Alphabet

Alphabet … finite (unempty) set of symbols
|A| … **size** of alphabet A

Examples: A = { ‘A’, ‘D’, ‘G’, ‘O’, ‘U’}, |A| = 5
A = {0,1}, |A| = 2
A = {○, □, △}, |A| = 3

word

Word (over alphabet A) … finite (maybe empty) sequence
also **string** of symbols of alhabet (A)
|w| … **length** of word w

Examples: w = OUAGADOUGOU, |w| = 11
w = 1001, |w| = 4
w = □△○△□, |w| = 5

Language

Language … set of words (=strings)
(not necessarily finite, can be empty)
|L| … cardinality of language L

---

1 **Language specification** -- List of all words of the language
(only for finite language!)

Examples: A_1 = {'A', 'D', 'G', 'O', 'U'}

L_1 = {ADA, DOG, GOUDA, D, GAG}, $|L_1| = 5$

A_2 = {0,1}

L_2 = {0, 1, 00, 01, 10, 11}, $|L_2| = 6$

A_3 = { ○, □, △}

L_3 = { △△, ○□○, □□△○ }, $|L_2| = 3$

② **Language specification** -- **Informal (but unambiguous) description in natural human language (usually for infinite language)**

Examples: $A_1 = \{$'A', 'D', 'G', 'O', 'U'$\}$

L_1: Set of all words over A_1, which begin with DA, end with G a and do not contain subsequence AA.

$L_1 = \{DAG, DADG, DAGG, DAOG, DAUG, DADAG, DADDG\ldots\}$

$|L_1| = \infty$

$A_2 = \{0,1\}$

L_2: Set of all words over A_2, which contain more 1s than 0s and each 0 is followed by at least two 1s.

$L_2 = \{ 1, 11, 011, 0111, 1011, 1111, \ldots, 011011, 011111, \ldots \}$

$|L_2| = \infty$

3 Language specification -- By finite automaton

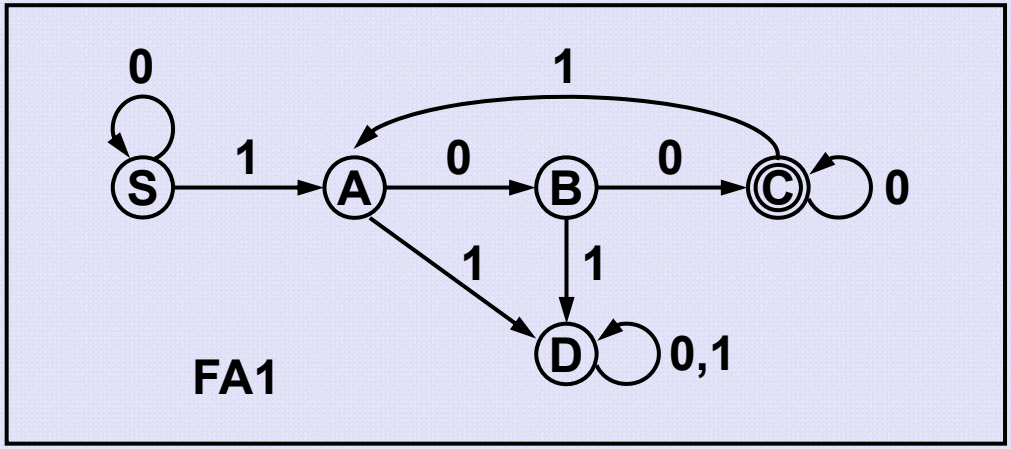
Finite automaton
is a five-tuple (A, Q, σ, S_0, Q_F), where:

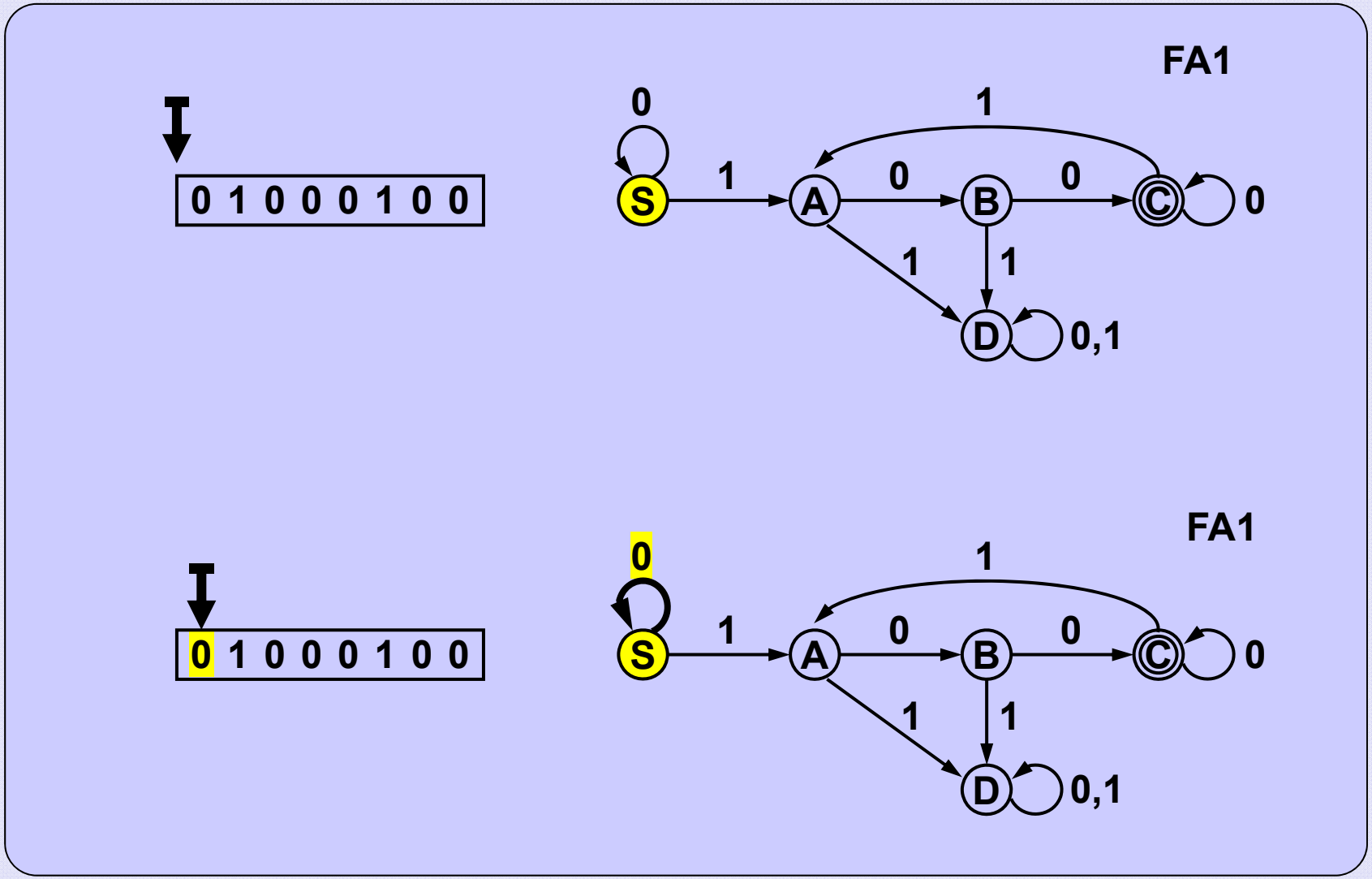
A … alphabet … finite set of symbols
|A| ... size of alphabet
Q ... set of states (often numbered) (what is „a state“ ?)
σ ... transition function ... $\sigma: Q \times A \rightarrow Q$
S_0 ... start state $S_0 \in Q$
Q_F ... unempty set of final states $\emptyset \neq Q_F \subseteq Q$

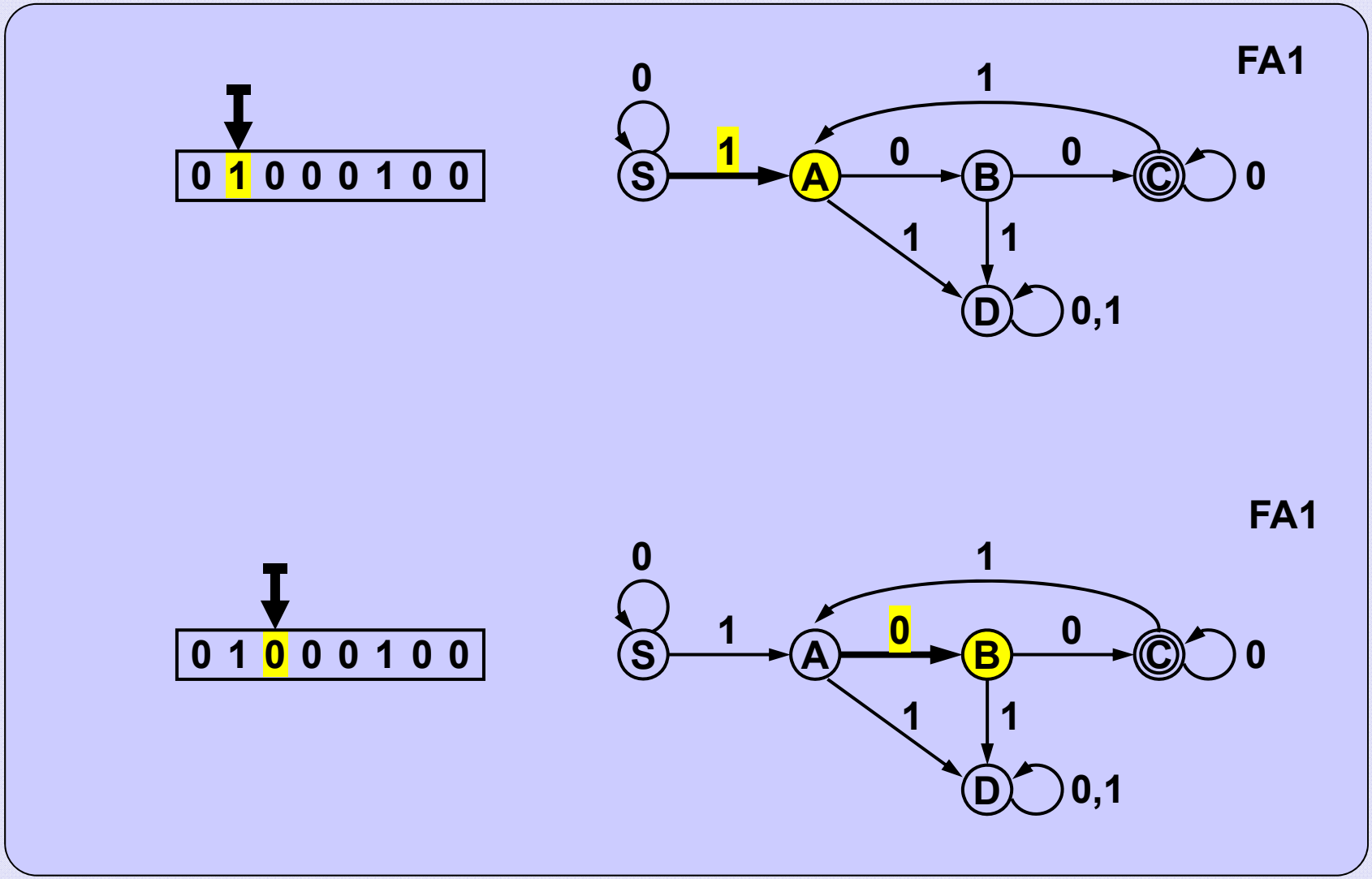
Automaton FA1:

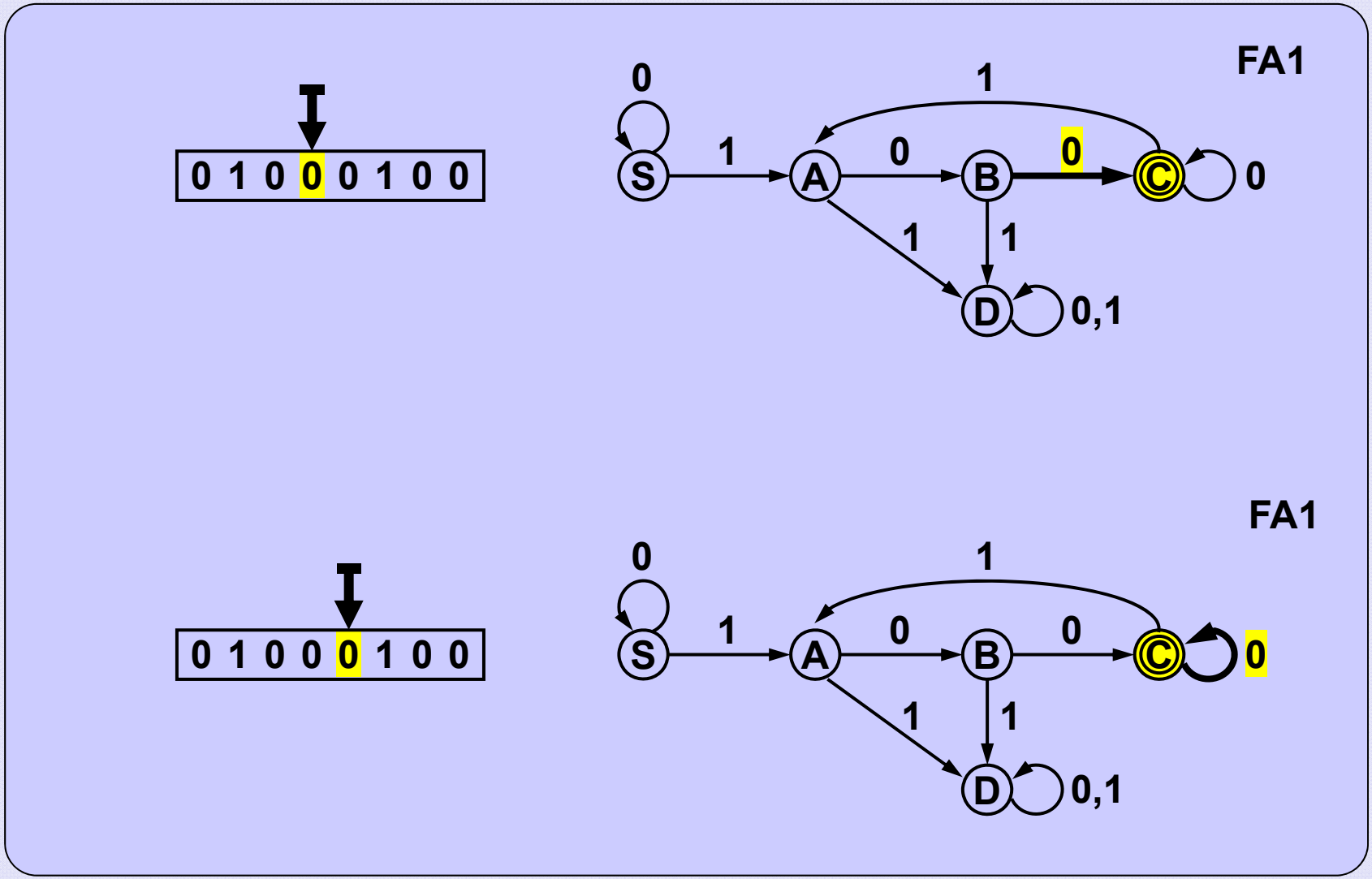
A … alphabet … {0,1}, |A| = 2
Q ... set of states {S, A, B, C, D }
σ ... transition function ... σ: Q×A → Q : {
σ(S,0) = S, σ(A,0) = B, σ(B,0) = C, σ(C,0) = C, σ(D,0) = D,
σ(S,1) = A, σ(A,1) = D, σ(B,1) = D, σ(C,1) = A, σ(D,1) = D }
S_0 ... start state S ∈ Q
Q_F … unempty set of final states ∅ ≠ { C } ⊆ Q

Transition diagram of the automaton FA1

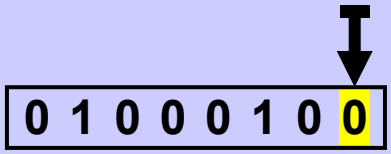


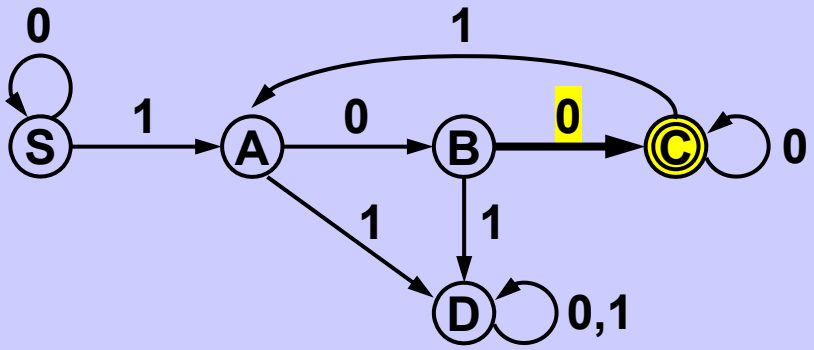

FA1
0 1 0 0 0 1 0 0
S
A
B
C
D
0
1
0,1
FA1
0 1 0 0 0 1 0 0
S
A
B
C
D
0
1
0,1


FA1
0 1 0 0 0 1 0 0
S
A
B
C
D
0
1
0,1
FA1
0 1 0 0 0 1 0 0
S
A
B
C
D
0
1
0,1


FA1
0 1 0 0 0 1 0 0
S
A
B
C
D
0
1
0,1
FA1
0 1 0 0 0 1 0 0
S
A
B
C
D
0
1
0,1

FA1

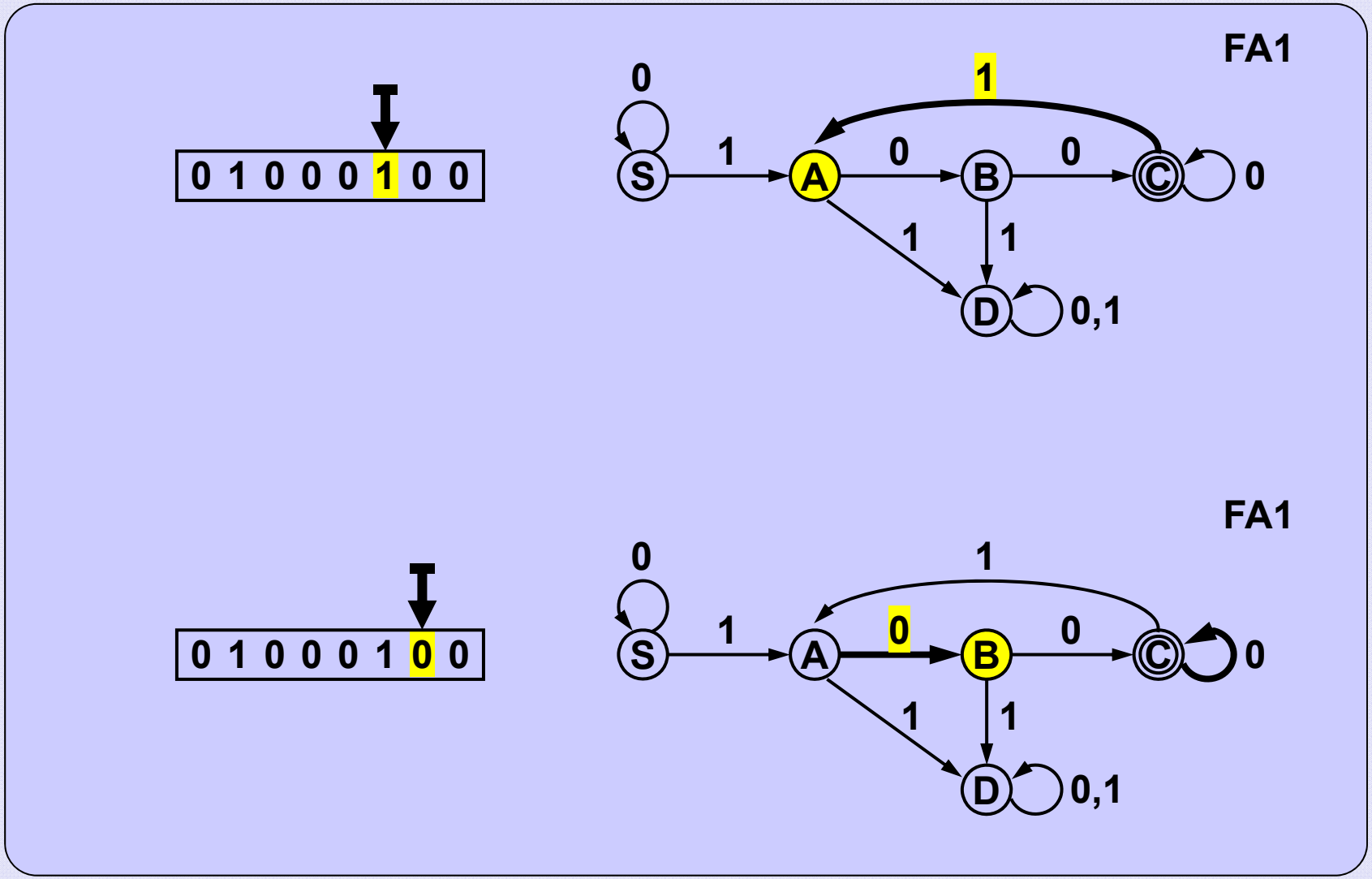


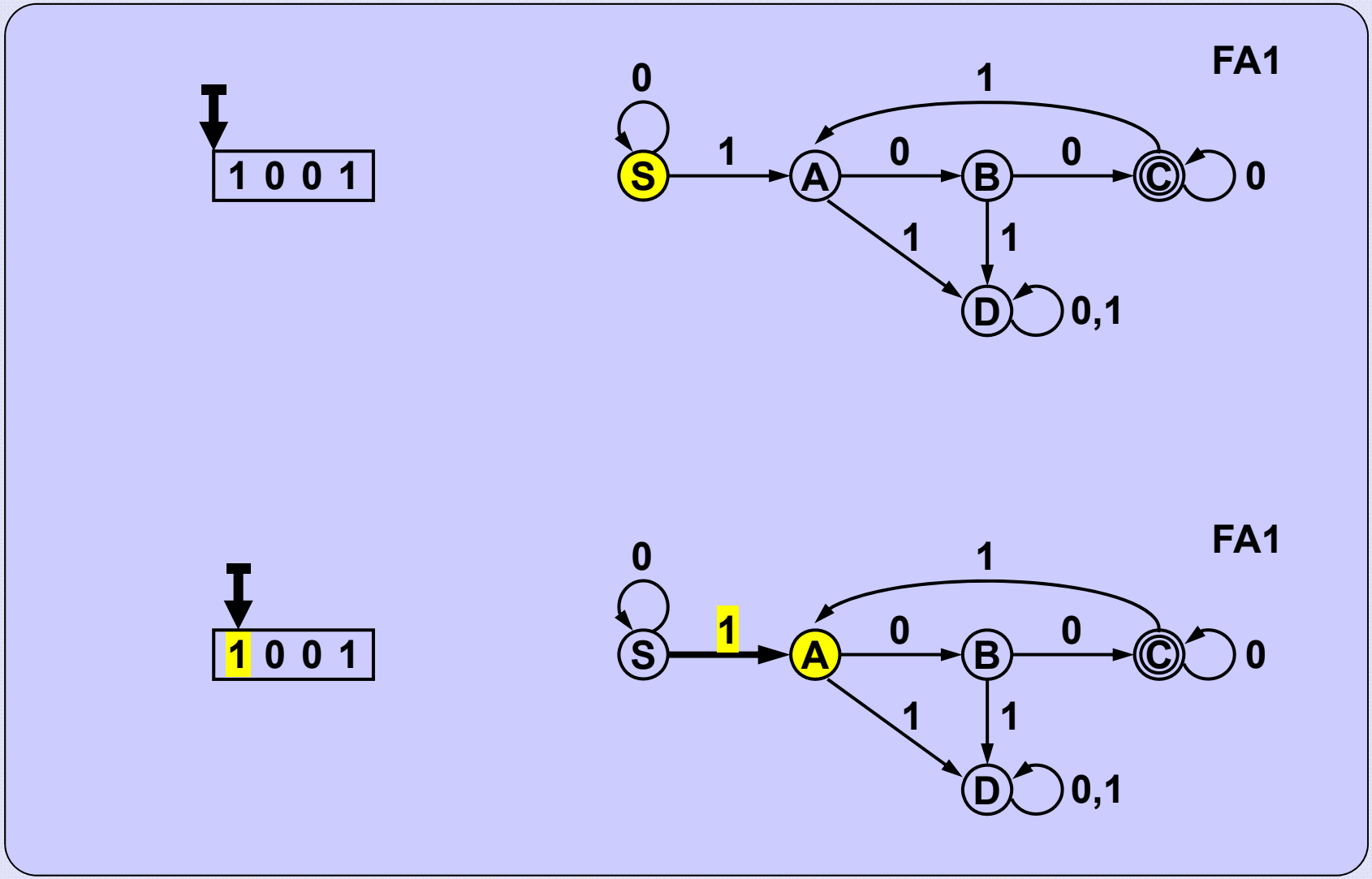


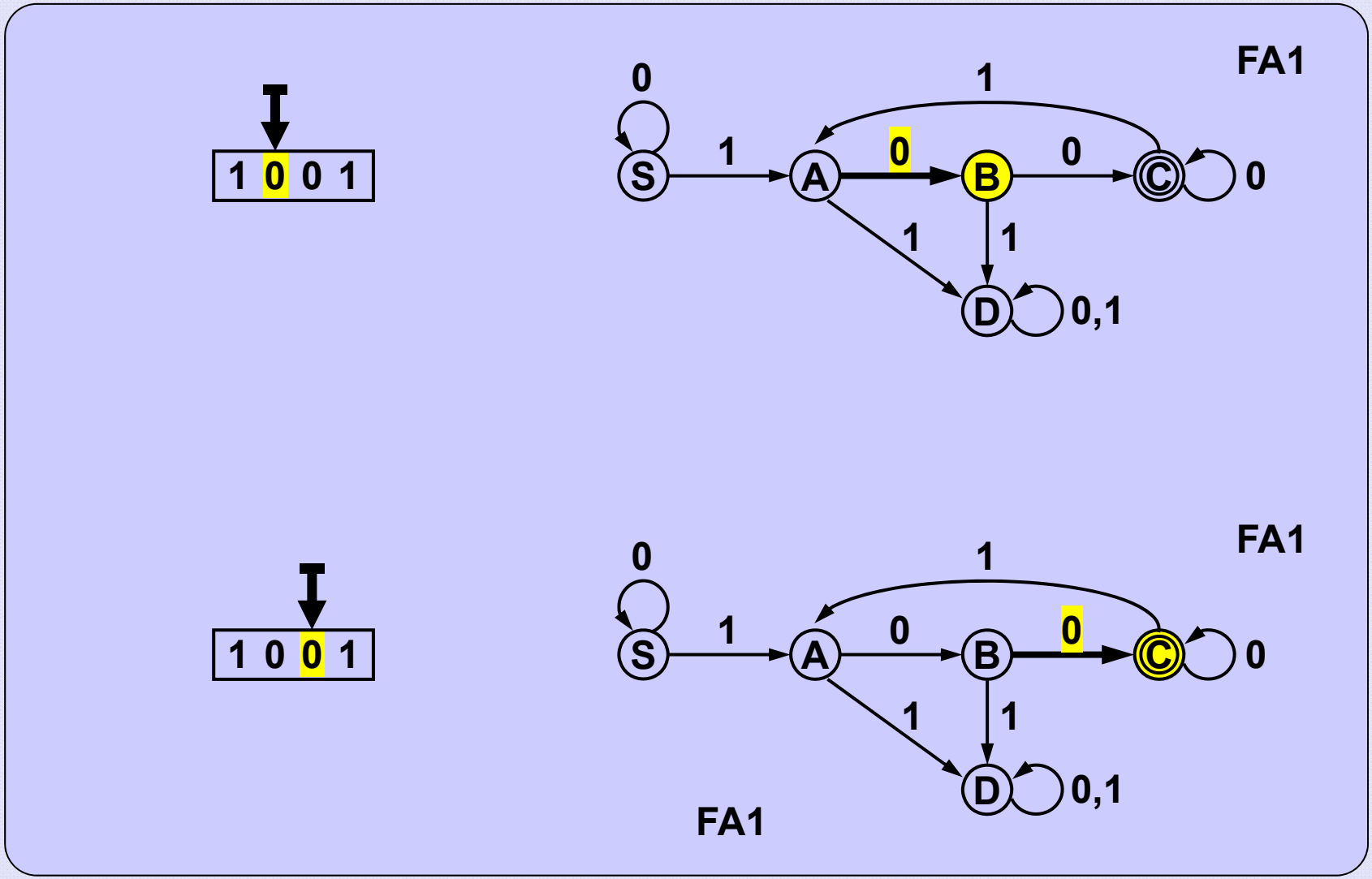
When the last word symbol is read automaton FA1 is in final state ◎

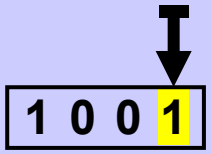
$\Rightarrow$

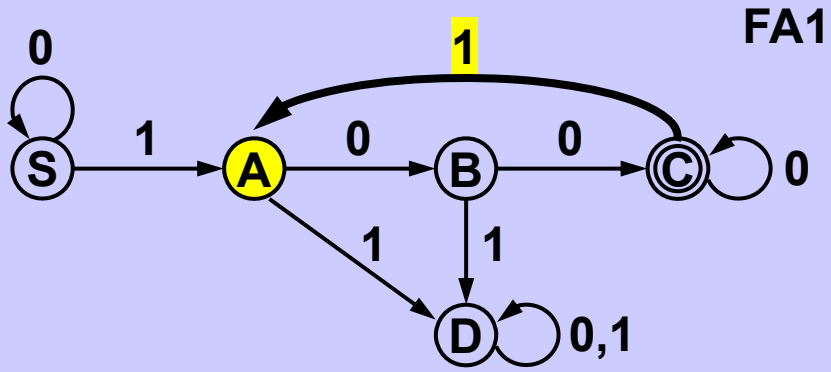
Word 0 1 0 0 0 1 0 0 is accepted by automaton FA1


FA1
0 1 0 0 0 1 0 0
S
A
B
C
D
0
1
0,1
FA1
0 1 0 0 0 1 0 0
S
A
B
C
D
0
1
0,1


1 0 0 1
FA1
S
A
B
C
D
0
1
0,1
1 0 0 1
FA1
S
A
B
C
D
0
1
0,1


1 0 0 1
FA1
0
1
S
1
A
0
B
0
C
0
1
1
D
0,1
1 0 0 1
FA1
0
1
S
1
A
0
B
0
C
0
1
1
D
0,1
FA1

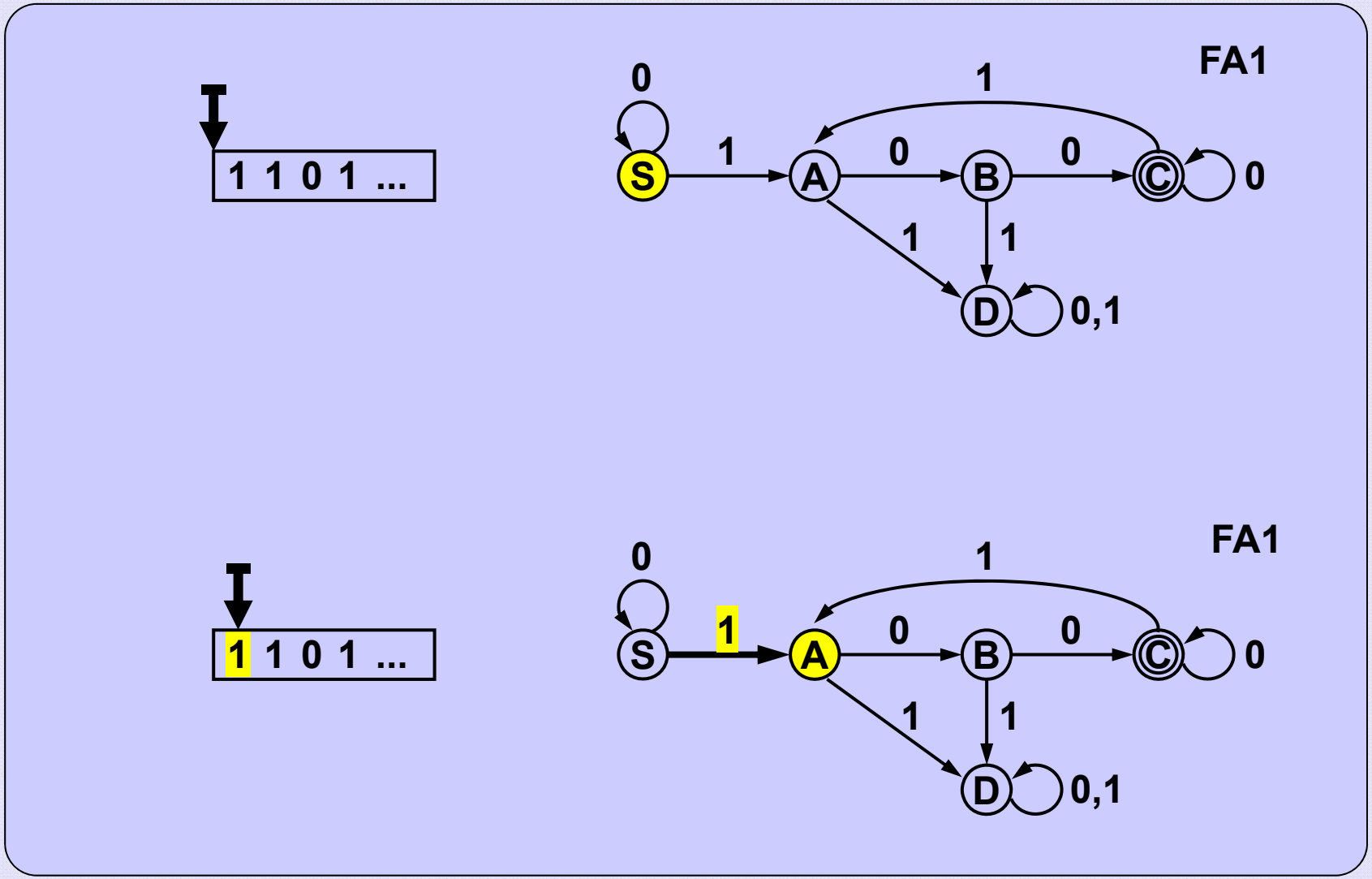


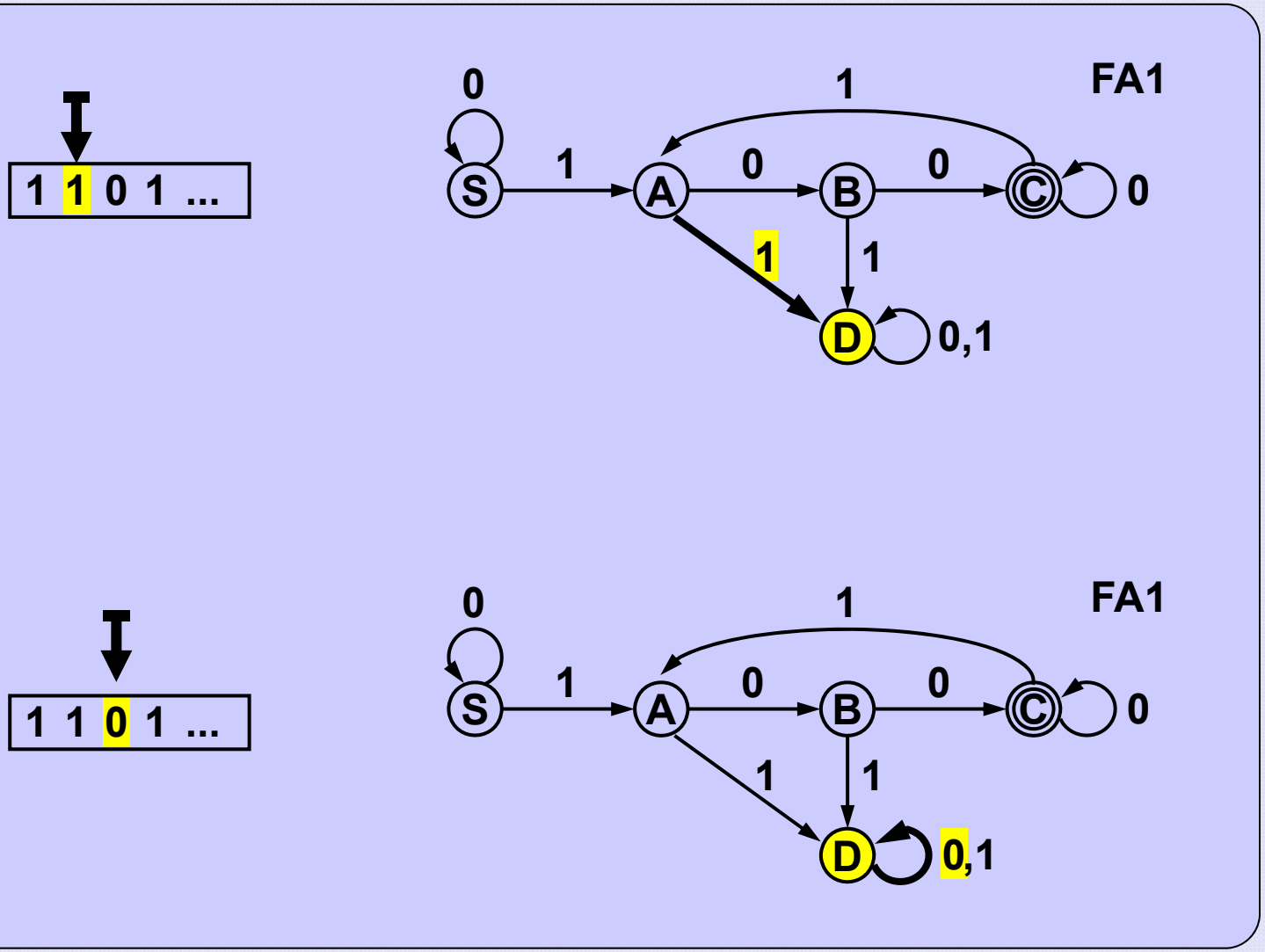


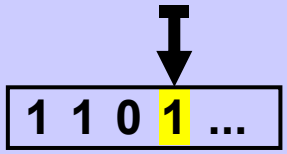
When the last word symbol is read automaton FA1 is in a state which is not final ◯

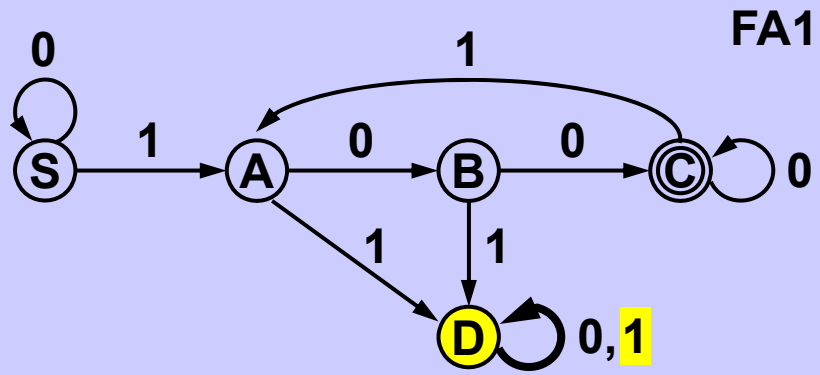
$\Rightarrow$

Word 1 0 0 1 **is not accepted by automaton FA1**


1 1 0 1 ...
FA1
0
1
S
A
B
C
D
0,1
1 1 0 1 ...
FA1
0
1
S
A
B
C
D
0,1


1 1 0 1 ...
FA1
S
A
B
C
D
0
1
0,1
1 1 0 1 ...
FA1
S
A
B
C
D
0
1
0,1





No word starting with	1 1 ...	is accepted by automaton FA1
No word containing	... 1 1 ...	is accepted by automaton FA1
No word containing	... 1 0 1 ...	is accepted by automaton FA1

Automaton FA1 accepts only words -- containing at least one 1
-- containing at least two 0s after each 1

Language accepted by automaton = set of all words accepted by automaton

Automaton activity:

At the begining the automaton is in the start state.

Next it reads the input word symbol by symbol and transits

to other states according to the transition function.

When the word is read the automaton is again in some state.

If it is in a final state, we say that it accepts the word,

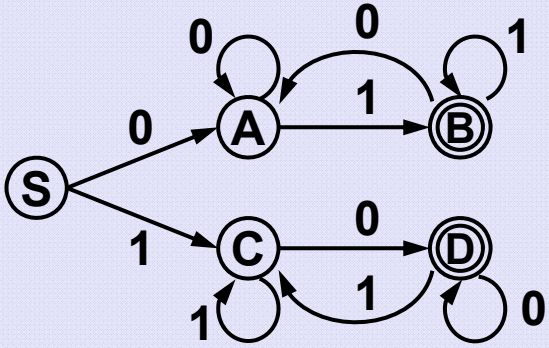
if it is not in a final state, we say that it does not accept the word.

All words accepted by the automaton represent

a language accepted (or recognized) by the automaton.

Language over alphabet {0,1} :

If the word starts with 0, it ends with 1,
If the word starts with 1, it ends with 0.



Automat FA2

Example of analysis of different words by FA2:

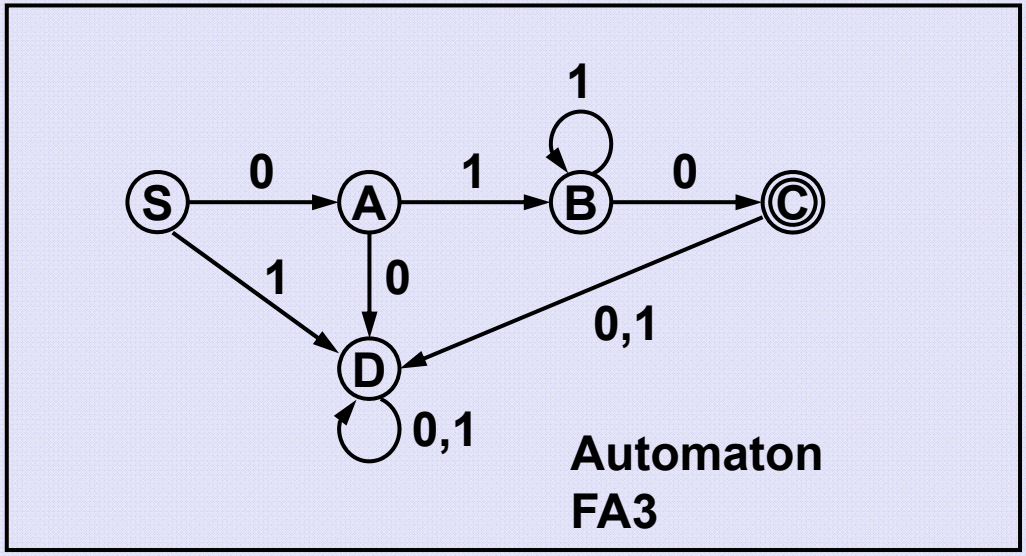
0 1 0 1 0 : (S),0 → (A),1 → (B),0 → (A),1 → (B),0 → (A)

(A) is not a final state, word 0 1 0 1 0 is rejected by FA2.

1 0 1 1 0 : (S),1 → (C),0 → (D),1 → (C),1 → (C),0 → (D)

(D) is a final state, word 1 0 1 1 0 is accepted by FA2.

Language:
{
0 1 0,
0 1 1 0,
0 1 1 1 0,
0 1 1 1 1 0,
0 1 1 1 1 1 0,
... **}**



Automaton FA3

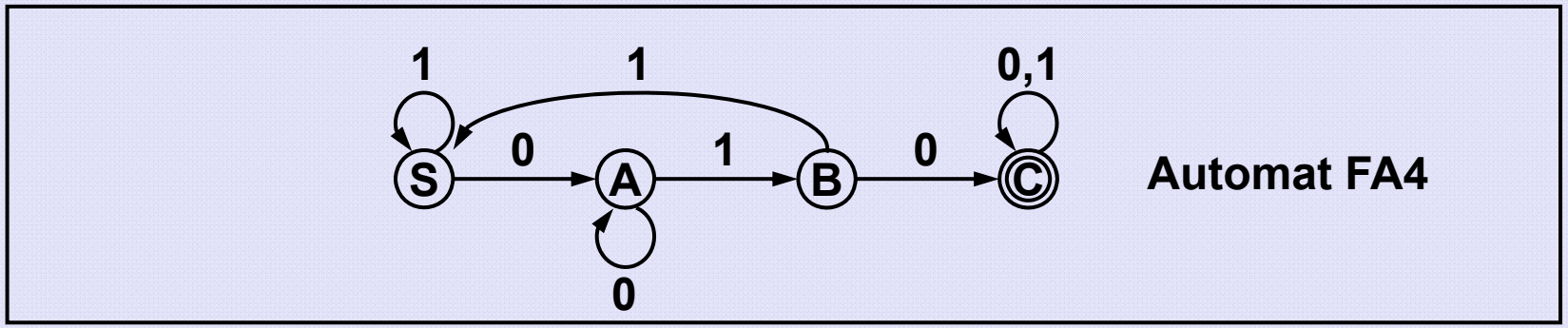
Example of analysis of different words by FA3:

0 1 0 1 0 : **(S),0 → (A),1 → (B),0 → (C),1 → (D),0 → (D)**

(D) is not a final state, word 0 1 0 1 0 is rejected by FA3.

0 1 1 1 0 : **(S),0 → (A),1 → (B),1 → (B),1 → (B),0 → (C)**

(C) is a final state, word 0 1 1 1 0 is accepted by FA3.



Automaton FA4 accepts each word over the alphabet {0,1} which contains subsequence ... 0 1 0 ...

Example of analysis of different words by FA4:

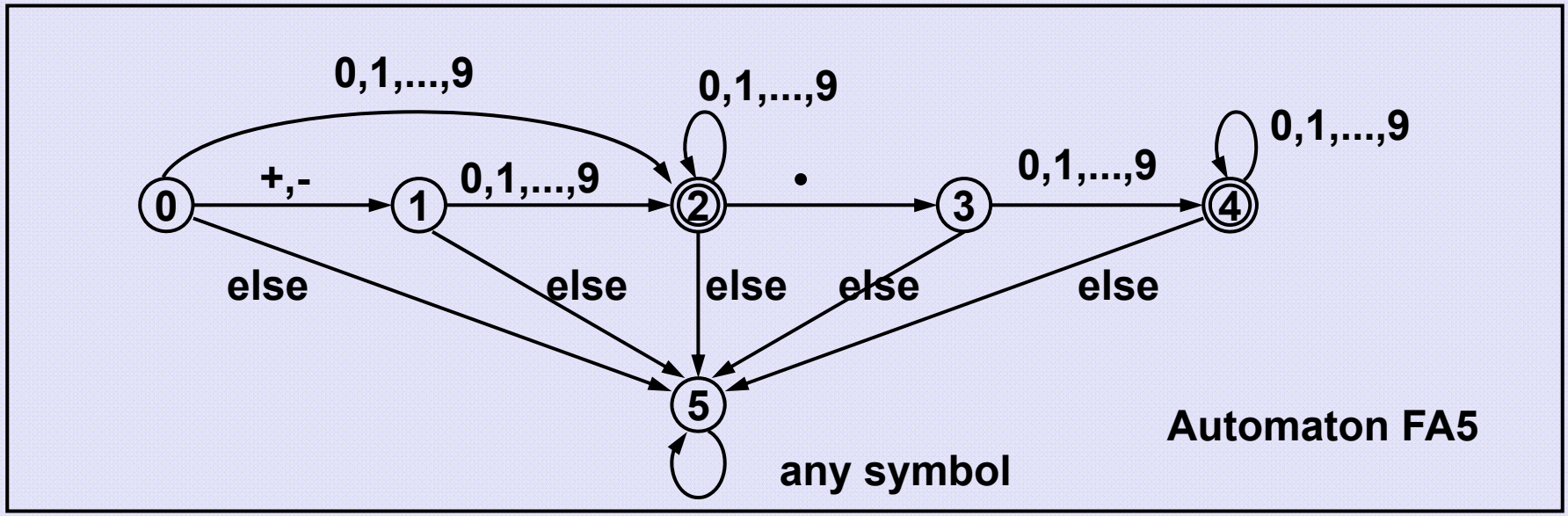
0 0 1 0 1 : (S),0 → (A),0 → (A),1 → (B),0 → (C),1 → (C)

(C) is a final state, word 0 0 1 0 1 is accepted by FA4.

0 1 1 1 0 : (S),0 → (A),1 → (B),1 → (S),1 → (S),0 → (A)

(A) is not a final state, word 0 1 1 1 0 is rejected by FA4.

Language over the alphabet { +, - , . , 0, 1, …, 8, 9, … } whose words represent a decimal numbers



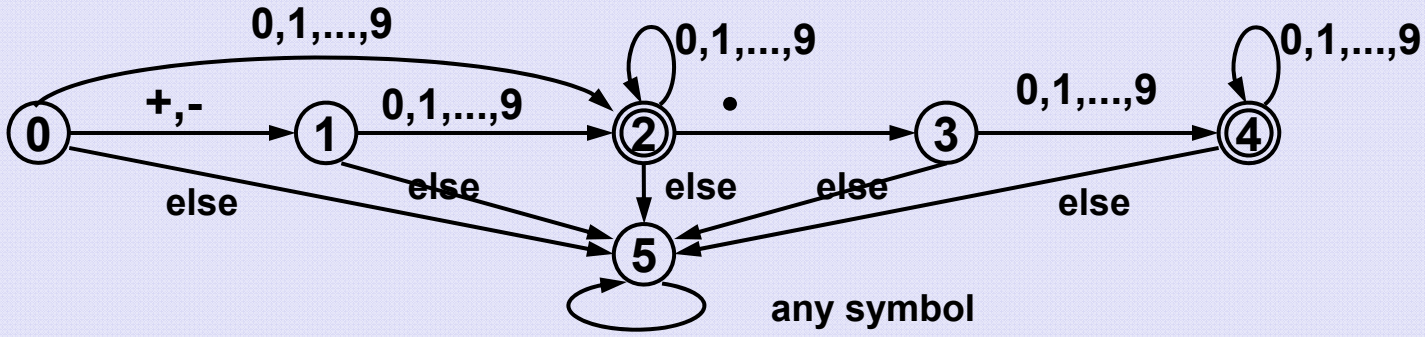
Example of word analysis

+87.09: (0),+ → (1),8 → (2),7 → (2), . → (3),0 → (4),9 → (4)

(4) is a final state, word +87.05 is accepted by FA5.

76+2: (0),7 → (2),6 → (2),+ → (5),2 → (5)

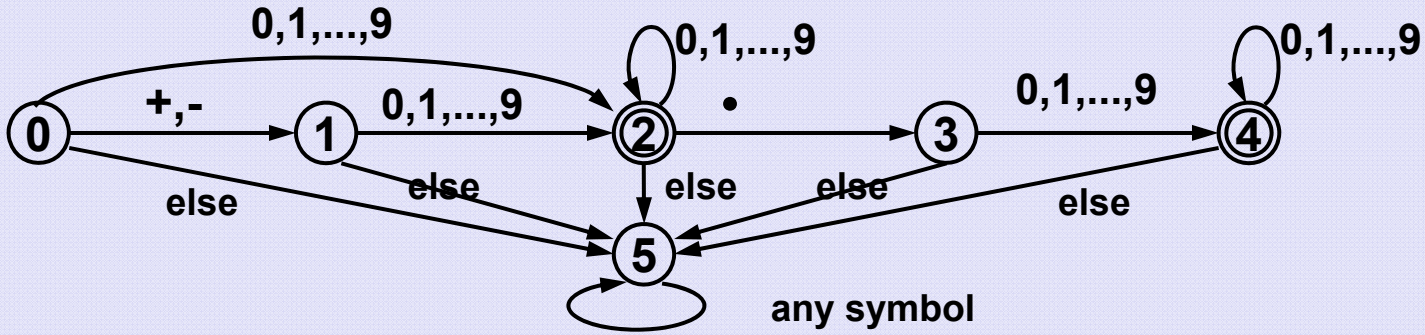
(5) is not a final state, word 76+2 is not accepted by FA5.



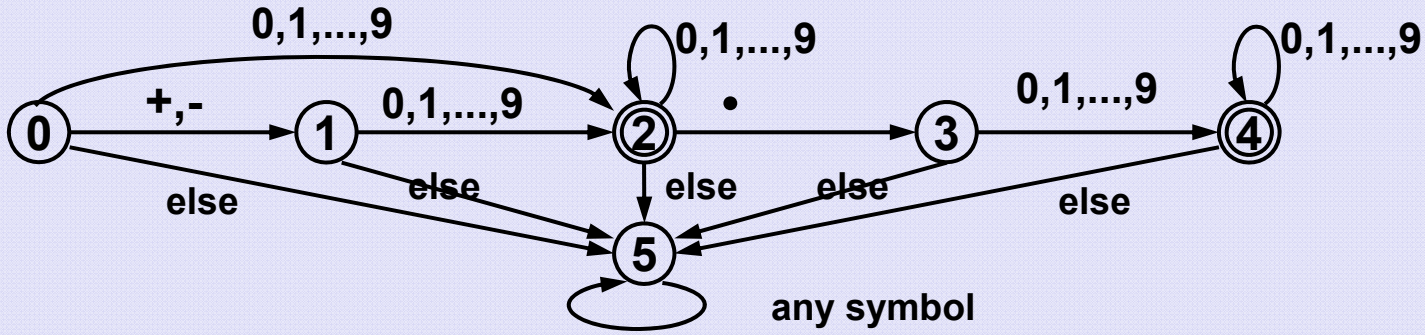
Code of the finite automaton

(The word which is being read is stored in the array arr[ ]):

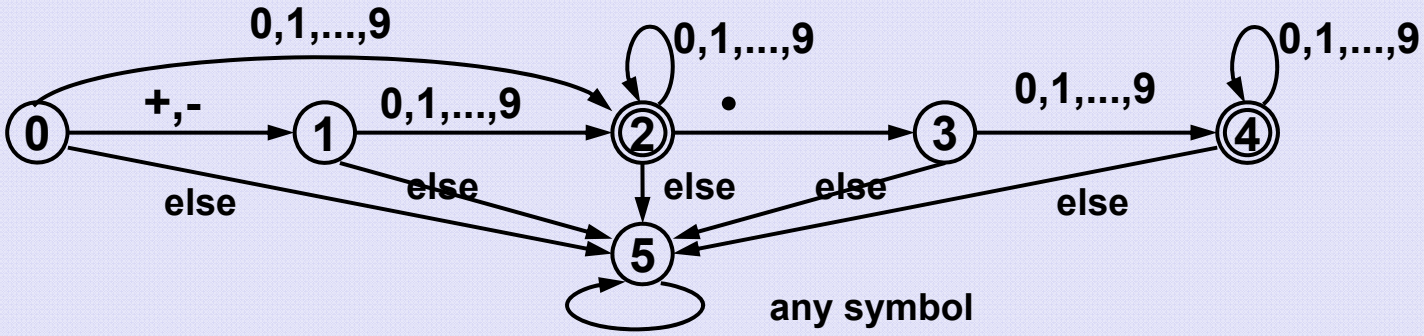
```
int isDecimal(char arr[], int length) {
int i;
int state = 0;

for(i = 0; i < length; i++) {  // check each symbol
  switch (state) {
  ...
```



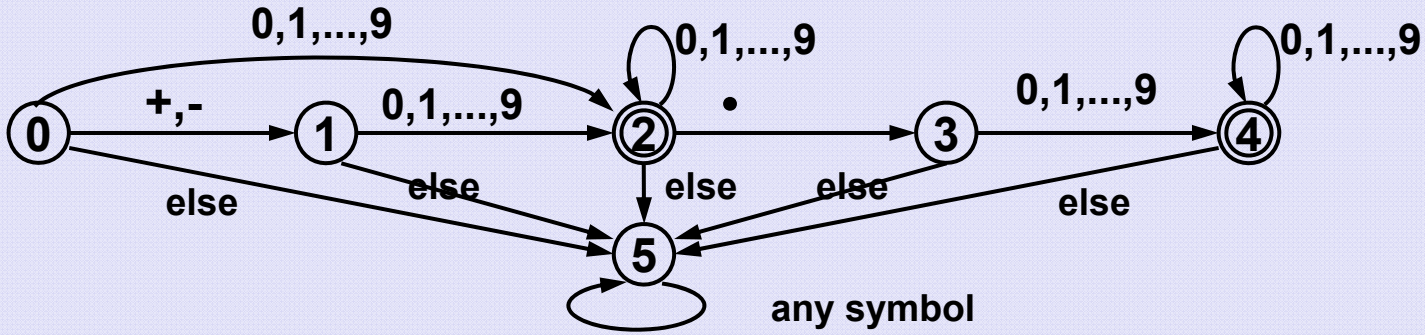

```
case 0:
  if ((arr[i] == '+') || (arr[i] == '-')) state = 1;
  else
  if ((arr[i] >= '0') && (arr[i] <= '9')) state = 2;
  else state = 5;
  break;
```



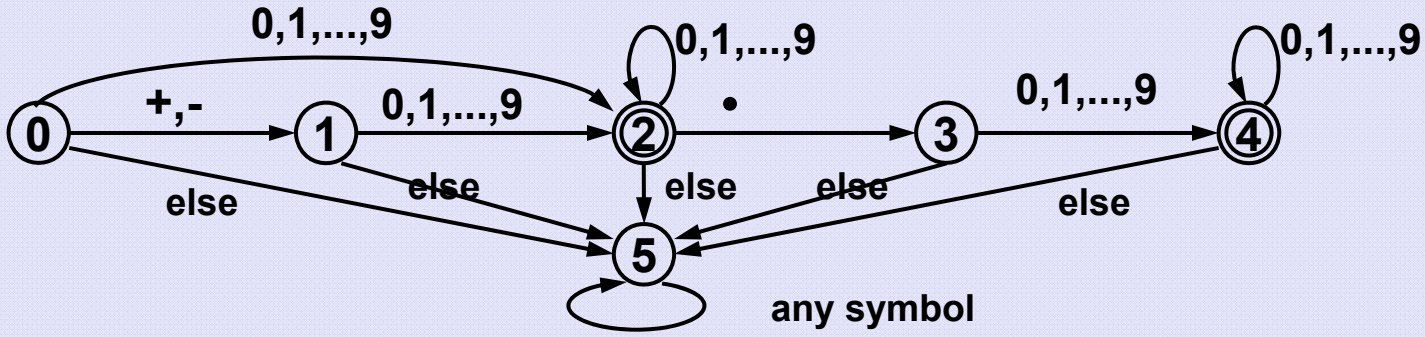

```
case 1:
  if ((arr[i] >= '0') && (arr[i] <= '9')) state = 2;
  else state = 5;
  break;
```




```
case 2:
  if ((arr[i] >= '0') && (arr[i] <= '9')) state = 2;
  else
  if (arr[i] == '.') state = 3;
  else state = 5;
  break;
```




```
③ case 3:
     if ((arr[i] >= '0') && (arr[i] <= '9')) state = 4;
     else state = 5;
     break;
④ case 4:
     if ((arr[i] >= '0') && (arr[i] <= '9')) state = 4;
     else state = 5;
     break;
⑤ case 5: break;  // no need to react anyhow
   default : break;
 } // end of switch
```




```
  } // end of for loop -- word has been read

if ((state == 2) || (state == 4)) // final states!!
     return 1;                    // success - decimal OK
else
     return 0;                    // not a decimal

} // end of function isDecimal()
```