Alphabet

Alphabet … finite (unempty) set of symbols

|A| … **size** of alphabet A

Examples:
A = { ‘A’, ‘D’, ‘G’, ‘O’, ‘U’}, |A| = 5
A = {0,1}, |A| = 2
A = {○, □, △}, |A| = 3

word

Word (over alphabet A) … finite (maybe empty) sequence of symbols of alhabet (A)

also **string**

|w| … **length** of word w

Examples:
w = OUAGADOUGOU, |w| = 11
w = 1001, |w| = 4
w = □△○△□, |w| = 5

Language

<u>Language</u> … set of words (=strings)
(not necessarily finite, can be empty)
|L| … <u>cardinality</u> of language L

---

① Language specification -- List of all words in the language (only for finite language!)

Examples:

A_1 = {'A', 'D', 'G', 'O', 'U'}

L_1 = {ADA, DOG, GOUDA, D, GAG}, $|L_1| = 5$

A_2 = {0,1}

L_2 = {0, 1, 00, 01, 10, 11}, $|L_2| = 6$

A_3 = {○, □, △}

L_3 = {△△, ○□○, □□△○}, $|L_2| = 3$

2 **Language specification** -- **Informal (but unambiguous) description in natural human language (usually for infinite language)**

Examples:

A_1 = {'A', 'D', 'G', 'O', 'U'}

L_1: Set of all words over A_1, which begin with DA, end with G a and do not contain subsequence AA.

L_1 = {DAG, DADG, DAGG, DAOG, DAUG, DADAG, DADDG… }

$|L_1| = \infty$

$A_2 = \{0,1\}$

L_2: Set of all words over A_2, which contain more 1s than 0s and where each 0 is followed by at least two 1s.

L_2 = { 1, 11, 011, 0111, 1011, 1111, … , 011011, 011111, … }

$|L_2| = \infty$

③ Language specification -- By finite automaton

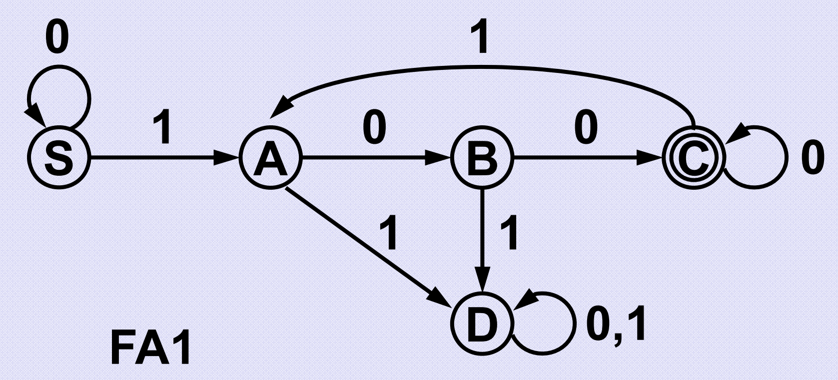
Finite automaton
is a five-tuple (A, Q, σ, S_0, Q_F), where:

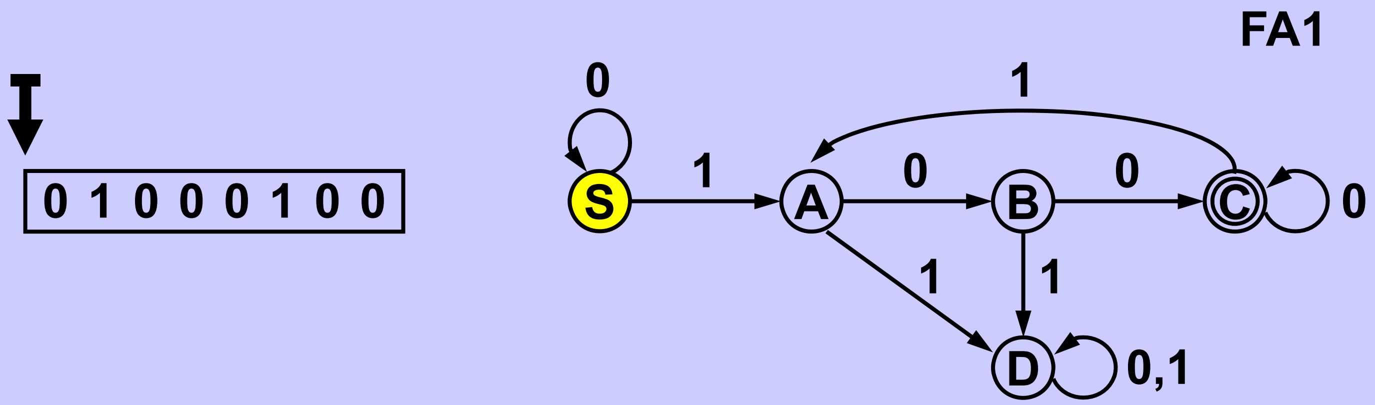
A … alphabet … finite set of symbols
|A| ... size of alphabet
Q ... set of states (often numbered) (what is „a state“ ?)
σ ... transition function ... $\sigma: Q \times A \to Q$
S_0 ... start state $S_0 \in Q$
Q_F ... unempty set of final states $\emptyset \neq Q_F \subseteq Q$

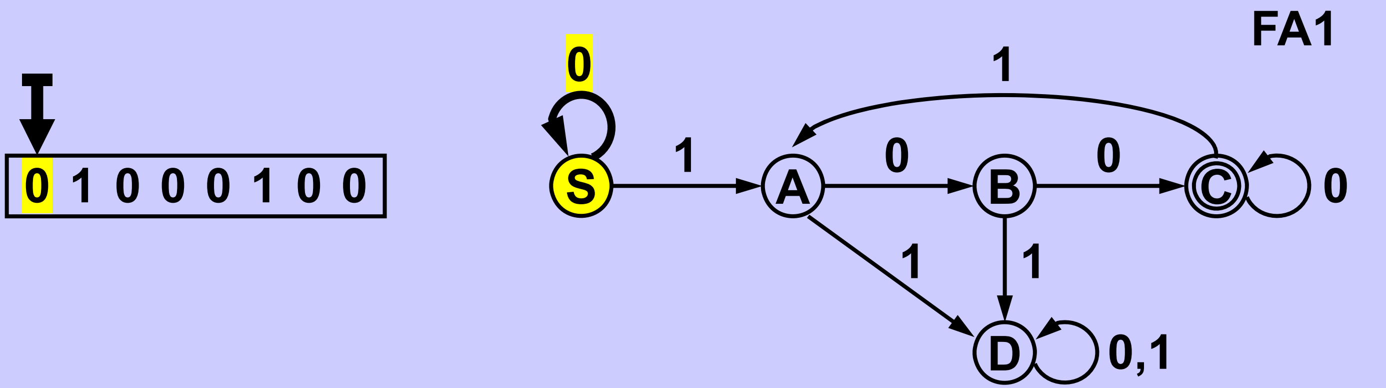
Automaton FA1:

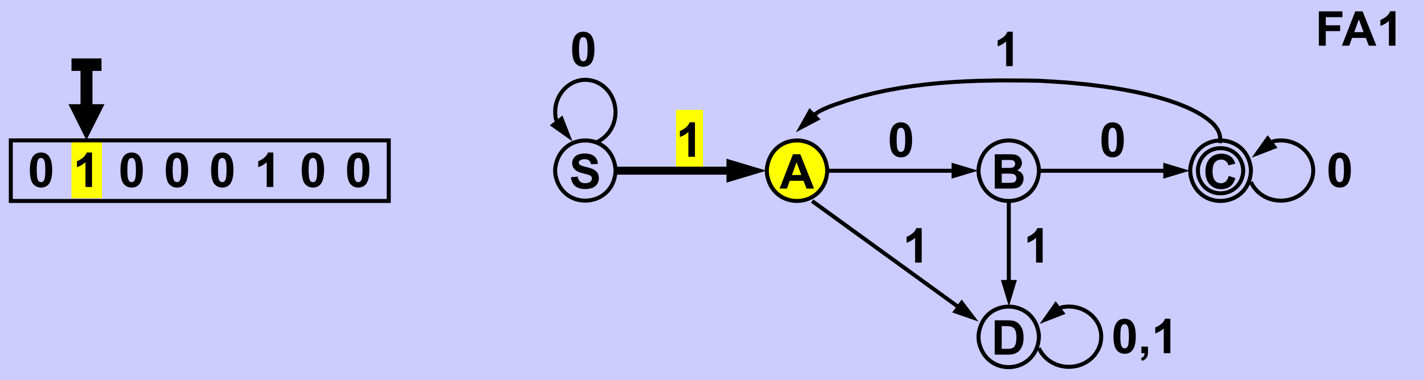
A … alphabet … {0,1}, |A| = 2
Q ... set of states {S, A, B, C, D }
σ ... transition function ... σ: Q×A → Q : {
σ(S,0) = S, σ(A,0) = B, σ(B,0) = C, σ(C,0) = C, σ(D,0) = D,
σ(S,1) = A, σ(A,1) = D, σ(B,1) = D, σ(C,1) = A, σ(D,1) = D }
S_0 ... start state S ∈ Q
Q_F … unempty set of final states ∅ ≠ { C } ⊆ Q

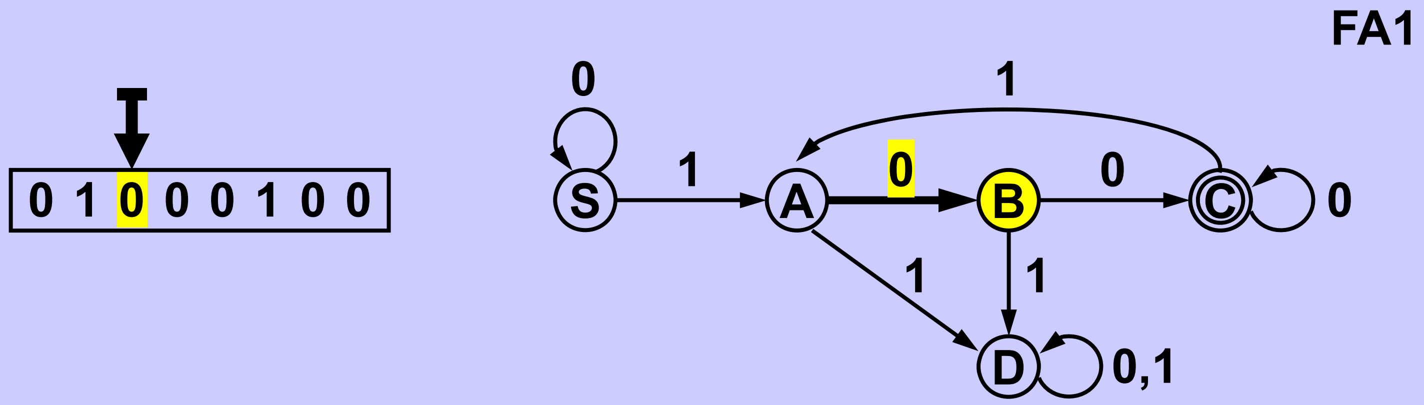
Transition diagram of the automaton FA1

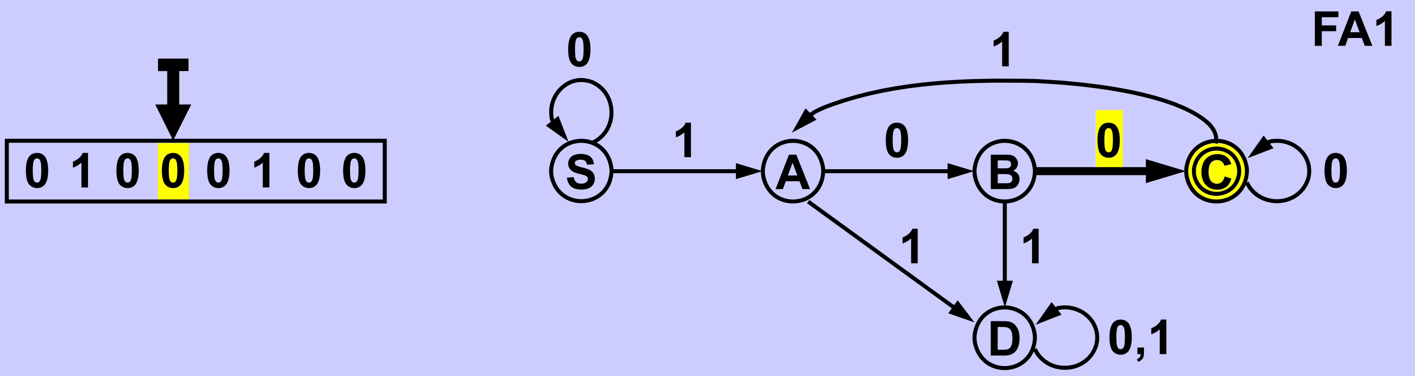


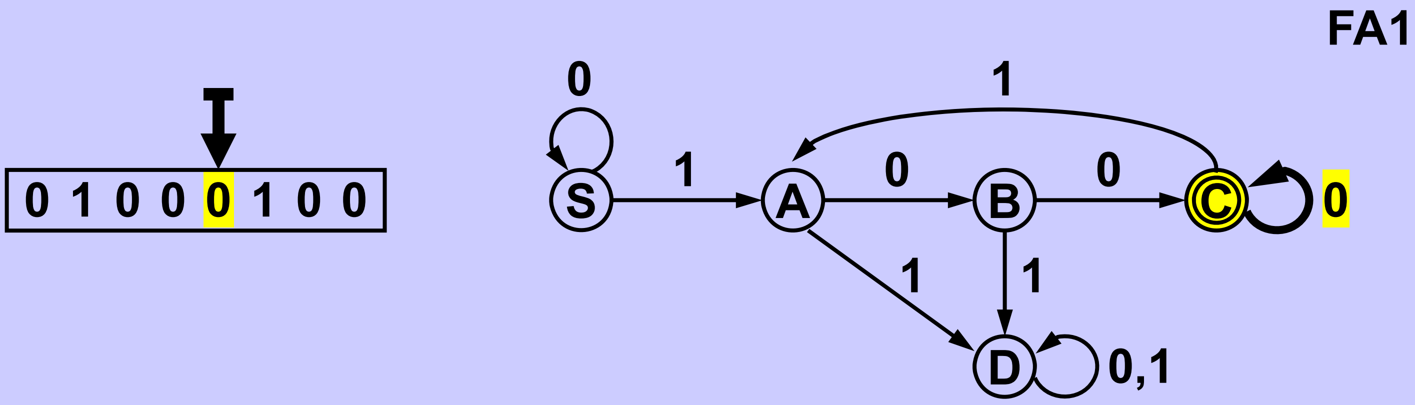

FA1
0 1 0 0 0 1 0 0
0
1
1
0
0
0
1
1
0,1
S
A
B
C
D

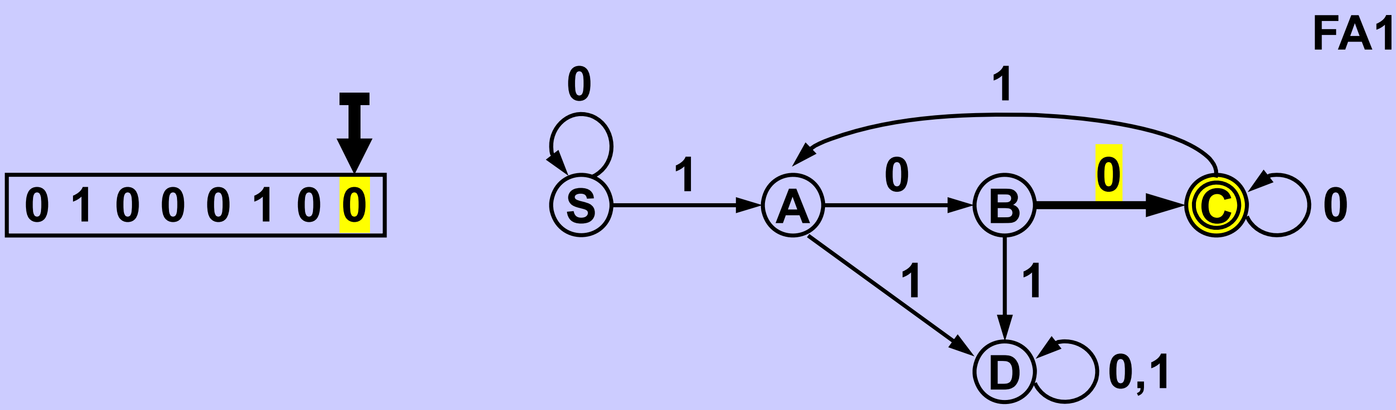

FA1
0 1 0 0 0 1 0 0
0
1
1
0
0
0
1
1
0,1
S
A
B
C
D


FA1
0 1 0 0 0 1 0 0
0
1
1
0
0
0
1
1
0,1
S
A
B
C
D


FA1
0 1 0 0 0 1 0 0
0
1
1
0
0
0
1
1
0,1
S
A
B
C
D


0 1 0 0 0 1 0 0
FA1
0
1
1
0
0
0
1
1
0,1
S
A
B
C
D

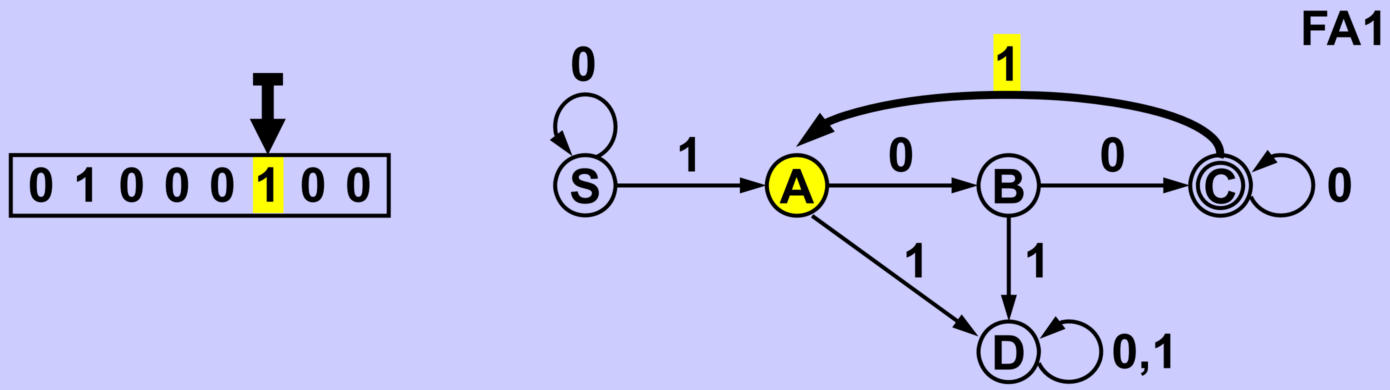

0 1 0 0 0 1 0 0
FA1
0
1
1
0
0
0
1
1
0,1
S
A
B
C
D

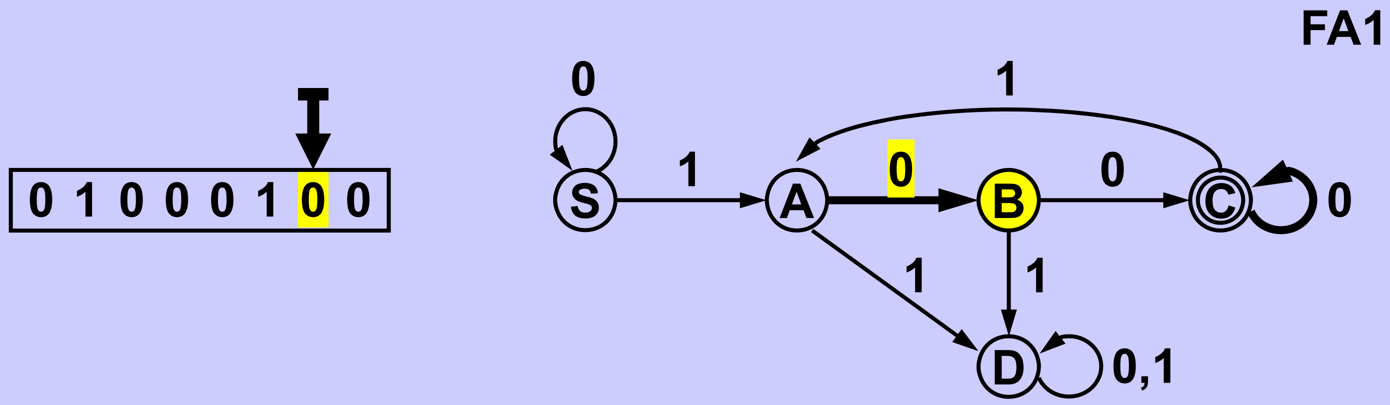


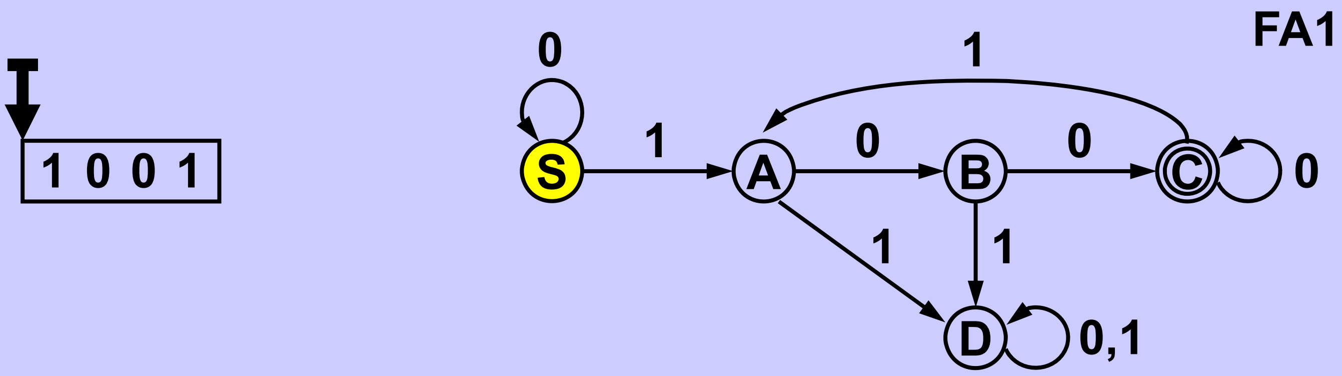
When the last word symbol is read automaton FA1 is in final state ◎

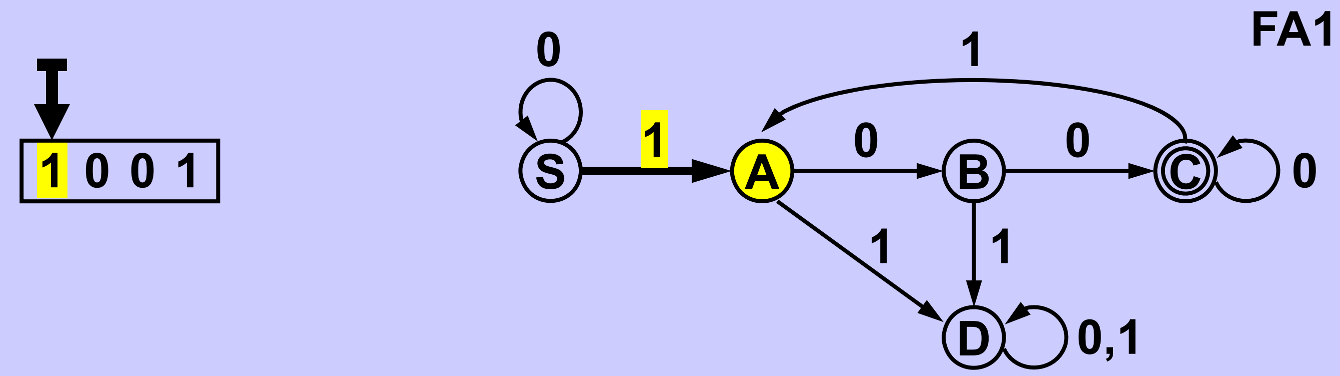
$\Rightarrow$

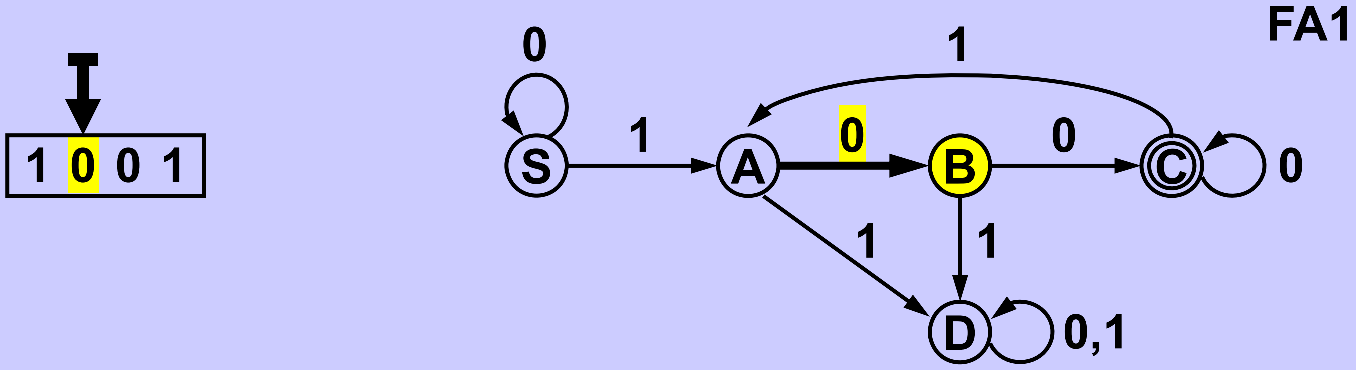
Word **0 1 0 0 0 1 0 0** is accepted by automaton FA1

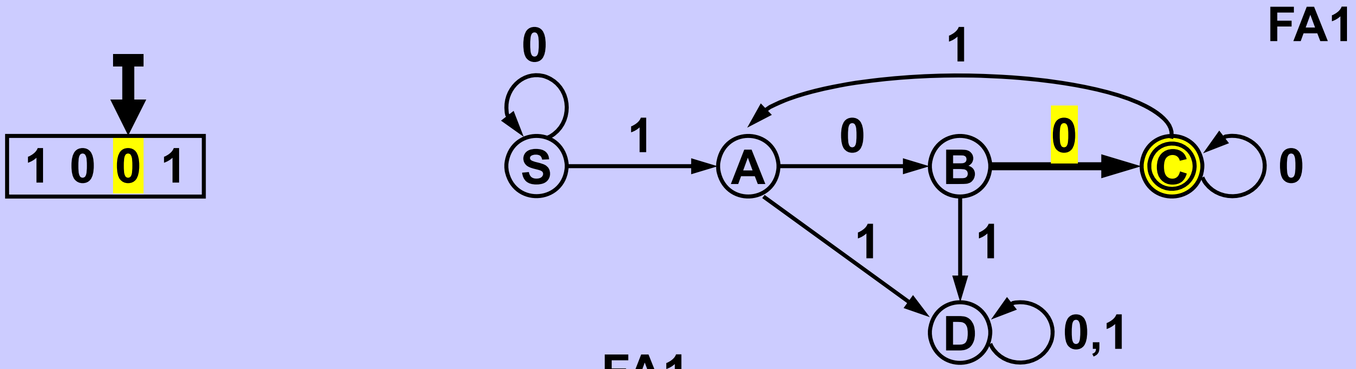

FA1
0 1 0 0 0 1 0 0
0
1
1
0
0
0
1
1
0,1
S
A
B
C
D


FA1
0 1 0 0 0 1 0 0
0
1
1
0
0
0
1
1
0,1
S
A
B
C
D

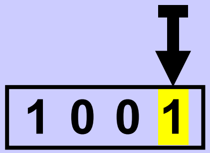

1 0 0 1
FA1
0
1
1
S
A
0
B
0
C
0
1
1
D
0,1

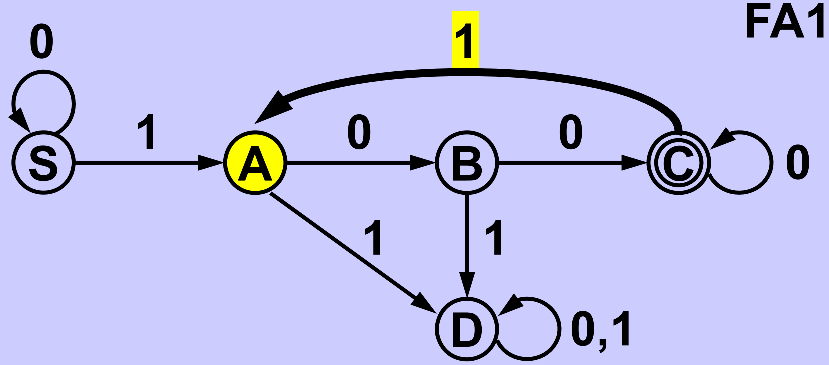

1 0 0 1
FA1
0
1
1
S
A
0
B
0
C
0
1
1
D
0,1





FA1

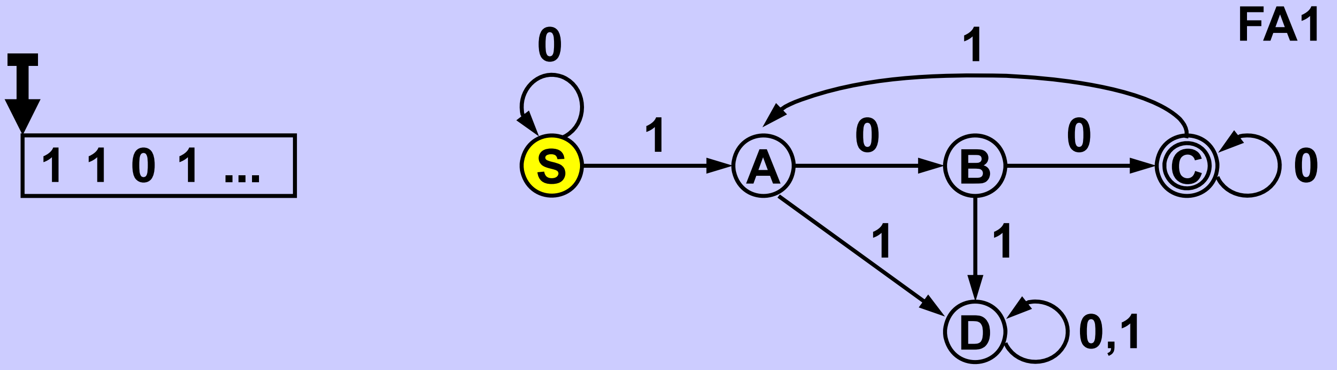


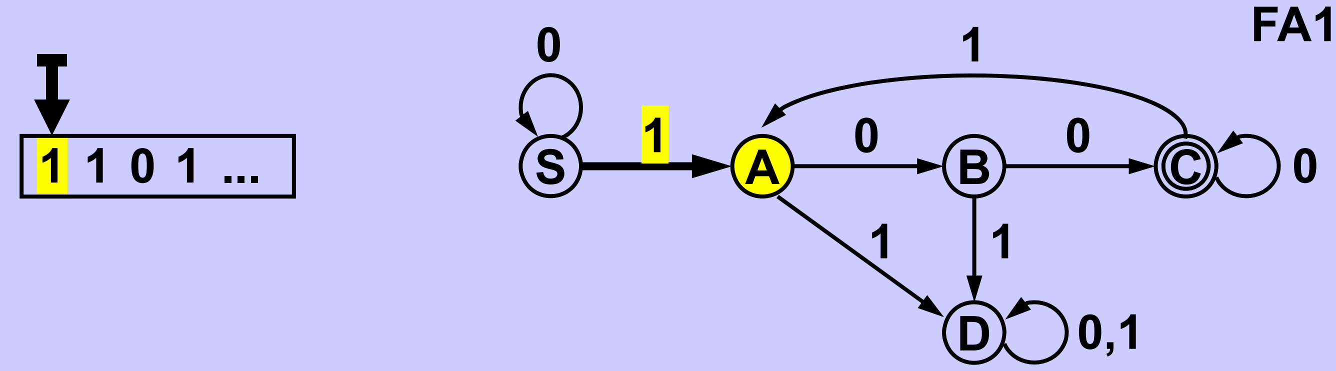


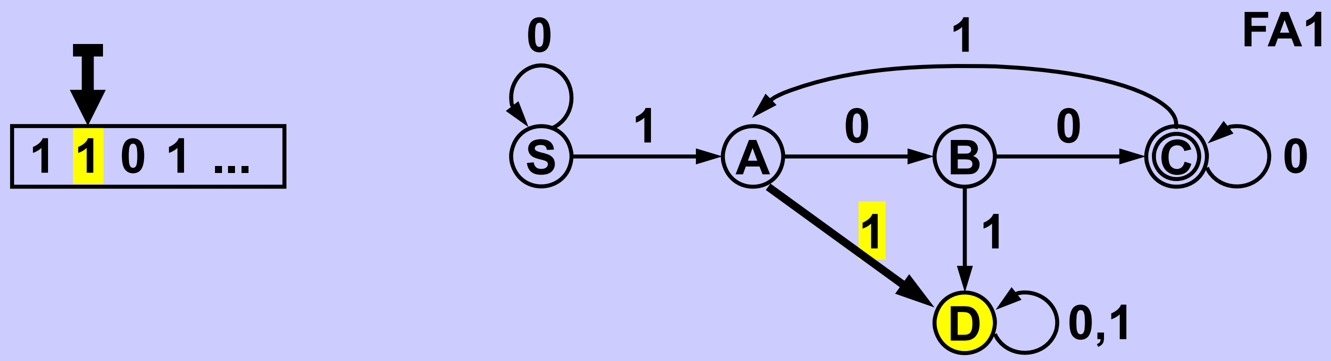
When the last word symbol is read automaton FA1 is in a state which is not final

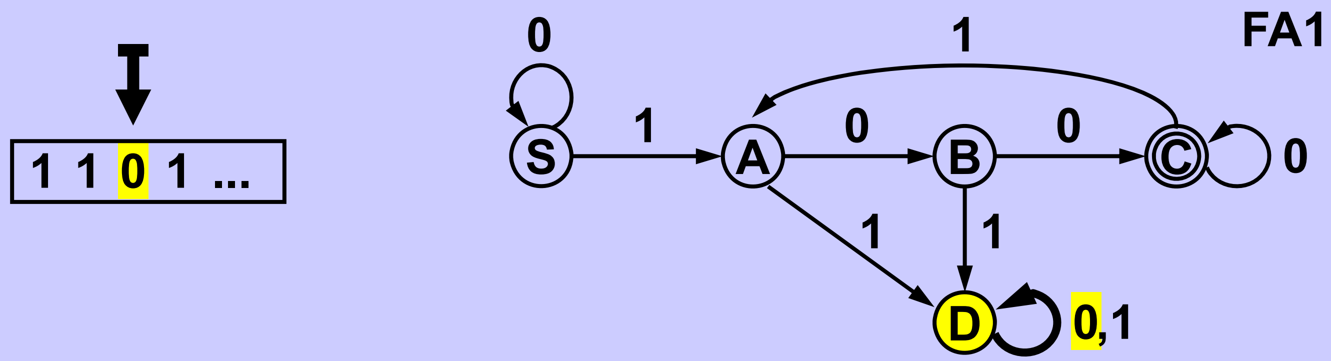
$\Rightarrow$

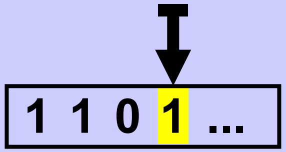
Word **1 0 0 1** **<u>is not accepted</u> by automaton FA1**

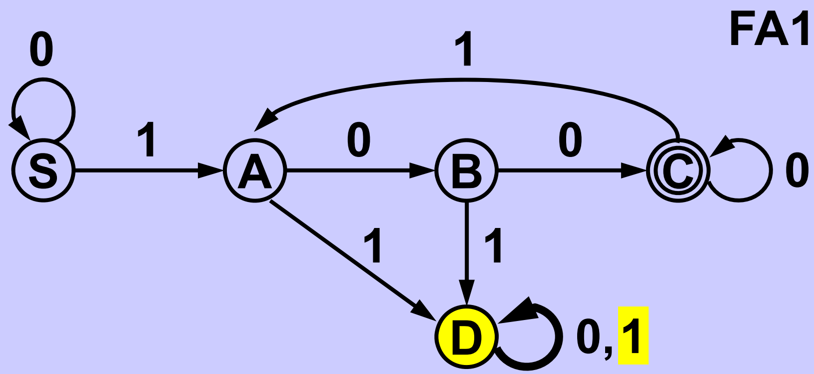

1 1 0 1 ...
FA1
0
1
1
0
0
0
1
1
0,1
S
A
B
C
D


1 1 0 1 ...
FA1
0
1
1
0
0
0
1
1
0,1
S
A
B
C
D


1 1 0 1 ...
FA1
0
1
1
0
0
0
1
1
0,1
S
A
B
C
D


1 1 0 1 ...
FA1
0
1
1
0
0
0
1
1
0,1
S
A
B
C
D





No word starting with 1 1 ... is accepted by automaton FA1

No word containing ... 1 1 ... is accepted by automaton FA1

No word containing ... 1 0 1 ... is accepted by automaton FA1

Automaton FA1 accepts only words
- containing at least one 1
- containing at least two 0s after each 1

Language accepted by automaton X = set of all words accepted by X

Automaton A activity:

At the begining, A is in the start state.

Next, A reads the input word symbol by symbol and transits

to other states according to its transition function.

When the word is read completely A is again in some state.

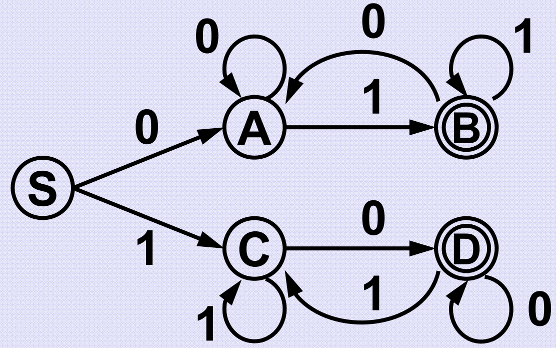
If A is in a final state, we say that <u>A accepts</u> the word,

if A is not in a final state, we say that <u>A does not accept</u> the word.

All words accepted by A represent
<u>a language accepted (or recognized) by A.</u>

Language over alphabet {0,1} :

If a word starts with 0, it ends with 1,
If a word starts with 1, it ends with 0.



Automat FA2

Example of analysis of different words by FA2:

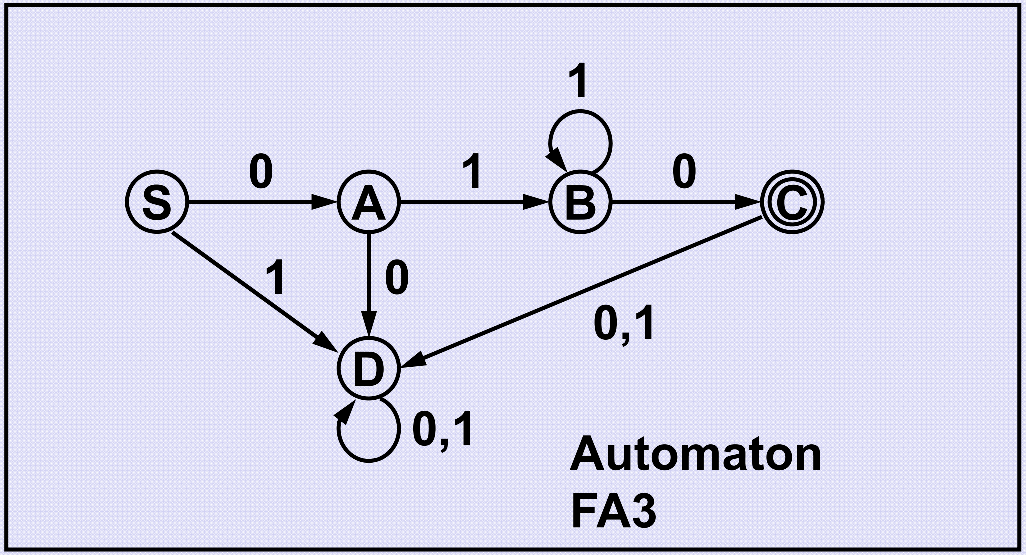
0 1 0 1 0 : (S),0 → (A),1 → (B),0 → (A),1 → (B),0 → (A)

(A) is not a final state, word 0 1 0 1 0 is rejected by FA2.

1 0 1 1 0 : (S),1 → (C),0 → (D),1 → (C),1 → (C),0 → (D)

(D) is a final state, word 1 0 1 1 0 is accepted by FA2.

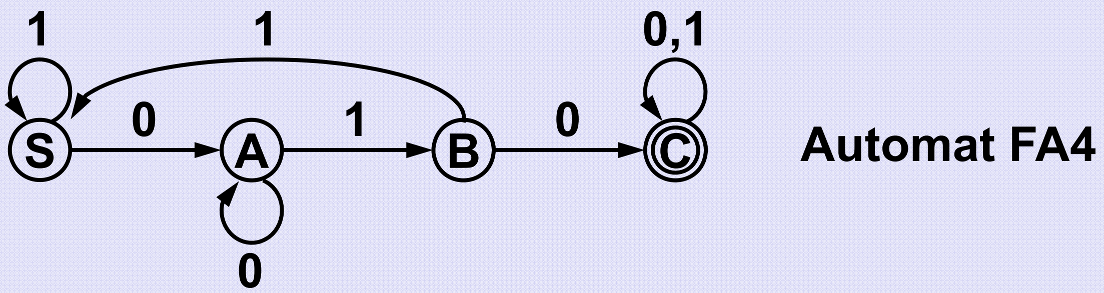
Language:
{
0 1 0,
0 1 1 0,
0 1 1 1 0,
0 1 1 1 1 0,
0 1 1 1 1 1 0,
... }



Example of analysis of different words by FA3:

0 1 0 1 0 : (S),0 → (A),1 → (B),0 → (C),1 → (D),0 → (D)
(D) is not a final state, word 0 1 0 1 0 is rejected by FA3.

0 1 1 1 0 : (S),0 → (A),1 → (B),1 → (B),1 → (B),0 → (C)
(C) is a final state, word 0 1 1 1 0 is accepted by FA3.



Automaton FA4 accepts each word over the alphabet {0,1} which contains subsequence ... 0 1 0 ...

Example of analysis of different words by FA4:

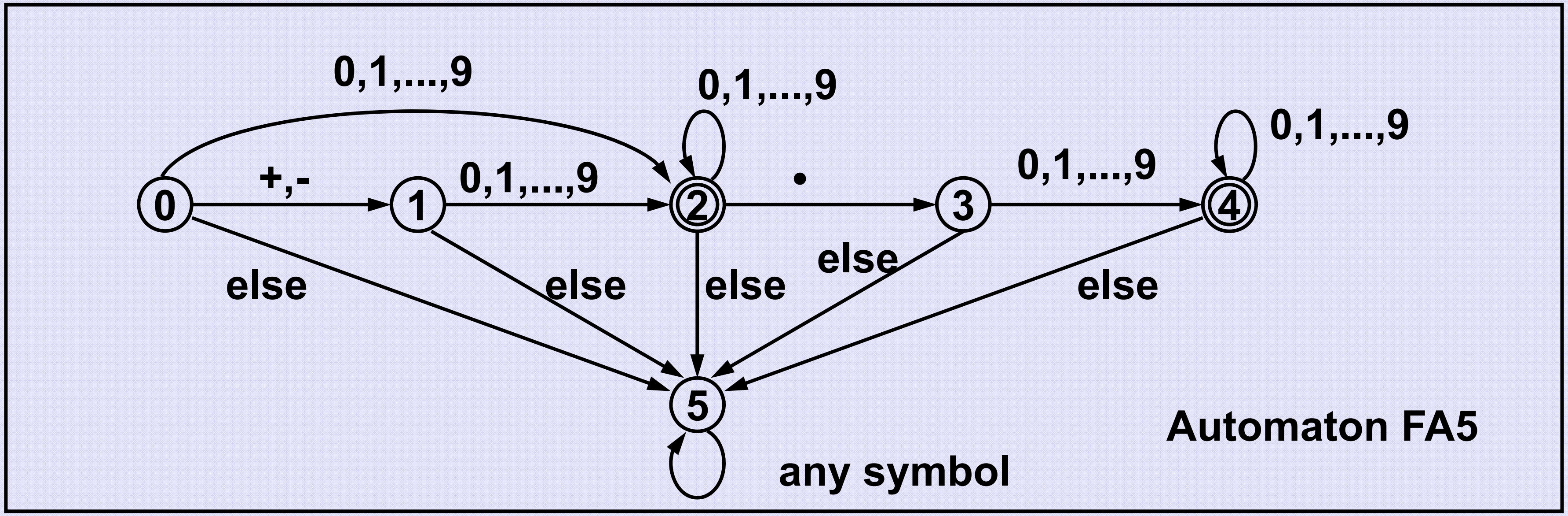
0 0 1 0 1 : (S),0 → (A),0 → (A),1 → (B),0 → (C),1 → (C)

(C) is a final state, word 0 0 1 0 1 is accepted by FA4.

0 1 1 1 0 : (S),0 → (A),1 → (B),1 → (S),1 → (S),0 → (A)

(A) is not a final state, word 0 1 1 1 0 is rejected by FA4.

Language over the alphabet { +, - , . , 0, 1, …, 8, 9, … } whose words represent decimal numbers



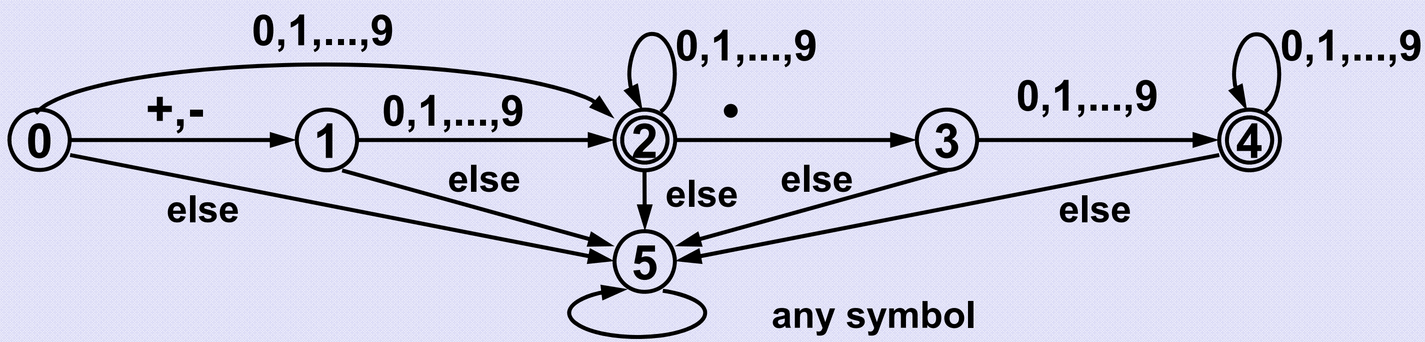
Example of word analysis

+87.09: (0),+ → (1),8 → (2),7 → (2), . → (3),0 → (4),9 → (4)

(4) is a final state, word +87.05 is accepted by FA5.

76+2: (0),7 → (2),6 → (2),+ → (5),2 → (5)

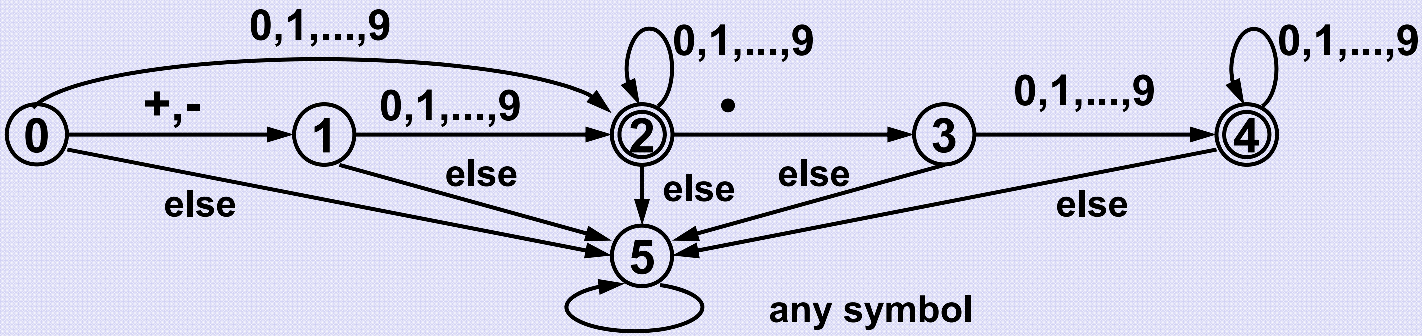
(5) is not a final state, word 76+2 is not accepted by FA5.



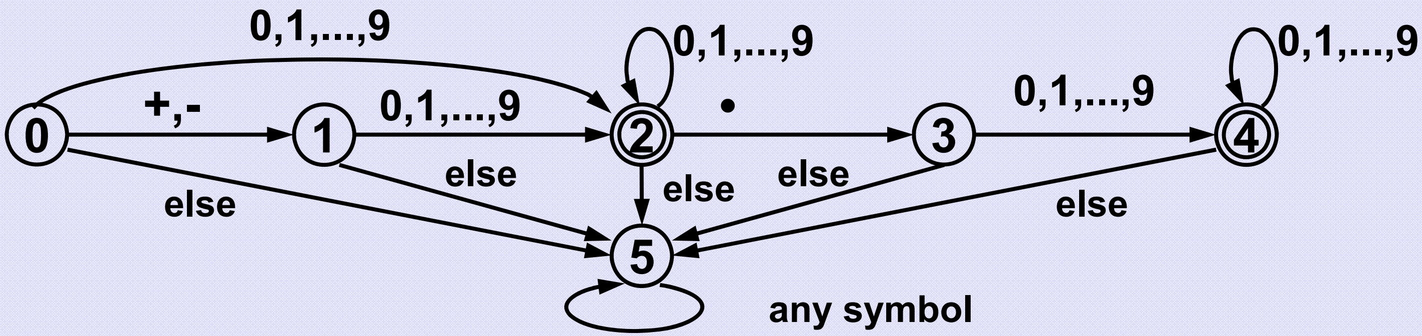
Code of the finite automaton

(The word which is being read is stored in the array arr[ ]):

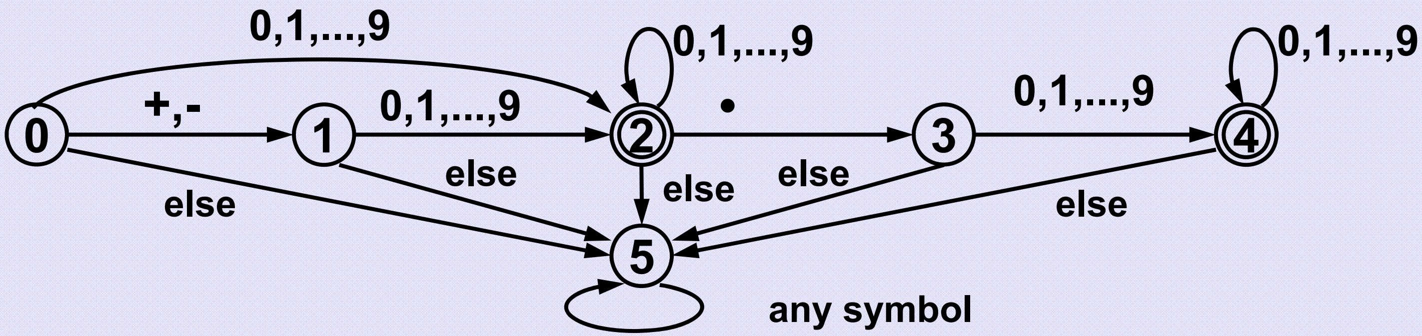
```
int isDecimal(char arr[], int length) {
int i;
int state = 0;

for(i = 0; i < length; i++) {  // check each symbol
  switch (state) {
  ...
```



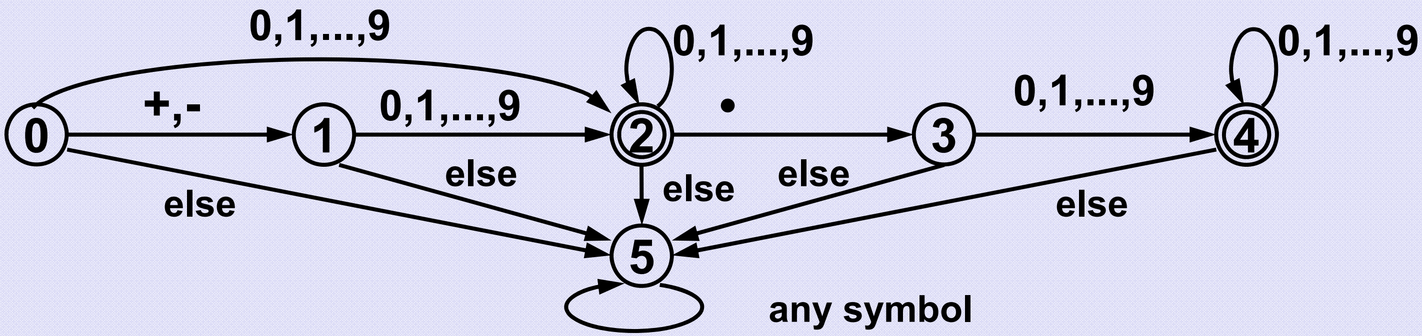

```
case 0:
  if ((arr[i] == '+') || (arr[i] == '-')) state = 1;
  else
  if ((arr[i] >= '0') && (arr[i] <= '9')) state = 2;
  else state = 5;
  break;
```



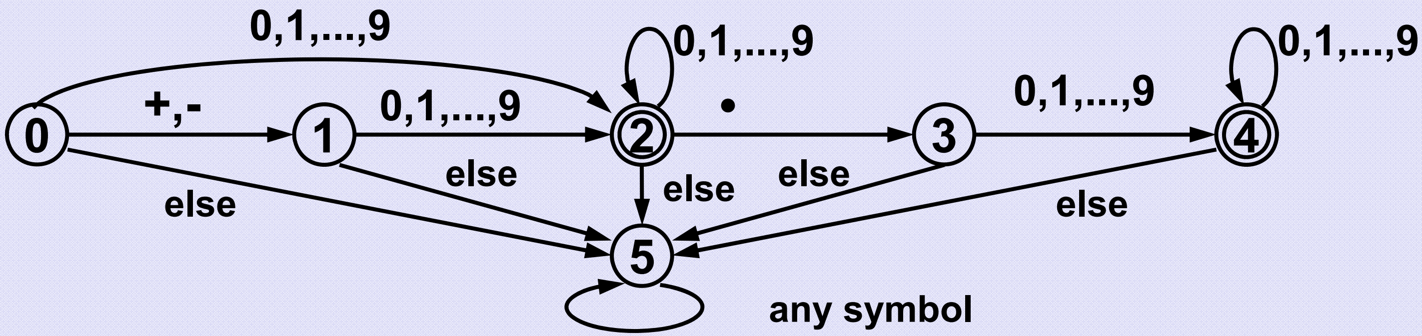

```
case 1:
  if ((arr[i] >= '0') && (arr[i] <= '9')) state = 2;
  else state = 5;
  break;
```




```
case 2:
  if ((arr[i] >= '0') && (arr[i] <= '9')) state = 2;
  else
  if (arr[i] == '.') state = 3;
  else state = 5;
  break;
```




```
(3) case 3:
       if ((arr[i] >= '0') && (arr[i] <= '9')) state = 4;
       else state = 5;
       break;
(4) case 4:
       if ((arr[i] >= '0') && (arr[i] <= '9')) state = 4;
       else state = 5;
       break;
(5) case 5: break;  // no need to react anyhow
    default : break;
 } // end of switch
```




```
  } // end of for loop -- word has been read

if ((state == 2) || (state == 4)) // final states!!
     return 1;                    // success - decimal OK
else
     return 0;                    // not a decimal

} // end of function isDecimal()
```