PLÁNOVÁNÍ A HRY - CV 1

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Course Preparation / Recap

- Algorithm Properties
- Searches
- Logics
- Satisfiability Problem

ALGORITHM PROPERTIES

Algorithm Properties

- **Soundness**
  - The result returned by the algorithm is a solution to the problem
- **Completeness**
  - If a solution exists, the algorithm finds it
- **Admissibility**
  - It is guaranteed that the algorithm finds the optimal solution
  - Optimality has to be defined

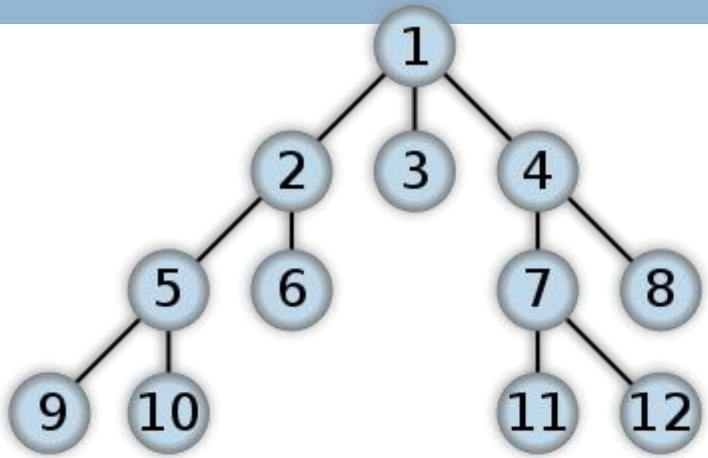
SEARCHING

Search Space

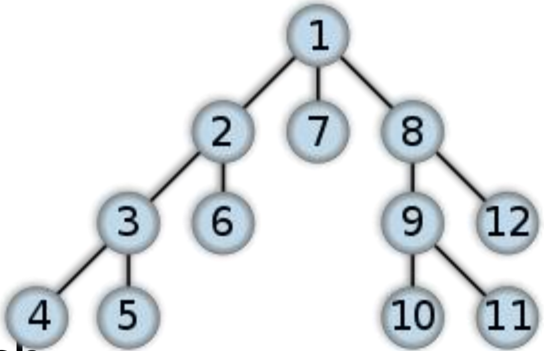
- **Search Space** S is a set of states, where the goal is to find the states that satisfy the condition g.
- Formally the **problem** is defined as a tuple (s_0,g, O), where:
  - s_0 is the initial state
  - g is the goal condition
  - O is a set of state – transition operators

Breadth – First Search

- Is **complete**
- **Complexity**
  - **Time** O(b^d)
  - **Space** O(b^d)
  - **b** is the number of siblings of each node
  - **d** is the depth of the search space



Depth-First Search



- Is complete
  - if no endless paths are present
- **Complexity**
  - **Time** depends on the way of the search
  - **Space** O(d)
  - **d** is the depth of the search space

A*

- **f'(n) = g(n) + h'(n)**
- **g(n)** - total distance it has taken to get from the starting position to the current location
- **h'(n)** - the estimated distance from the current position to the goal destination/state. A heuristic function is used to create this estimate on how far away it will take to reach the goal state.

First-order logic

- Whereas propositional logic assumes the world contains **facts**,
- first-order logic (like natural language) assumes the world contains
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  - **Objects**: people, houses, numbers, colors, baseball games, wars, …
  - **Relations**: red, round, prime, brother of, bigger than, part of, comes between, …
  - **Functions**: father of, best friend, one more than, plus, …

Syntax of FOL: Basic elements

- Constants KingJohn, 2, NUS,...
- Predicates Brother, >,...
- Functions Sqrt, LeftLegOf,...
- Variables x, y, a, b,...
- Connectives $\neg, \Rightarrow, \wedge, \vee, \Leftrightarrow$
- Equality $=$
- Quantifiers $\forall, \exists$

Atomic sentences

Atomic sentence = *predicate* ($term_1$,...,$term_n$)
or $term_1$ = $term_2$

Term = *function* ($term_1$,...,$term_n$)
or *constant* or *variable*

- E.g., *Brother(KingJohn,RichardTheLionheart)* > *(Length(LeftLegOf(Richard)), Length(LeftLegOf(KingJohn)))*

Complex sentences

- Complex sentences are made from atomic sentences using connectives
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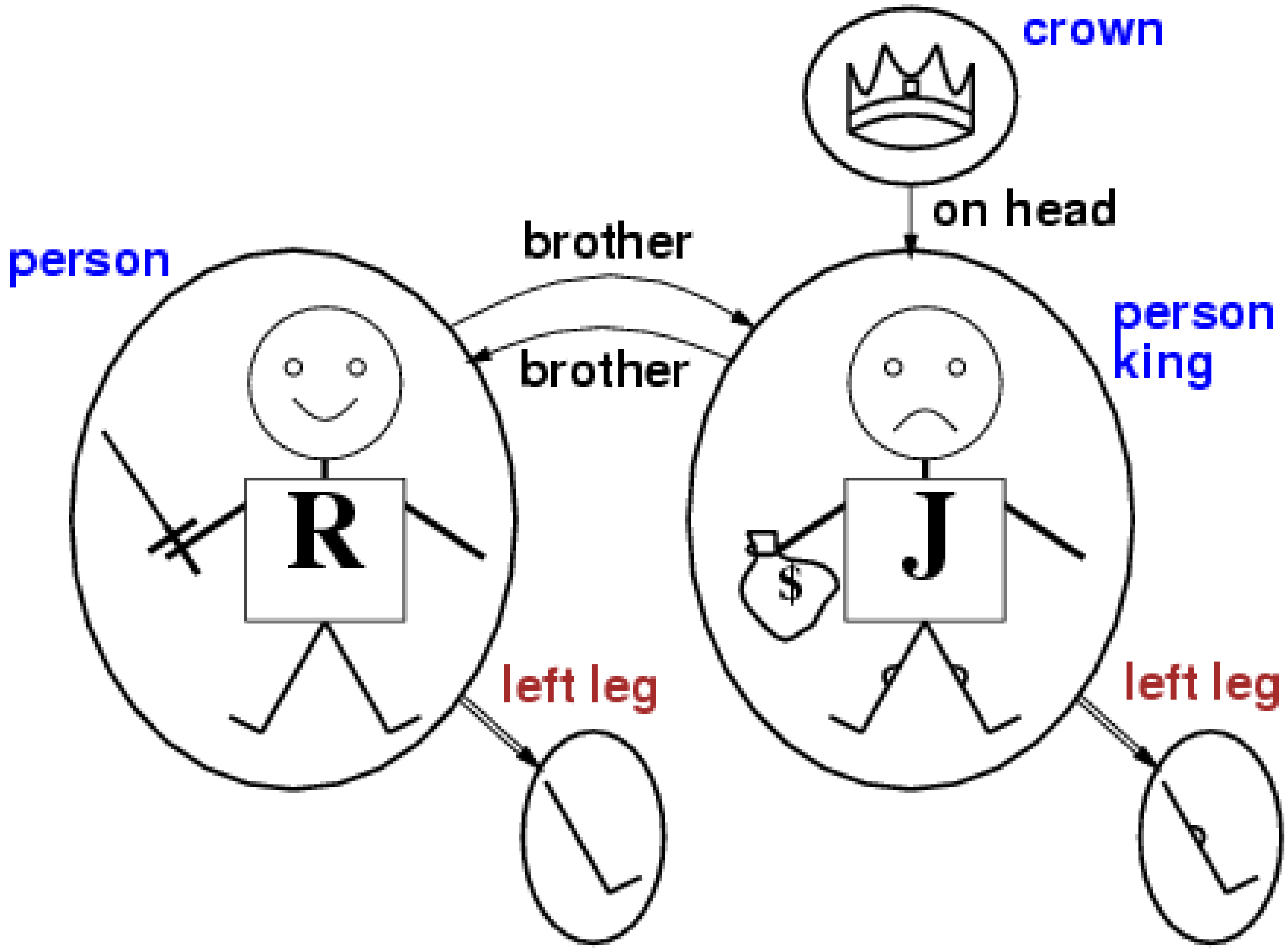
$$\neg S,\ S_1 \wedge S_2,\ S_1 \vee S_2,\ S_1 \Rightarrow S_2,\ S_1 \Leftrightarrow S_2,$$

E.g. $Sibling(KingJohn,Richard) \Rightarrow Sibling(Richard,KingJohn)$

$>(1,2) \vee \leq (1,2)$

$>(1,2) \wedge \neg >(1,2)$

Models for FOL: Example



Universal quantification

- $\forall$<*variables*> <*sentence*>

- Everyone at NUS is smart: $\forall x\ At(x,CVUT) \Rightarrow Smart(x)$

- $\forall x\ P$ is true in a model *m* iff *P* is true with *x* being each possible object in the model

- Roughly speaking, equivalent to the conjunction of instantiations of *P*

A common mistake to avoid

- Typically, $\Rightarrow$ is the main connective with $\forall$
-
- Common mistake: using $\wedge$ as the main connective with $\forall$:

  $\forall x\ At(x, CVUT) \wedge Smart(x)$

  means "Everyone is at CVUT and everyone is smart"

Existential quantification

- ∃<*variables*> <*sentence*>
- Someone at CVUT is smart:
- ∃x At(x, CVUT) ∧ Smart(x
-
- ∃*x P* is true in a model *m* iff *P* is true with *x* being some possible object in the model
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- Roughly speaking, equivalent to the disjunction of instantiations of *P*

Another common mistake to avoid

- Typically, $\wedge$ is the main connective with $\exists$

- Common mistake: using $\Rightarrow$ as the main connective with $\exists$:
- $\exists x\ At(x, CVUT) \Rightarrow Smart(x)$
  - is true if there is anyone who is not at CVUT!

Equality

- $term_1 = term_2$ is true under a given interpretation if and only if $term_1$ and $term_2$ refer to the same object

- E.g., definition of *Sibling* in terms of *Parent*:
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- $\forall x,y\ Sibling(x,y) \Leftrightarrow [\neg(x = y) \land \exists m,f\ \neg (m = f) \land Parent(m,x) \land Parent(f,x) \land Parent(m,y) \land Parent(f,y)]$

Satisfiability

- **Model** of the formula is a set of assignments of the true/false values to the variables in a way that the formula is evaluated to be **true.**
  - $\neg p$ is true iff p is false
  - $p \land q$ is true iff p is true and q is true
- Satisfiability problem (SAT) is a problem of evaluating, whether a model for the given formula exists.

3-SAT problem

- Conjunctive normal form
  - 3-CNF
- First known NP-complete problem
- $(x_{11}$ OR x_{12} OR $x_{13})$ AND
  $(x_{21}$ OR x_{22} OR $x_{23})$ AND
  $(x_{31}$ OR x_{32} OR $x_{33})$ AND
  …..