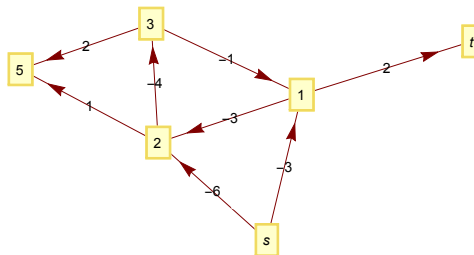


Figure 3: The graph with a negative cycle for the example.

References

- [1] R. K. Ahuja, T. L. Magnanti, and J. B. Orlin, *Network Flows: Theory, Algorithms, and Applications*. Prentice Hall; United States Ed edition, 1993.