Advanced algorithms

Seminar 9/19/2012

1. Light bulbs

Imagine a hall with N light bulbs on the ceiling along with N switches. One would expect that each operates a corresponding light bulb, yet it is not the case – the i-th switch operates not just the i-th bulb, but the $2i$-th, $3i$-th, ..., and the $\lfloor \frac{N}{i} \rfloor i$-th light bulb as well. Each press of a switch toggles the state of a light bulb, i.e., it turns it on if it was off and vice versa. Now imagine yourself going through the hall pressing the switches one after another, how many light bulbs remain on if initially they were all off?

2. Chocolate

Everyone loves chocolate, right?! Consider a chocolate bar of size $N \times M$ (in one piece). How would you break it such that you ended up with $N \times M$ pieces as fast as possible? How many breaks would it take?

3. Garden

Peter is an enthusiastic gardener. This year he decided on planting only carrots and cabbages in his garden. The garden is divided into N patches, and on any of them can be planted either carrots or cabbages, but he made himself a rule such that no two neighbouring patches contain cabbage, because then the garden looks ugly. Now, thinking more deeply about the rule, he asked himself a question in how many ways he can plant his garden. Can you make it up for him?

4. Tutor[1]

Marko is extremely happy to teach his class of Advanced algorithms. This semester he came up with a novelty – on each seminar he calls up a student to come to the blackboard and solve a really difficult problem. He will pick students completely randomly[2] which may (naturally) lead to one student being asked more then once and some not at all. So as one of his problems he prepared the following question: How many seminars (on average) the class should have such that all of the N students have been at the blackboard at least once? Could you help your fellow students and come up with an answer?

[1] Be careful — Maths!

[2] We are assuming a uniform distribution