Patch tracking based on comparing its pixels[1]

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Last update: April 2, 2014

Talk Outline

- comparing patch pixels
- normalized cross-correlation, ssd . . .
- KLT - gradient based optimization
- good features to track

[1] Please note that the lecture will be accompanied be several sketches and derivations on the blackboard and few live-interactive demos in Matlab

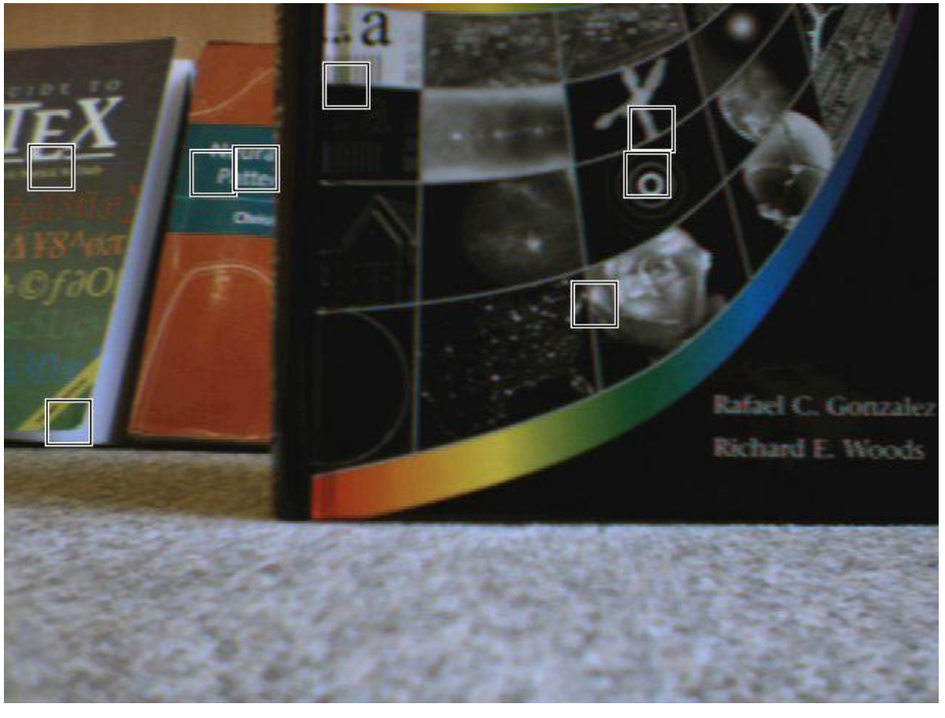
What is the problem?

video: CTU campus, door of G building

Tracking of dense sequences — camera motion

T - Template

I - Image



Scene static, camera moves.

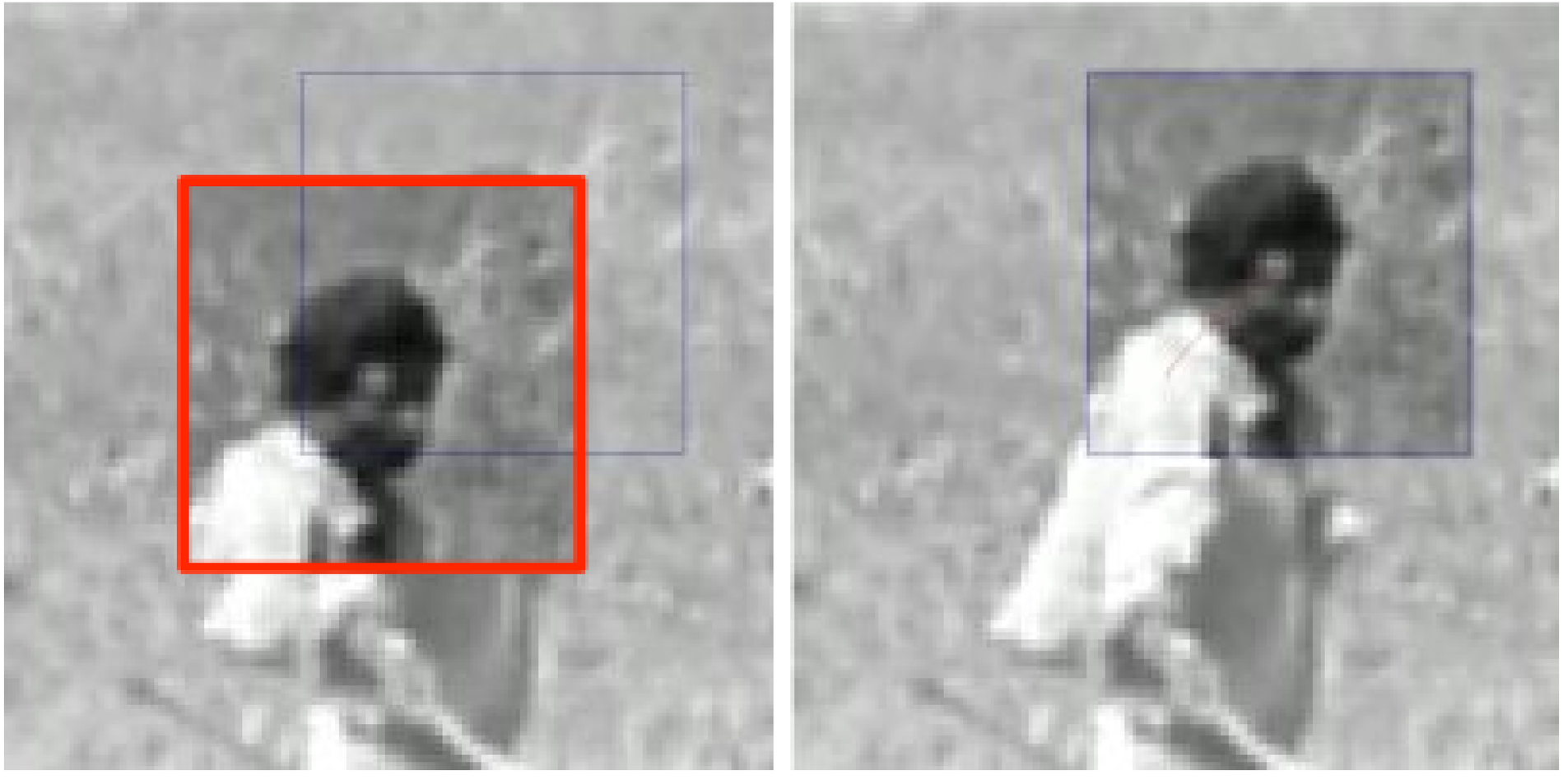
Tracking of dense sequences — object motion

T - Template

I - Image

Camera static, object moves.

Alignment of an image (patch)



Goal is to align a template image $T(\mathbf{x})$ to an input image $I(\mathbf{x})$. $\mathbf{x}$ column vector containing image coordinates $[x, y]^\top$. The $I(\mathbf{x})$ could be also a small subwindow within an image.

How to measure the alignment?

- What is the best criterial function?
- How to find the best match, in other words, how to find extremum of the criterial function?

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Criterial function

What are the desired properties (on a certain domain)?

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Criterial function

What are the desired properties (on a certain domain)?

- convex (remember the optimization course?)
- discriminative
- ...

Normalized cross-correlation

You may know it as correlation coefficient (from statistics)

$$\rho_{X,Y} = \frac{\operatorname{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$

where σ means standard deviation.

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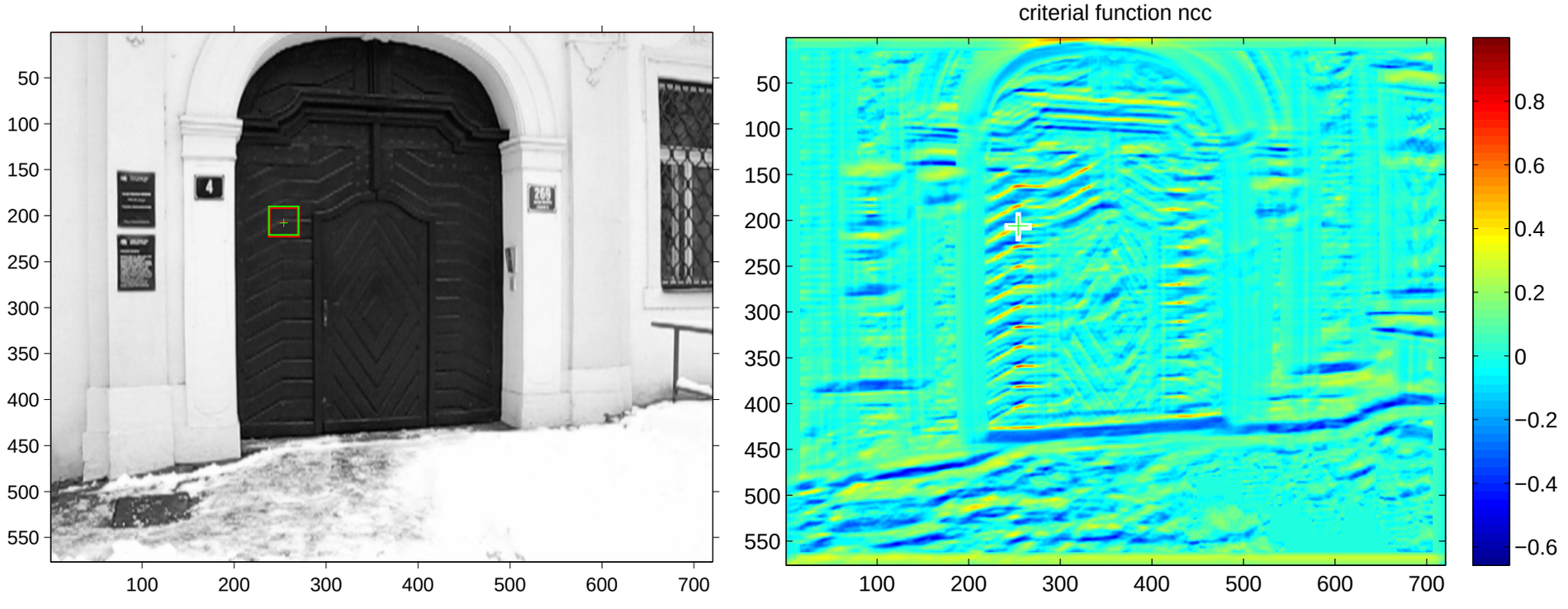
where σ means standard deviation.

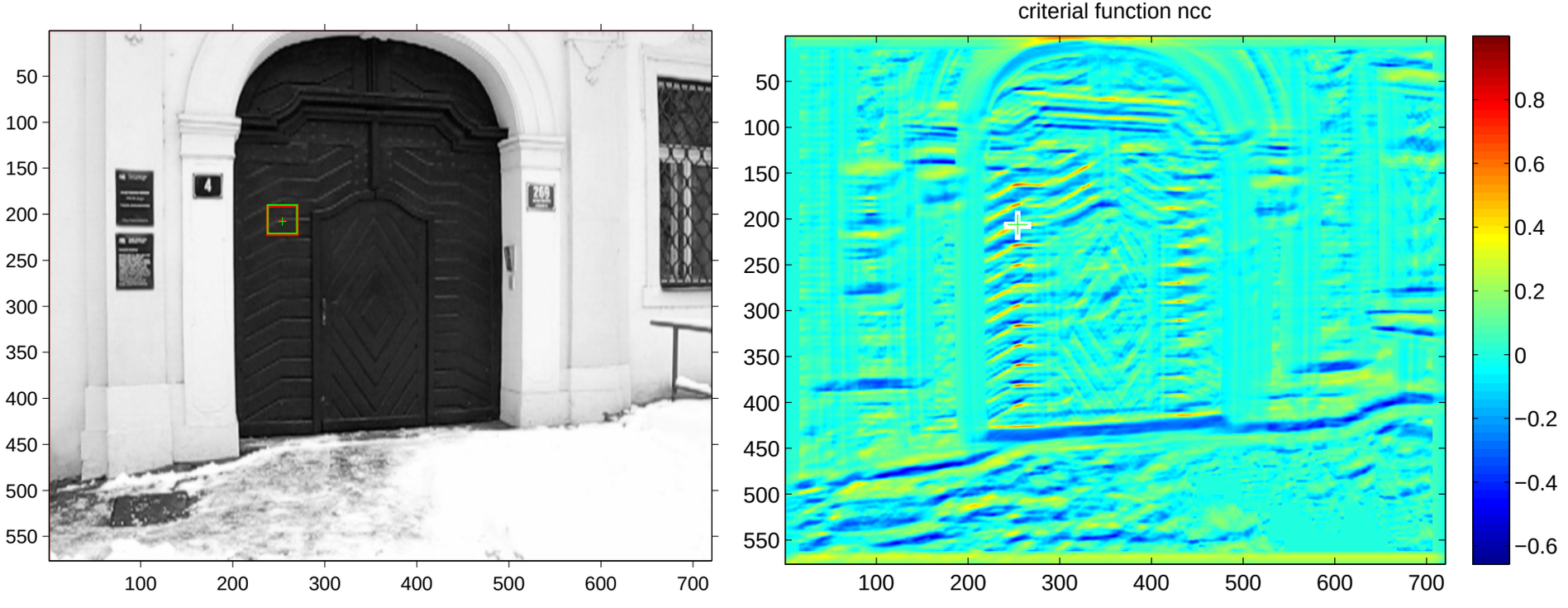
Having template $T(k,l)$ and image $I(x,y)$,

$$r(x,y) = \frac{\sum_k \sum_l \left(T(k,l) - \overline{T}\right)\left(I(x+k,y+l) - \overline{I(x,y)}\right)}{\sqrt{\sum_k \sum_l \left(T(k,l) - \overline{T}\right)^2} \sqrt{\sum_k \sum_l \left(I(x+k,y+l) - \overline{I(x,y)}\right)^2}}$$

Normalized cross-correlation – in picture



Normalized cross-correlation – in picture

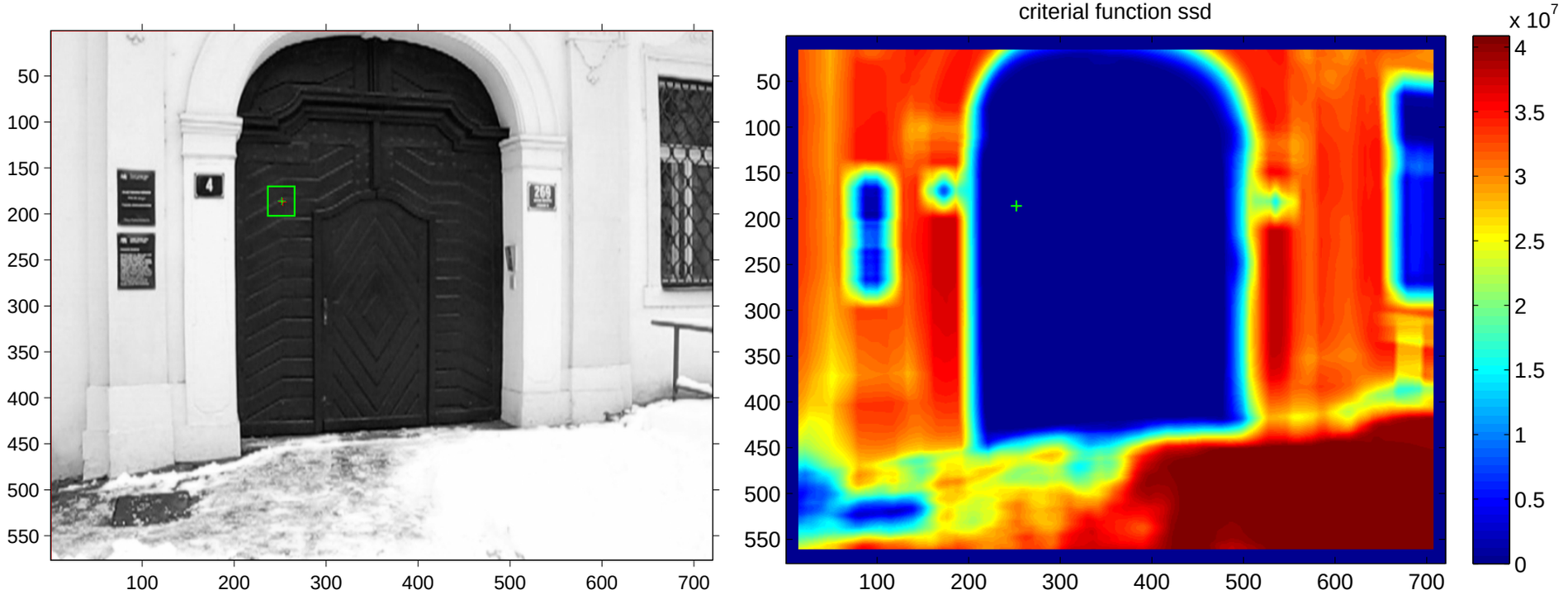


- well, definitely not convex
- but the discriminability looks promising
- very efficient in computation, see [3][2].

[2] check also `normxcorr2` in Matlab

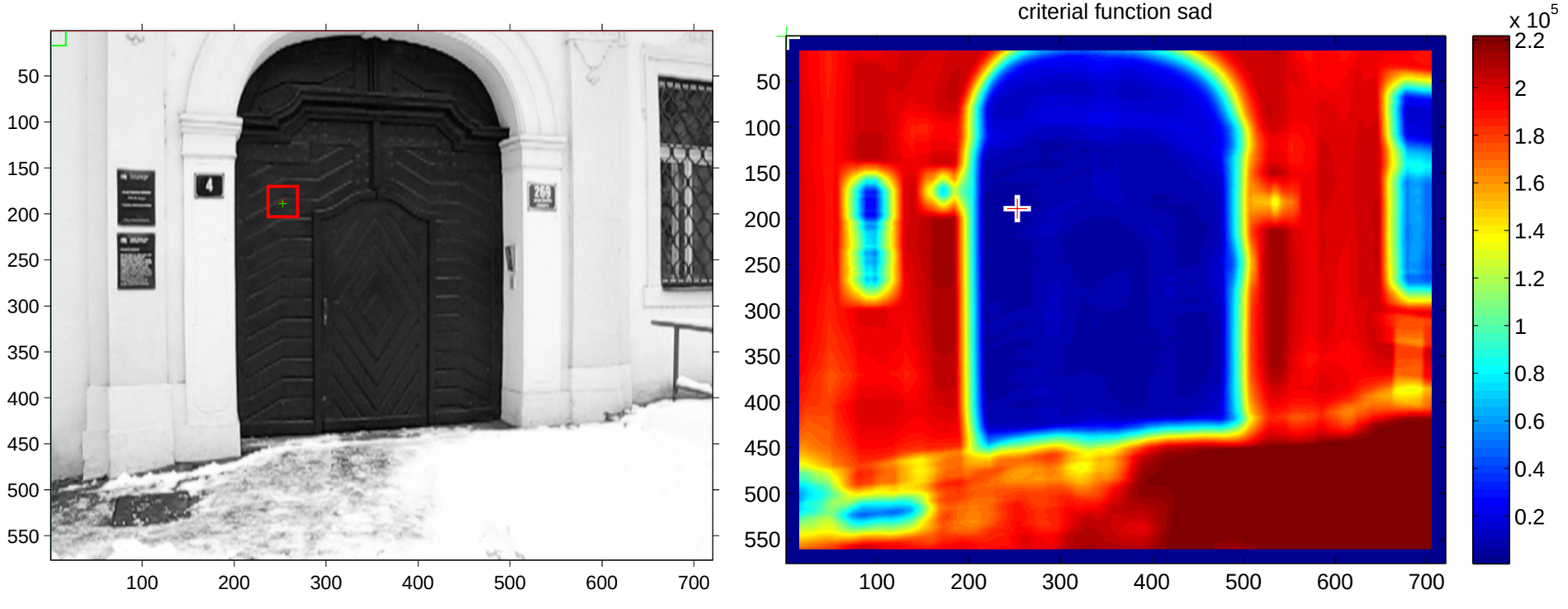
Sum of squared differences

$$ssd(x,y) = \sum_k \sum_l \left(T(k,l) - I(x+k, y+l)\right)^2$$

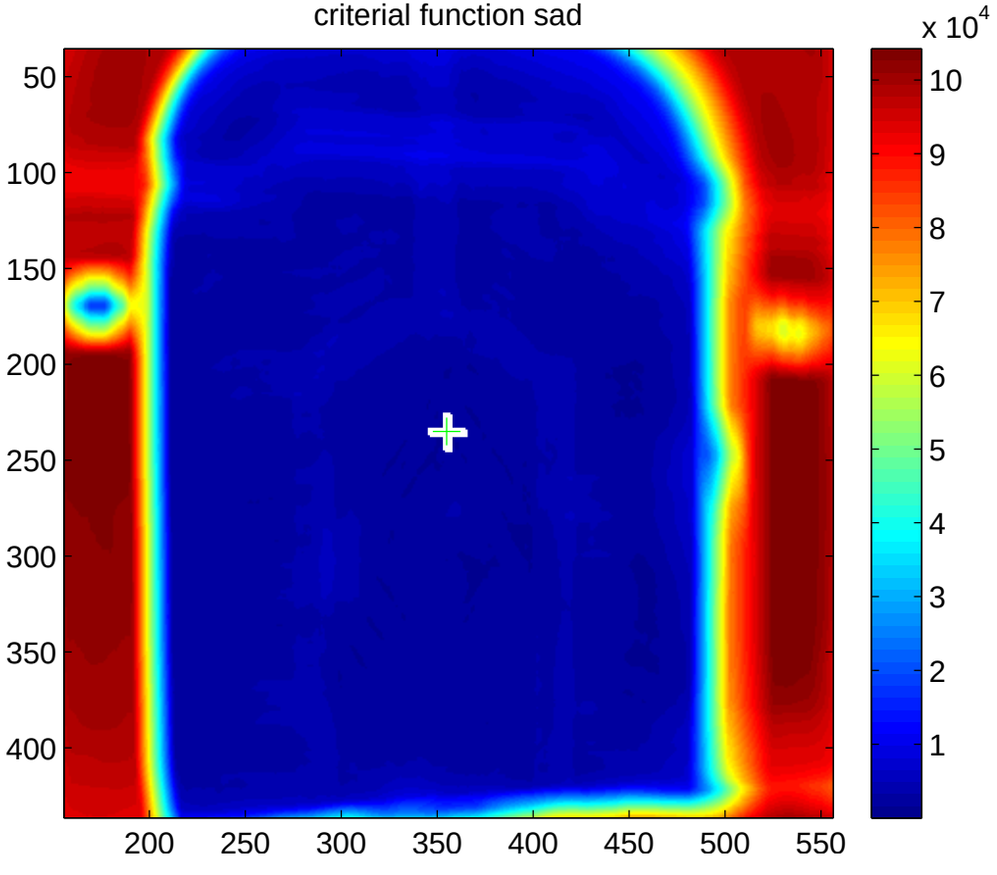


Sum of absolute differences

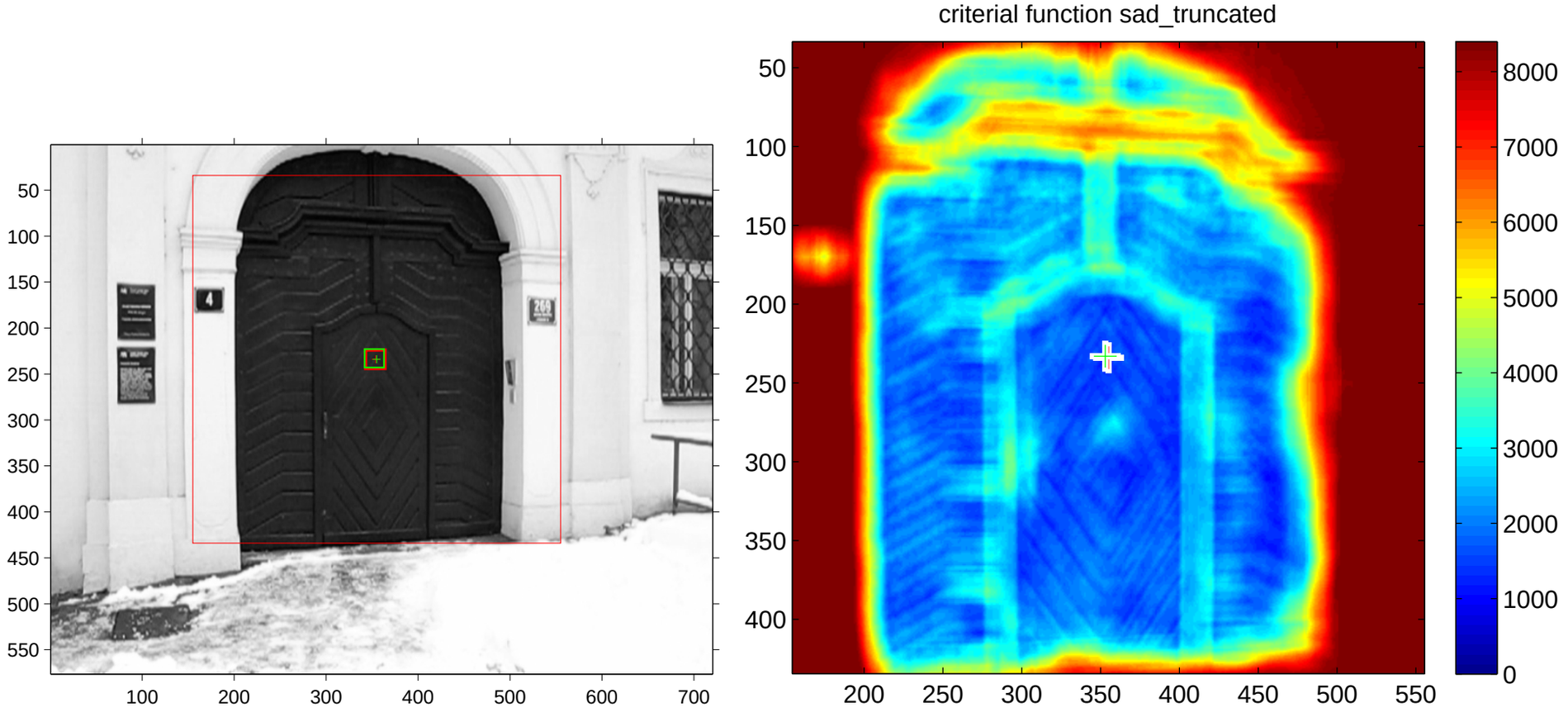
$$sad(x,y) = \sum_k \sum_l |T(k,l) - I(x+k, y+l)|$$



SAD for the door part



SAD for the door part – truncated

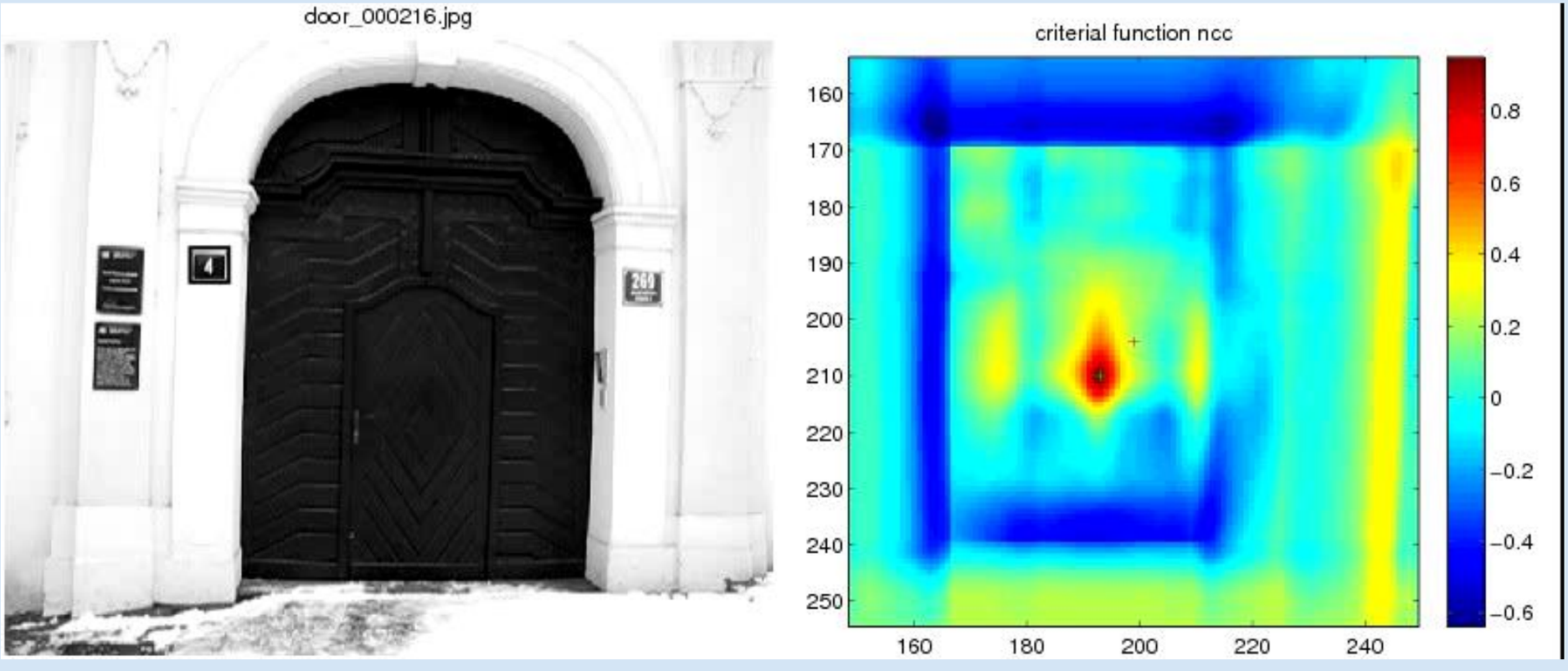


Differences greater than 20 intensity levels are counted as 20.

Normalized cross-correlation: how it works

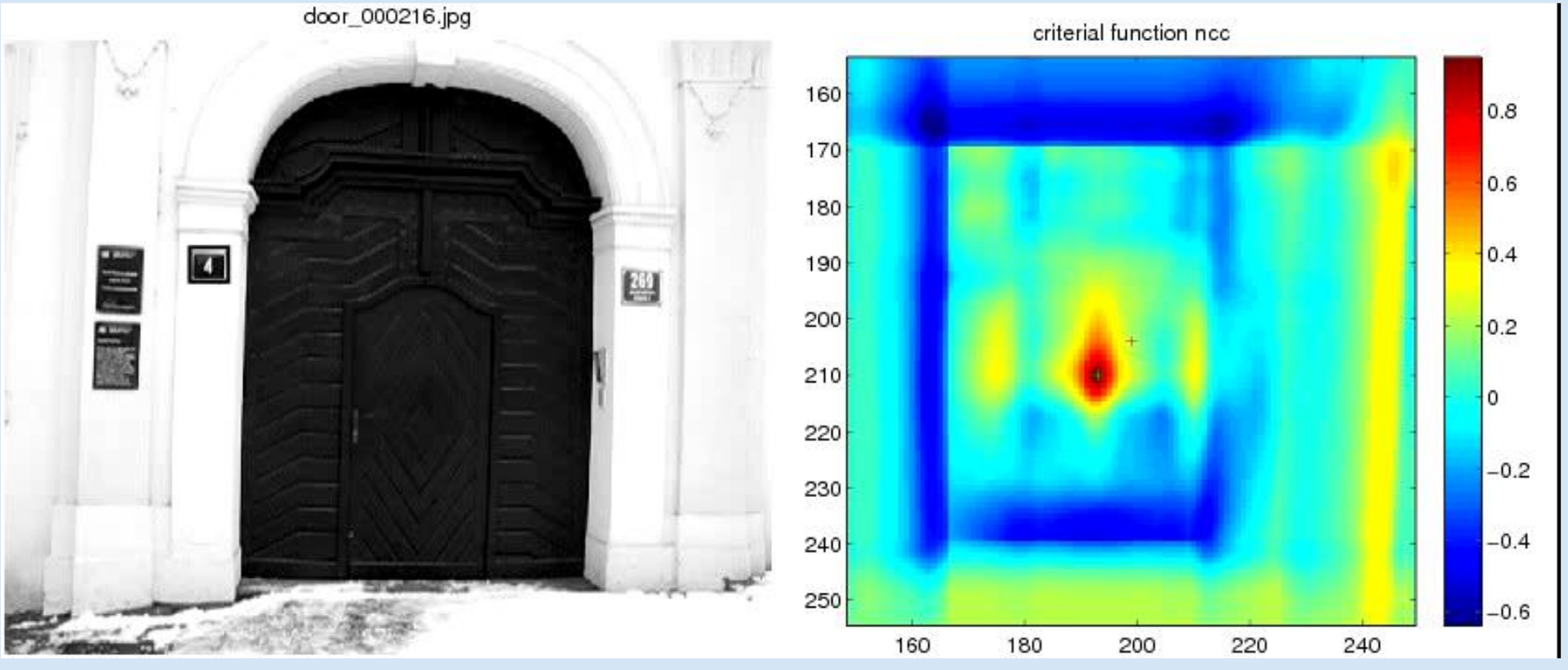
live demo for various patches

Normalized cross-correlation: tracking



video

Normalized cross-correlation: tracking



video

- What went wrong?
- Why did it failed?

Suggestions for improvement?

Tracking as an optimization problem

- finding extrema of a criterial function . . .

Tracking as an optimization problem

- finding extrema of a criterial function . . .
- . . . sounds like an optimization problem

Kanade–Lucas–Tomasi (KLT) tracker

- Iteratively minimizes sum of square differences.
- It is a Gauss-Newton gradient algorithm.

Importance in Computer Vision

- Firstly published in 1981 as an image registration method [4].
- Improved many times, most importantly by Carlo Tomasi [5, 6]
- Free implementation(s) available[3]. Also part of the OpenCV library[4].
- After more than two decades, a project[5] at CMU dedicated to this single algorithm and results published in a premium journal [1].
- Part of plethora computer vision algorithms.

Our explanation follows mainly the paper [1]. It is a good reading for those who are also interested in alternative solutions.

[3] http://www.ces.clemson.edu/~stb/klt/
[4] http://opencv.willowgarage.com/wiki/
[5] http://www.ri.cmu.edu/projects/project_515.html

Original Lucas-Kanade algorithm I

Goal is to align a template image $T(\mathbf{x})$ to an input image $I(\mathbf{x})$. $\mathbf{x}$ column vector containing image coordinates $[x, y]^\top$. The $I(\mathbf{x})$ could be also a small subwindow withing an image.

Set of allowable warps $\mathbf{W}(\mathbf{x}; \mathbf{p})$, where $\mathbf{p}$ is a vector of parameters. For translations

$$\mathbf{W}(\mathbf{x}; \mathbf{p}) = \begin{bmatrix} x + p_1 \\ y + p_2 \end{bmatrix}$$

$\mathbf{W}(\mathbf{x}; \mathbf{p})$ can be arbitrarily complex

The best alignment, $\mathbf{p}^*$, minimizes image dissimilarity

$$\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x}; \mathbf{p})) - T(\mathbf{x})]^2$$

Original Lucas-Kanade algorithm II

$$\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x};\mathbf{p})) - T(\mathbf{x})]^2$$

$I(\mathbf{W}(\mathbf{x};\mathbf{p})$ is nonlinear! The warp $\mathbf{W}(\mathbf{x};\mathbf{p})$ may be linear but the pixels value are, in general, non-linear. In fact, they are essentially unrelated to $\mathbf{x}$.

Linearization of the image: It is assumed that some $\mathbf{p}$ is known and best increment $\Delta\mathbf{p}$ is sought. The modified problem

$$\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x};\mathbf{p}+\Delta\mathbf{p})) - T(\mathbf{x})]^2$$

is solved with respect to $\Delta\mathbf{p}$. When found then $\mathbf{p}$ gets updated

$$\mathbf{p} \leftarrow \mathbf{p} + \Delta\mathbf{p}$$

...

Original Lucas-Kanade algorithm III

$$\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x};\mathbf{p}+\Delta\mathbf{p})) - T(\mathbf{x})]^2$$

linearization by performing first order Taylor expansion[6]

$$\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x})]^2$$

$\nabla I = [\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}]$ is the gradient image[7] computed at $\mathbf{W}(\mathbf{x};\mathbf{p})$. The term $\frac{\partial \mathbf{W}}{\partial \mathbf{p}}$ is the Jacobian of the warp.

[6] Detailed explanation on the blackboard.

[7] As a vector it should have been a column wise oriented. However, for sake of clarity of equations row vector is exceptionally considered here.

Original Lucas-Kanade algorithm IV

Derive $\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x})]^2$ with respect to $\Delta\mathbf{p}$

Original Lucas-Kanade algorithm IV

Derive $\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x})]^2$ with respect to $\Delta\mathbf{p}$

$$2 \sum_{\mathbf{x}} \left[\nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^{\top} \left[I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x}) \right]$$

setting equality to zero yields

Original Lucas-Kanade algorithm IV

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setting equality to zero yields

$$\Delta\mathbf{p} = \mathtt{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^{\top} [T(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p}))]$$

where $\mathtt{H}$ is (Gauss-Newton) approximation of Hessian matrix.

$$\mathtt{H} = \sum_{\mathbf{x}} \left[\nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^{\top} \left[\nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]$$

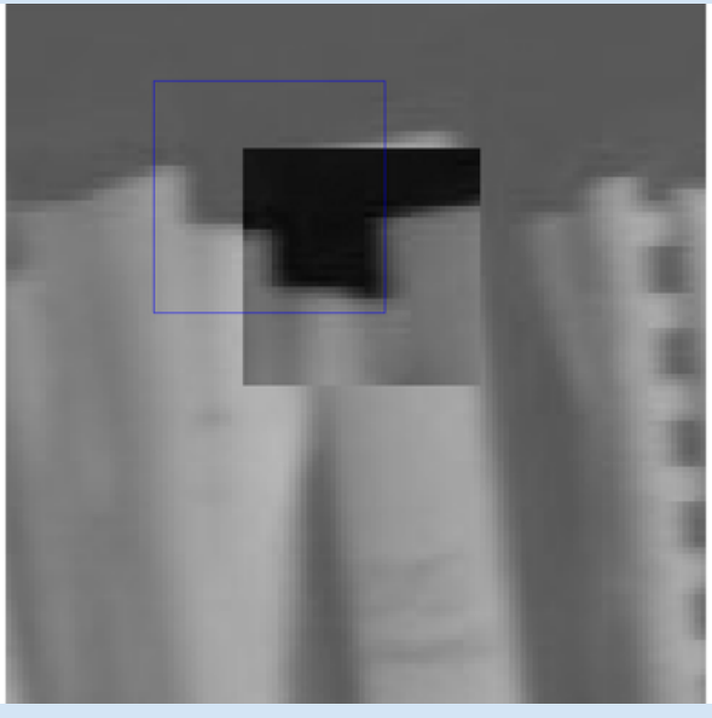
The Lucas-Kanade algorithm—Summary

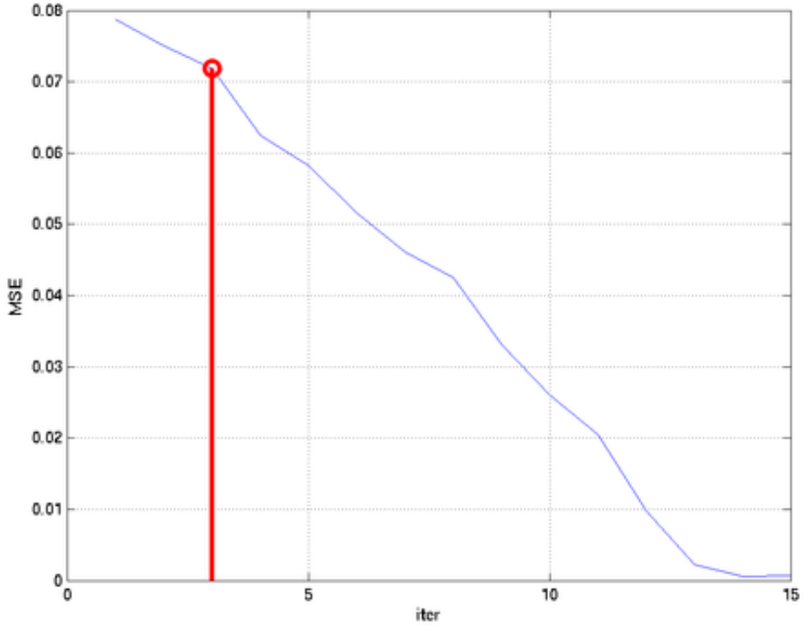
Iterate:

1. Warp I with $\mathbf{W}(\mathbf{x};\mathbf{p})$
2. Warp the gradient ∇I with $\mathbf{W}(\mathbf{x};\mathbf{p})$
3. Evaluate the Jacobian $\frac{\partial \mathbf{W}}{\partial \mathbf{p}}$ at $(\mathbf{x};\mathbf{p})$ and compute the steepest descent image $\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}}$
4. Compute the $\mathtt{H} = \sum_{\mathbf{x}} \left[\nabla I \ \frac{\partial \mathbf{W}}{\partial \mathbf{p}}\right]^{\top} \left[\nabla I \ \frac{\partial \mathbf{W}}{\partial \mathbf{p}}\right]$
5. Compute $\Delta \mathbf{p} = \mathtt{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \ \frac{\partial \mathbf{W}}{\partial \mathbf{p}}\right]^{\top} [T(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p}))]$
6. Update the parameters $\mathbf{p} \leftarrow \mathbf{p} + \Delta \mathbf{p}$

until $\|\Delta \mathbf{p}\| \leq \epsilon$

Example of convergence





video

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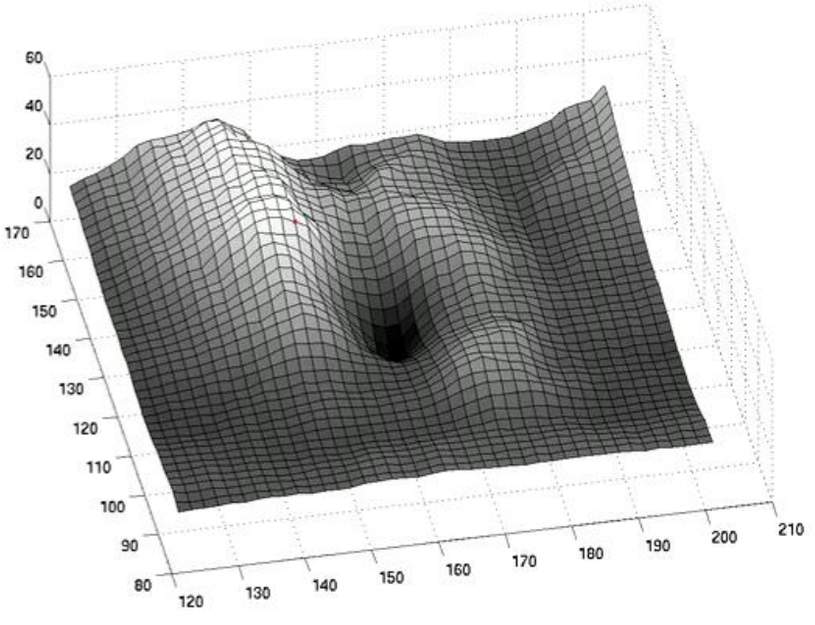
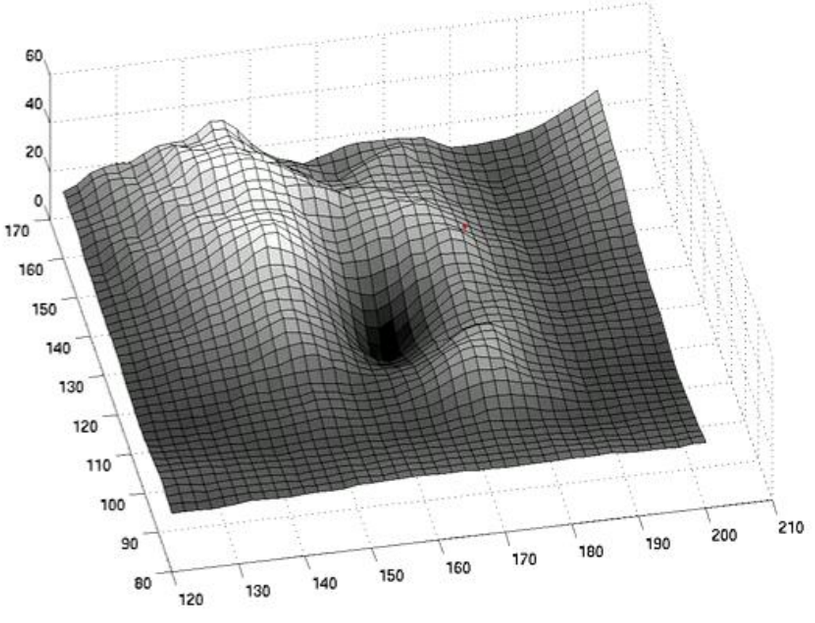


Convergence video: Initial state is within the basin of attraction

Example of divergence



Divergence video: Initial state is outside the basin of attraction

Example – on-line demo

Let play and see . . .

What are good features (windows) to track?

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How to select good templates $T(\mathbf{x})$ for image registration, object tracking.

$$\Delta \mathbf{p} = \mathtt{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^{\top} [T(\mathbf{x}) - I(\mathbf{W}(\mathbf{x}; \mathbf{p}))]$$

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$$\mathtt{H} = \sum_{\mathbf{x}} \left[\nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^{\top} \left[\nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]$$

The stability of the iteration is mainly influenced by the inverse of Hessian. We can study its eigenvalues. Consequently, the criterion of a good feature window is $\min(\lambda_1, \lambda_2) > \lambda_{min}$ (texturedness).

What are good features for translations?

Consider translation $\mathbf{W}(\mathbf{x};\mathbf{p}) = \begin{bmatrix} x+p_1 \\ y+p_2 \end{bmatrix}$. The Jacobian is then

$$\frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{aligned} \mathtt{H} &= \sum_{\mathbf{x}} \left[\nabla I \ \frac{\partial \mathbf{W}}{\partial \mathbf{p}}\right]^{\top} \left[\nabla I \ \frac{\partial \mathbf{W}}{\partial \mathbf{p}}\right] \\ &= \sum_{\mathbf{x}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial y} \end{bmatrix} [\frac{\partial I}{\partial x}, \frac{\partial I}{\partial x}] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\ &= \sum_{\mathbf{x}} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix} \end{aligned}$$

The image windows with varying derivatives in both directions. Homeogeneous areas are clearly not suitable. Texture oriented mostly in one direction only would cause instability for this translation.

What are the good points for translations?

The matrix

$$\mathtt{H} = \sum_{\mathbf{x}} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}$$

Should have large eigenvalues. We have seen the matrix already, where?

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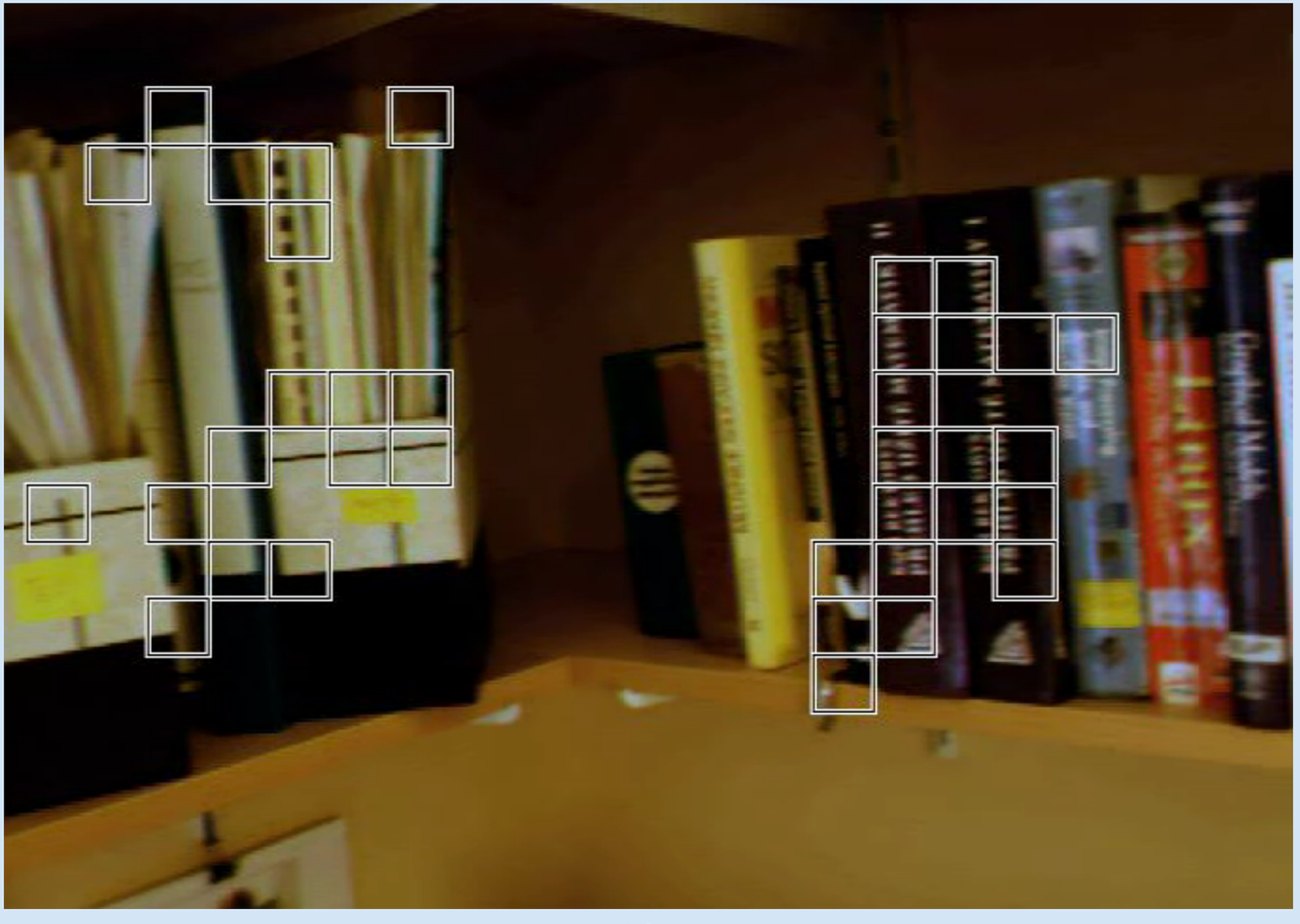
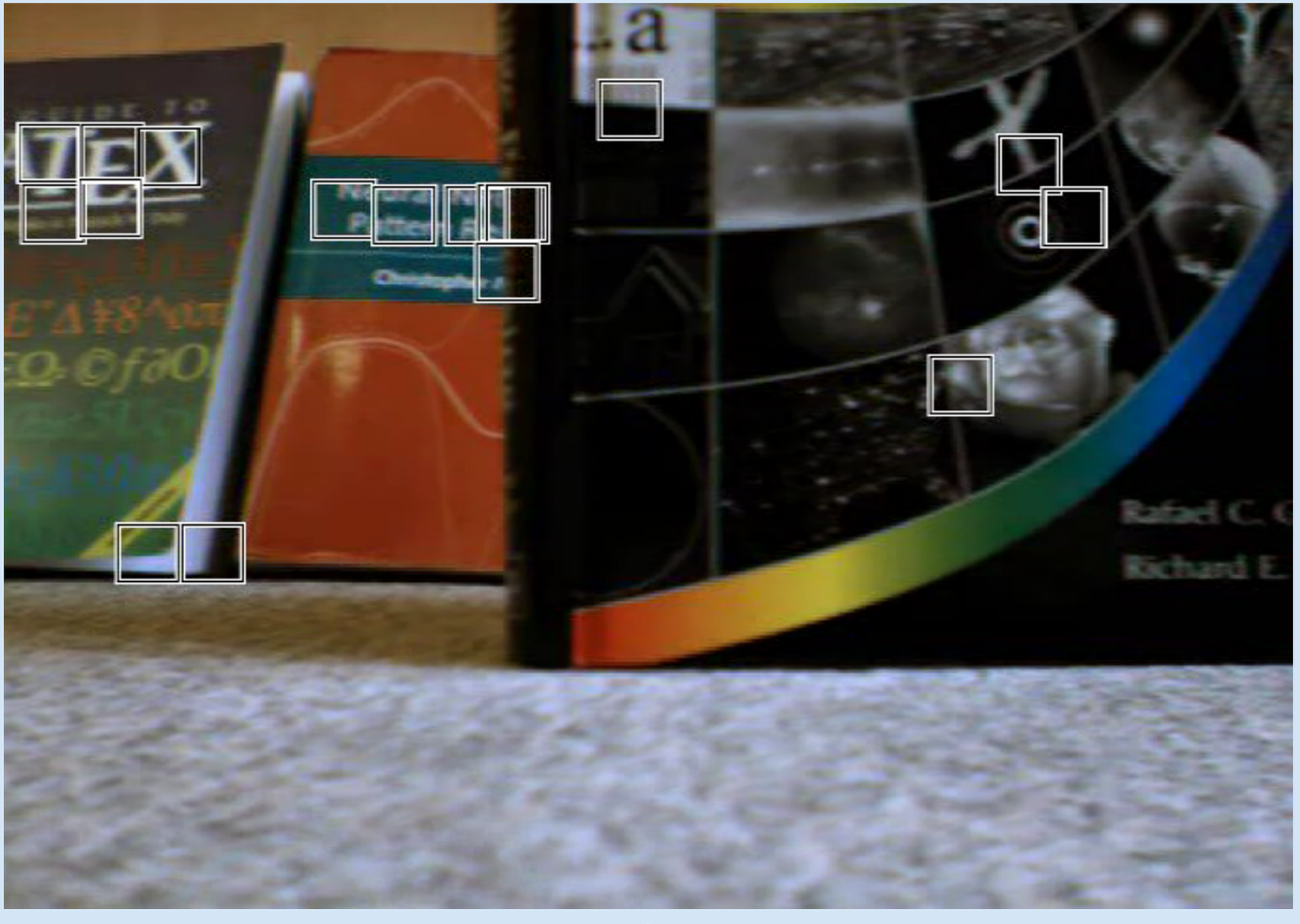
Harris corner detector [2]! The matrix is sometimes called Harris matrix.

Experiments - no occlusions



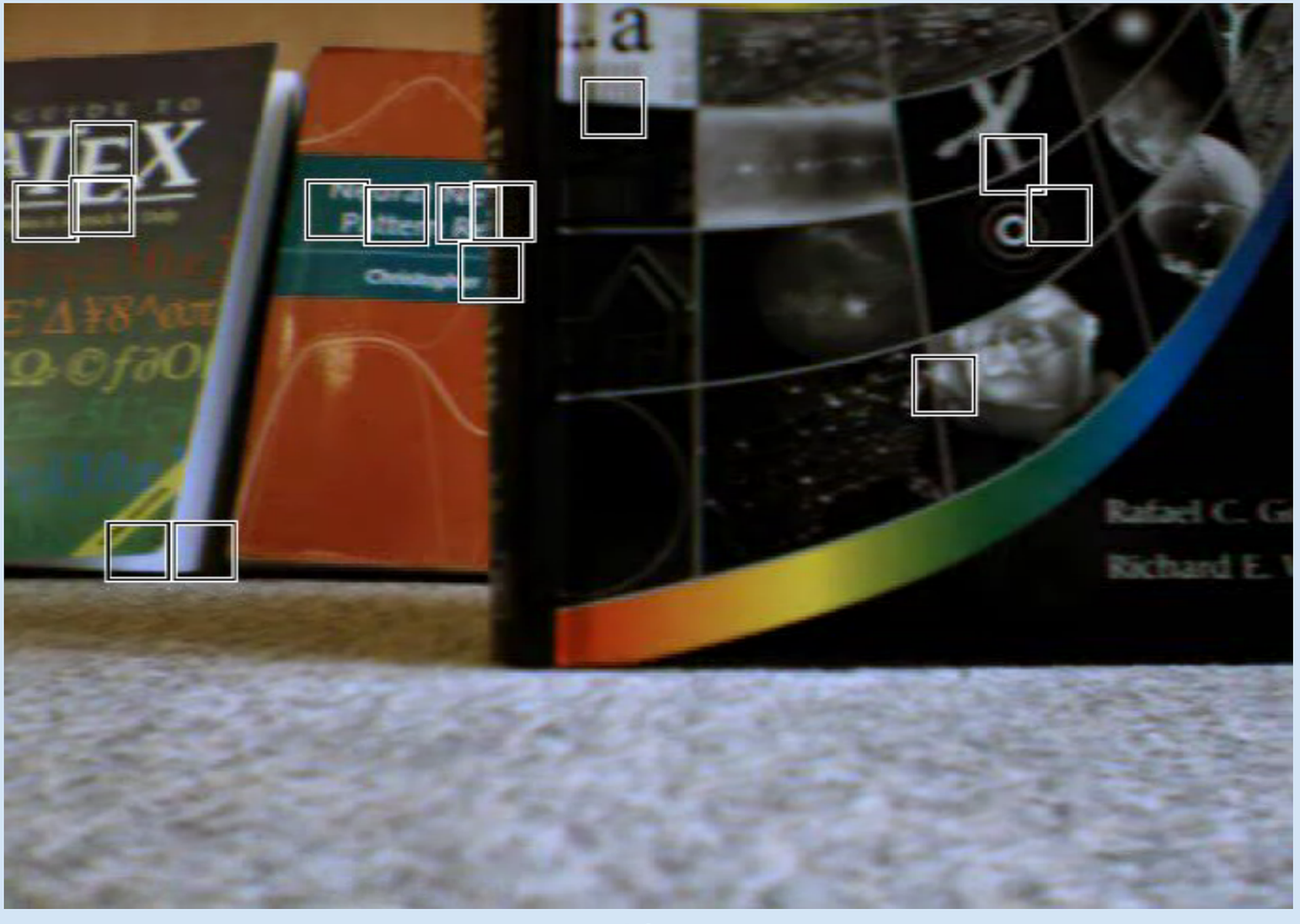
video

Experiments - occlusions



video

Experiments - occlusions with dissimilarity

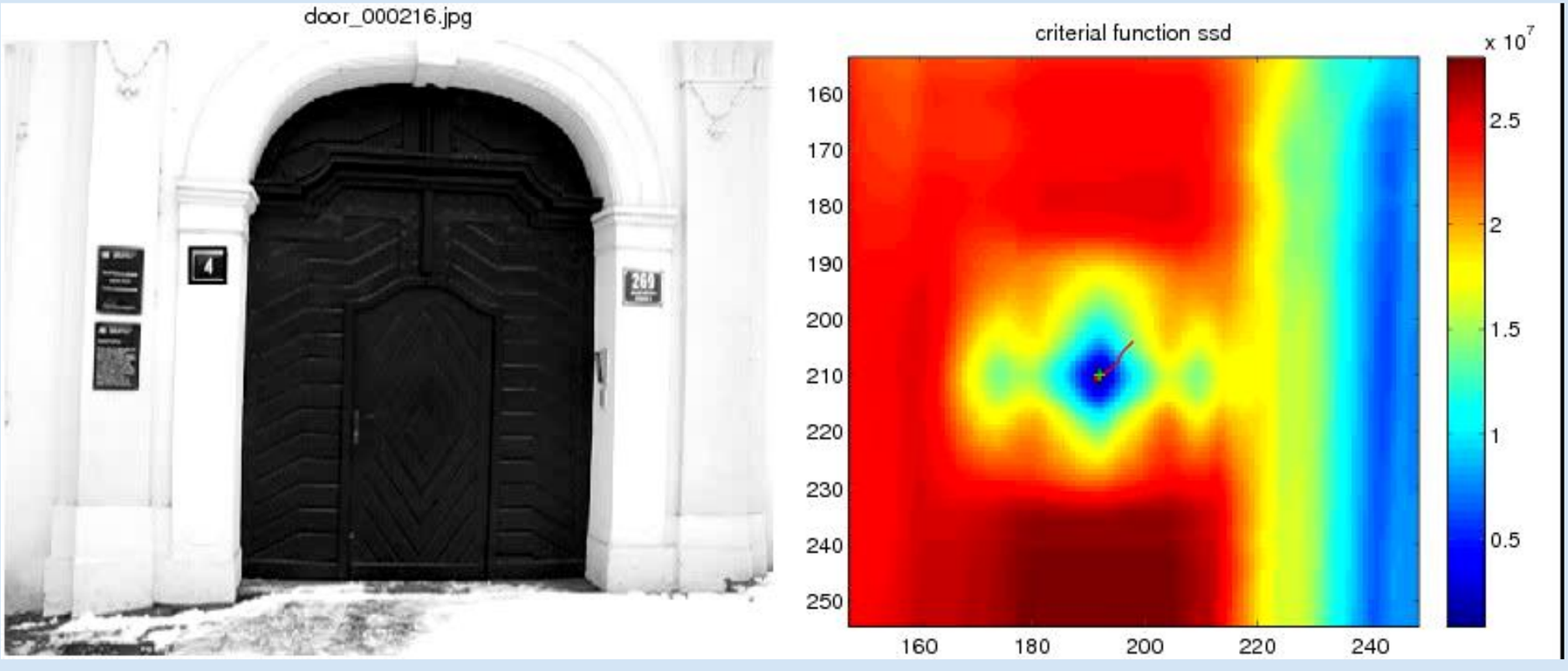


video

Experiments - object motion

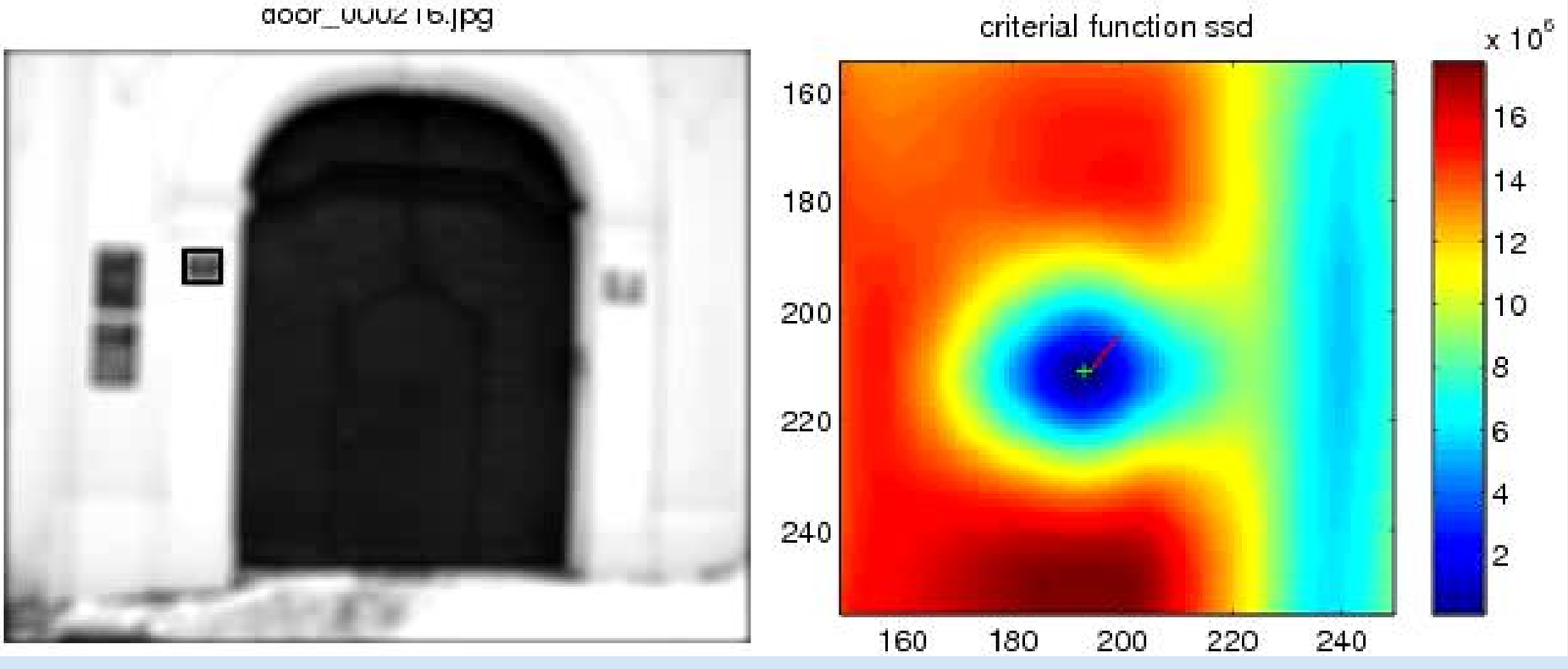
video

Experiments – door tracking



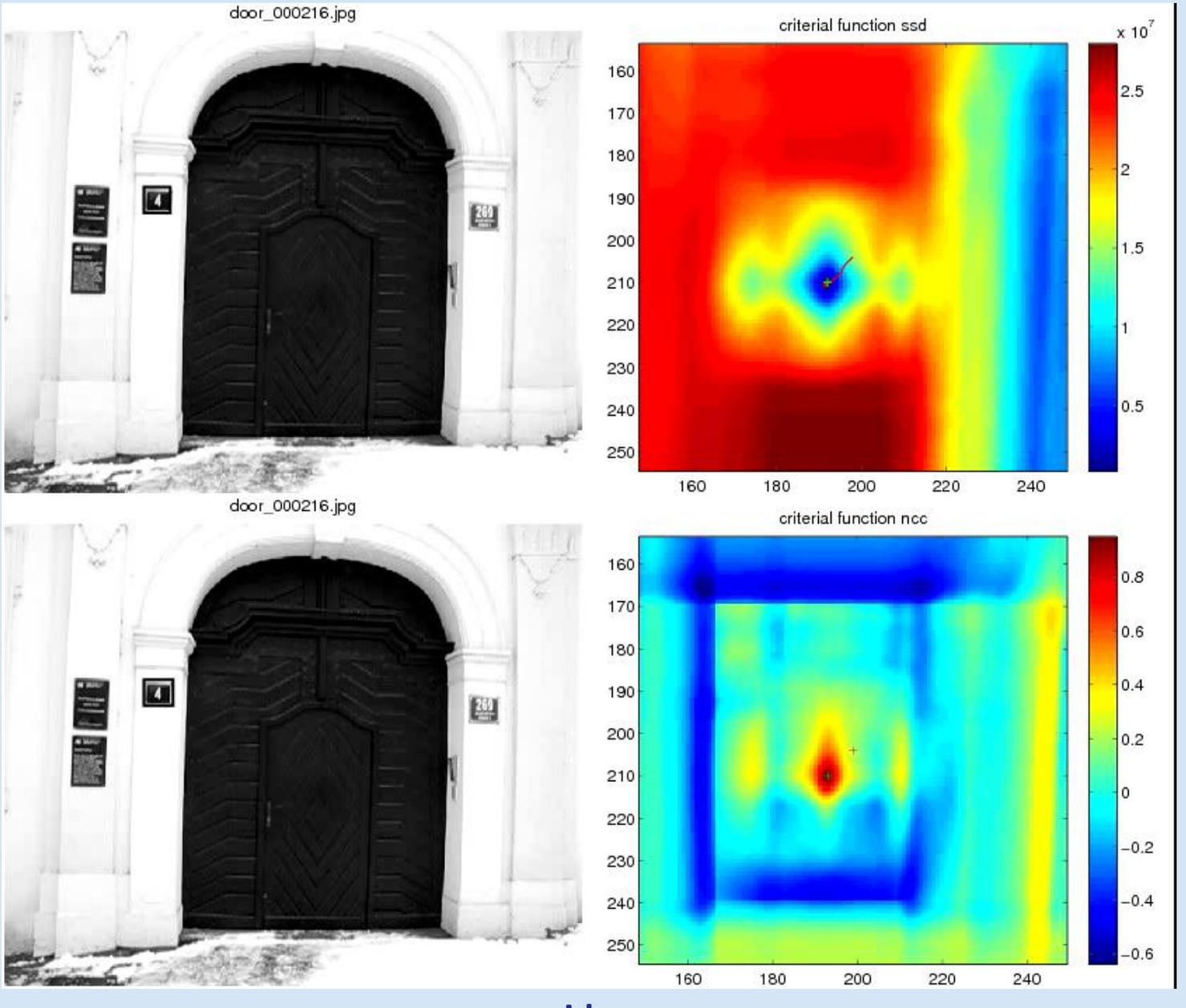
video

Experiments – door tracking – smoothed



video

Comparison of ncc vs KLT tracking



video

References

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[2] C. Harris and M. Stephen. A combined corner and edge detection. In M. M. Matthews, editor, Proceedings of the 4th ALVEY vision conference, pages 147–151, University of Manchaster, England, September 1988. on-line copies available on the web.

[3] J.P. Lewis. Fast template matching. In Vision Interfaces, pages 120–123, 1995. Extended version published on-line as "Fast Normalized Cross-Correlation" at http://scribblethink.org/Work/nvisionInterface/nip.html.

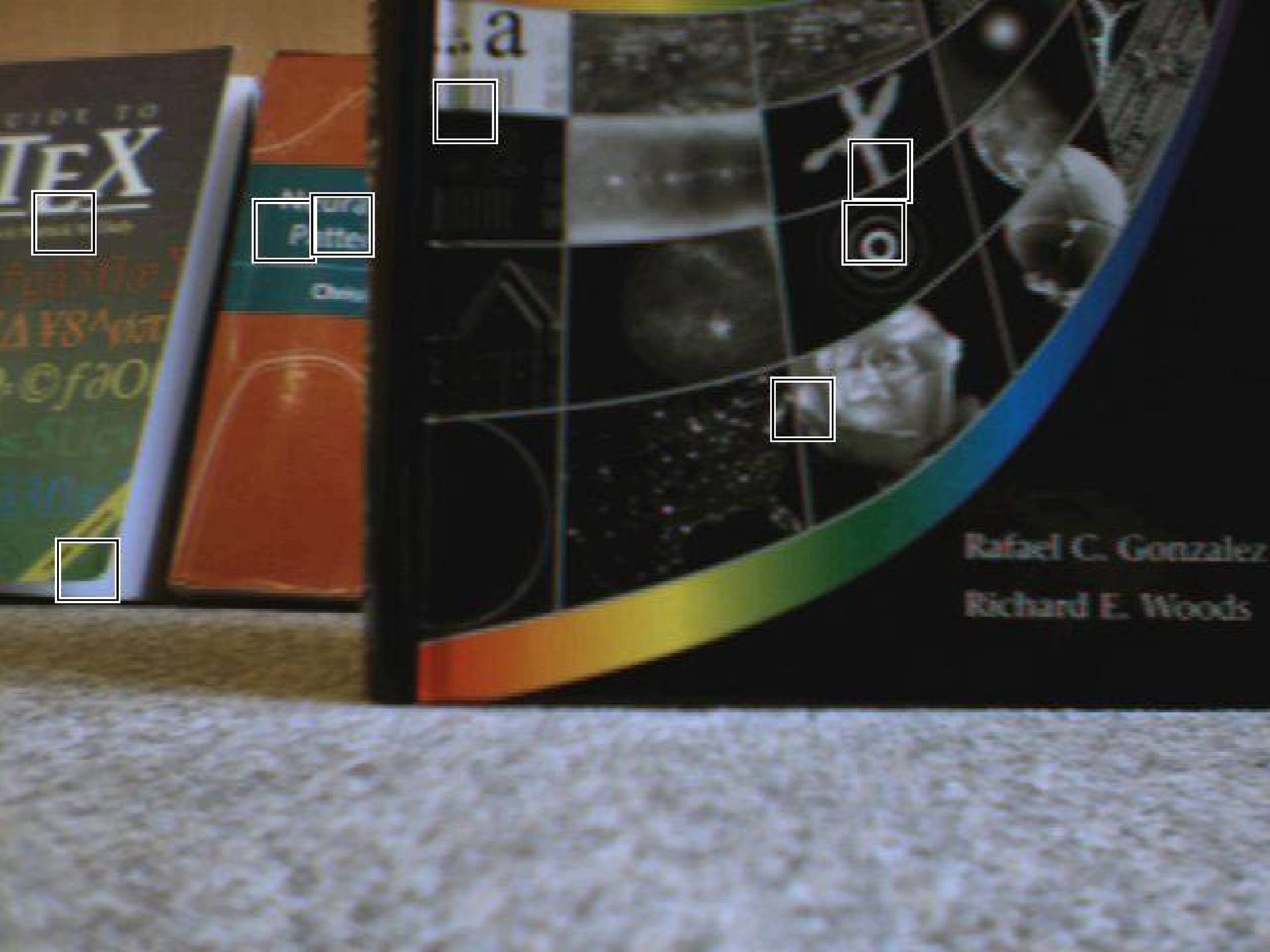
[4] Bruce D. Lucas and Takeo Kanade. An iterative image registration technique with an application to stereo vision. In Proceedings of the 7th International Conference on Artificial Intelligence, pages 674–679, August 1981.

[5] Jianbo Shi and Carlo Tomasi. Good features to track. In IEEE Conference on Computer Vision and Pattern Recognition (CVPR), pages 593–600, 1994.

[6] Carlo Tomasi and Takeo Kanade. Detection and tracking of point features. Technical Report CMU-CS-91-132, Carnegie Mellon University, April 1991.

End

TEX
Rafael C. Go
Richard E. W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Rafael C. Gonzalez
Richard E. Woods

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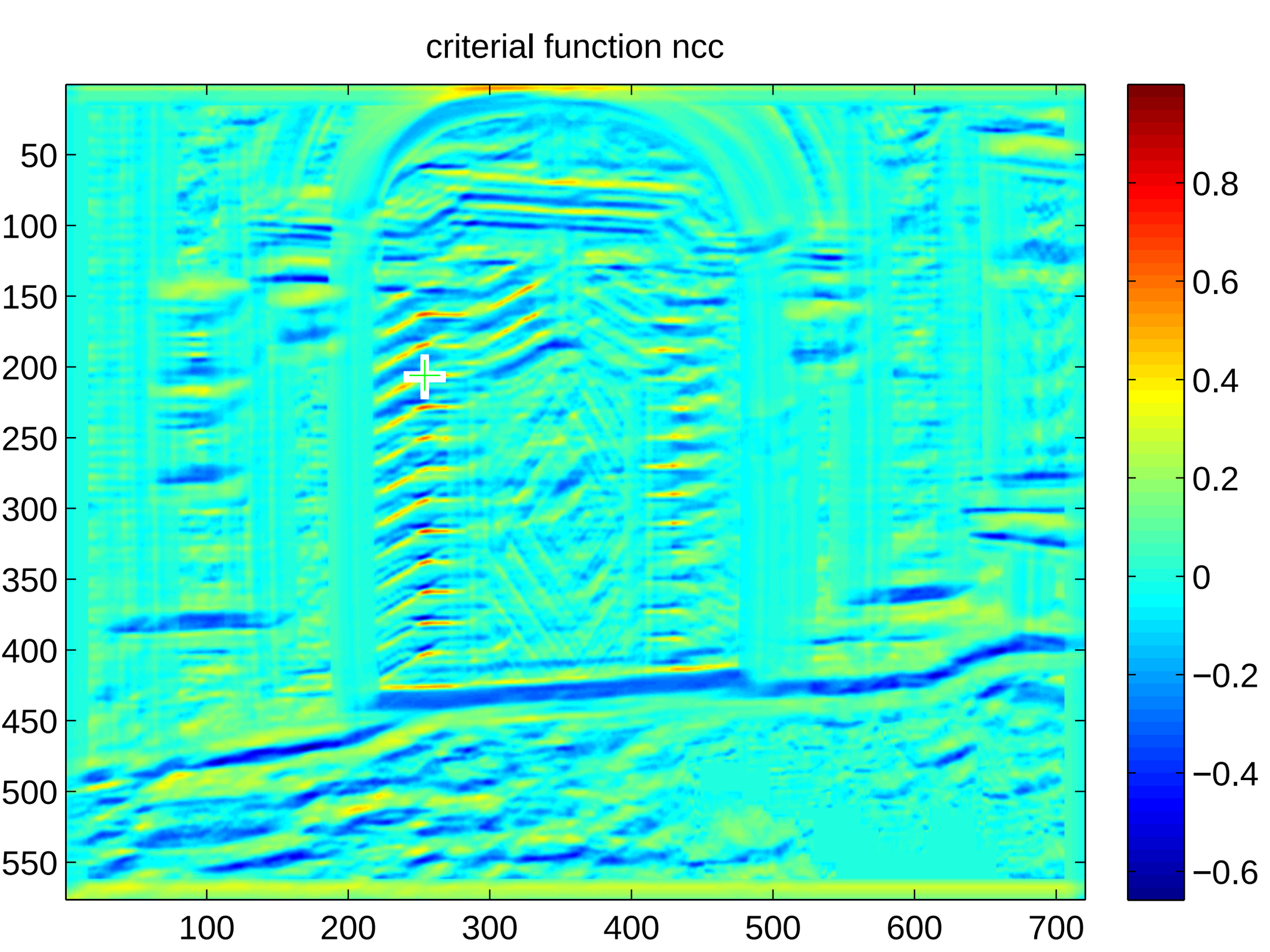
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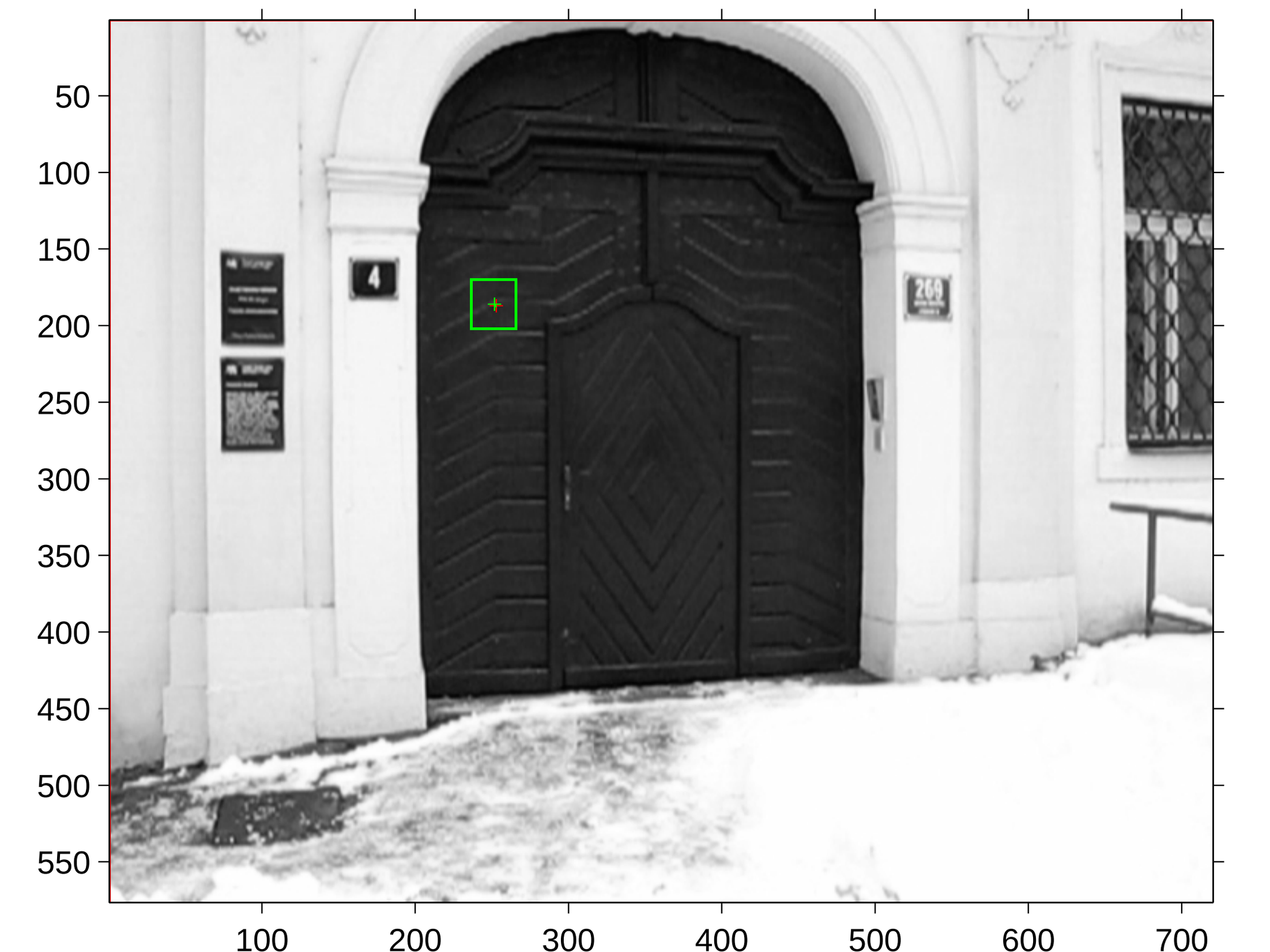
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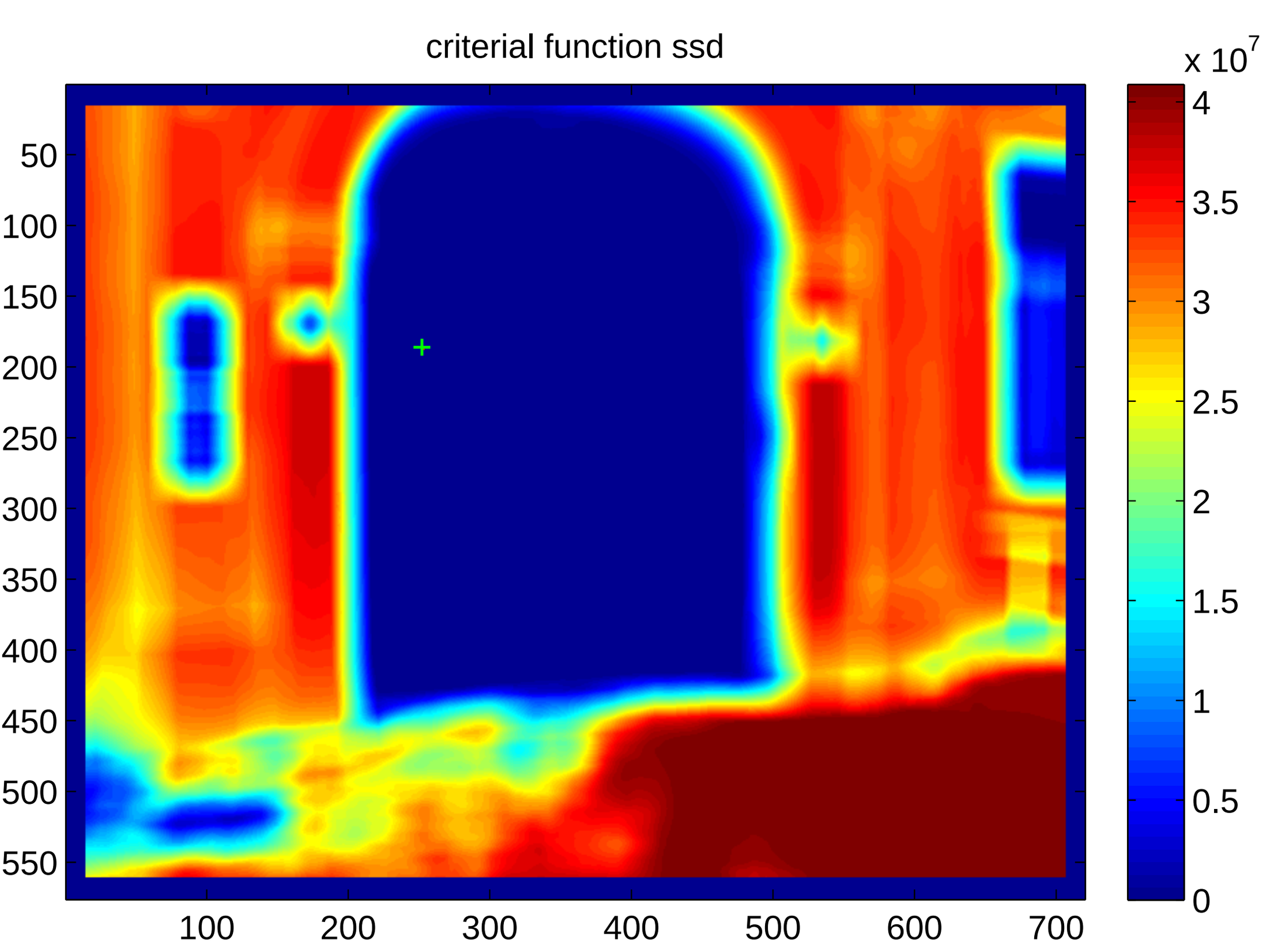
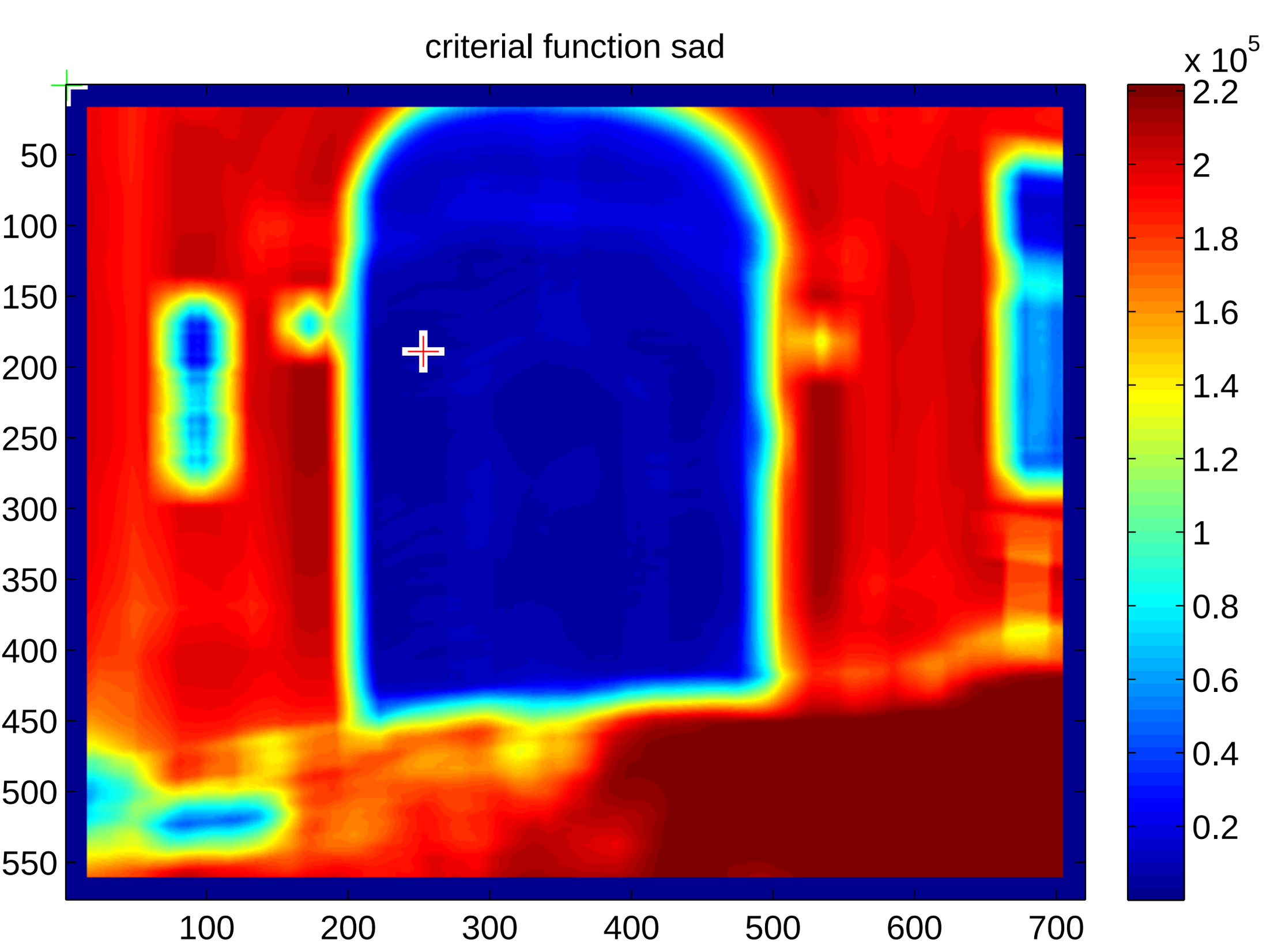
4
269


criterial function ssd
x 10^7
4
3.5
3
2.5
2
1.5
1
0.5
0
50
100
150
200
250
300
350
400
450
500
550
100
200
300
400
500
600
700

50
100
150
200
250
300
350
400
450
500
550
100
200
300
400
500
600
700
4
269


criterial function sad
x 10^5
2.2
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
50
100
150
200
250
300
350
400
450
500
550
100
200
300
400
500
600
700

50
100
150
200
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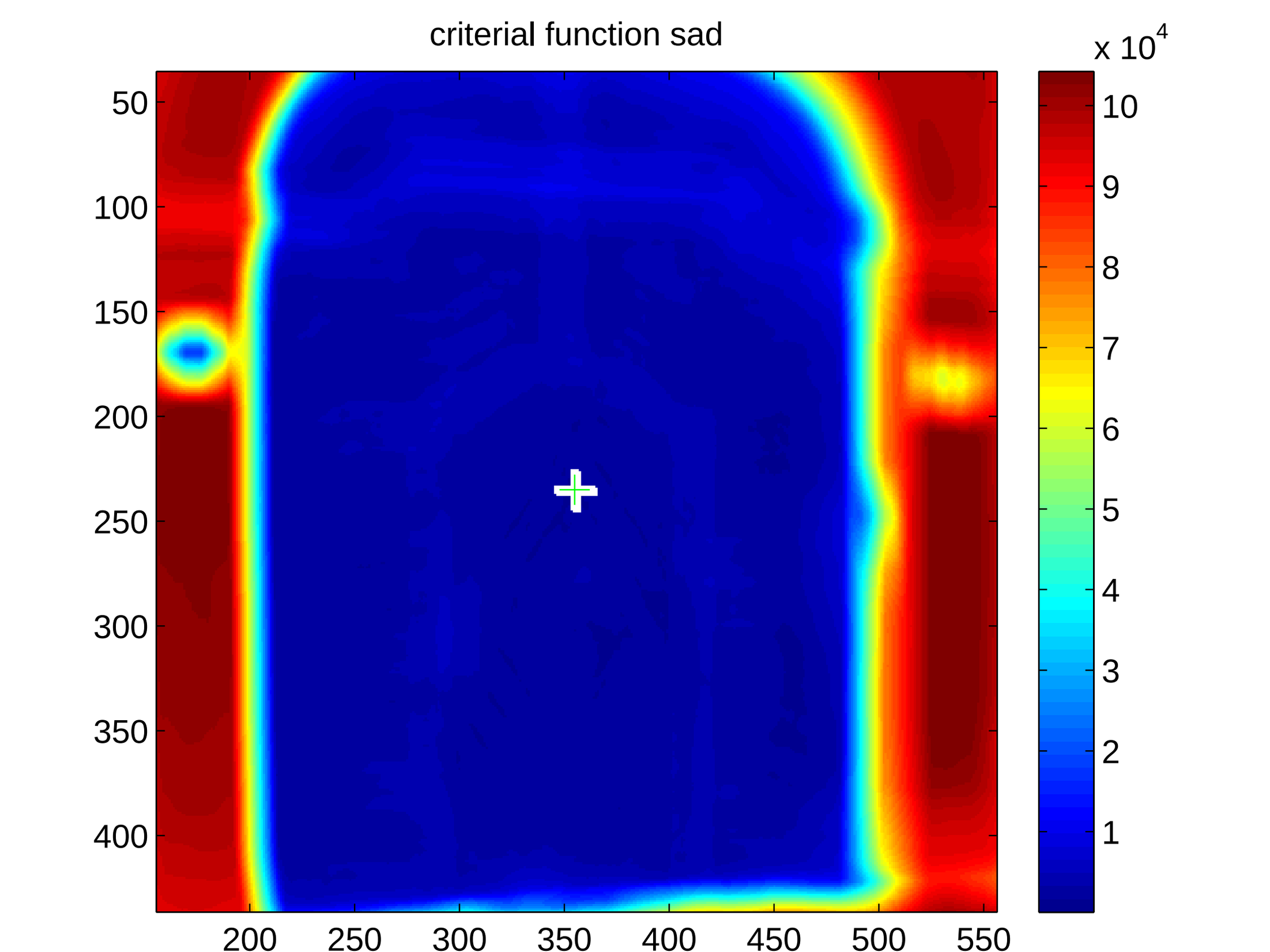
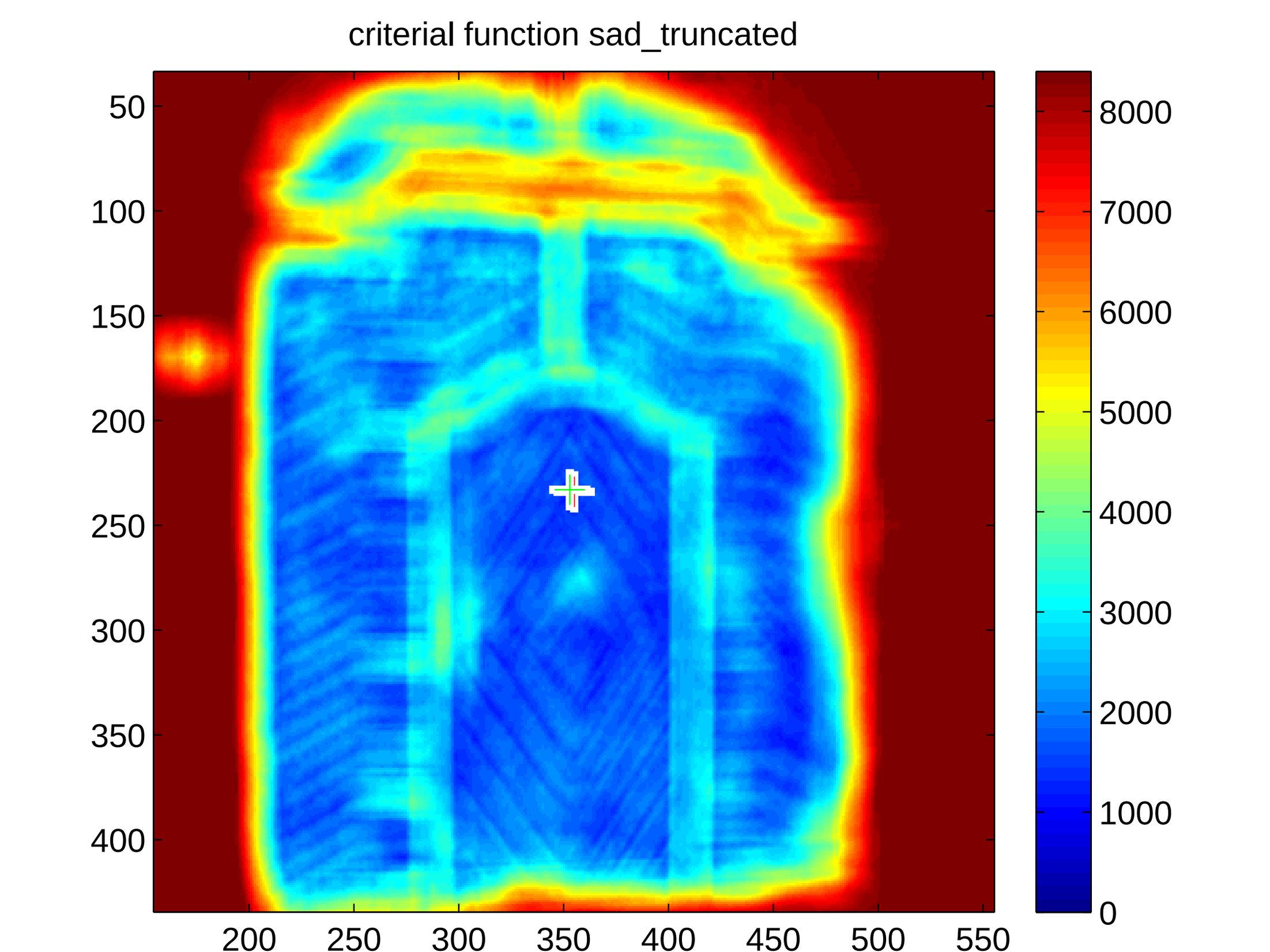
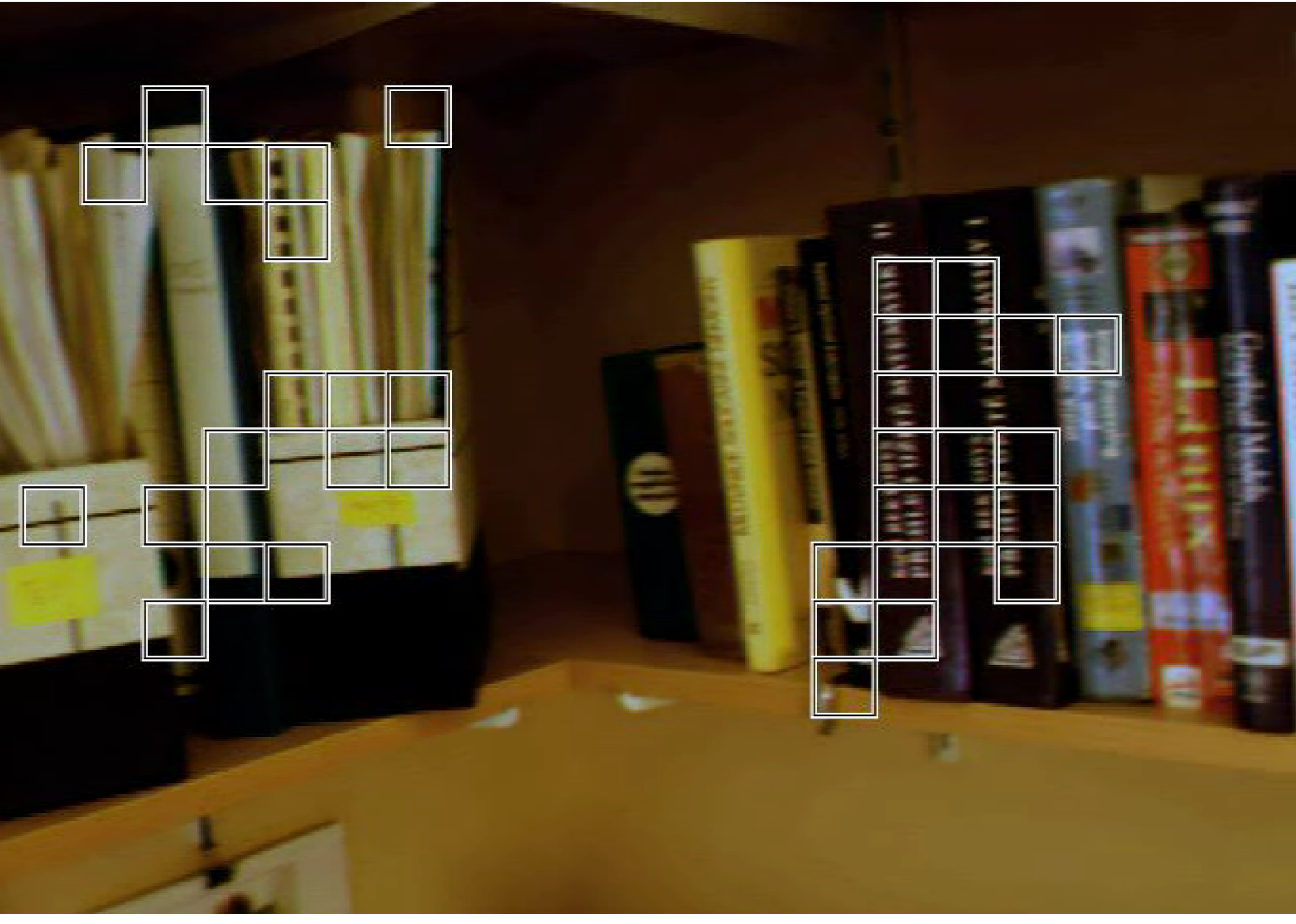
300
350
400
450
500
550
100
200
300
400
500
600
700
4
269


criterial function sad
x 10^4
10
9
8
7
6
5
4
3
2
1
50
100
150
200
250
300
350
400
200
250
300
350
400
450
500
550

50
100
150
200
250
300
350
400
450
500
550
100
200
300
400
500
600
700
4
269


criterial function sad_truncated
50
100
150
200
250
300
350
400
200
250
300
350
400
450
500
550
8000
7000
6000
5000
4000
3000
2000
1000
0