Patch tracking based on comparing its pixels[1]

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Talk Outline

- comparing patch pixels
- normalized cross-correlation, ssd . . .
- KLT - gradient based optimization
- good features to track

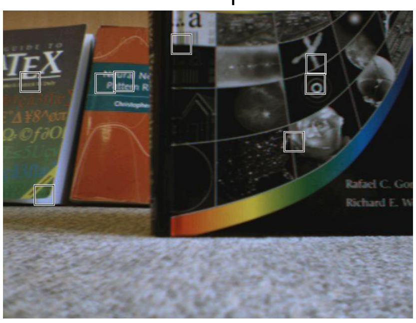
[1]Please note that the lecture will be accompanied be several sketches and derivations on the blackboard and few live-interactive demos in Matlab

What is the problem?

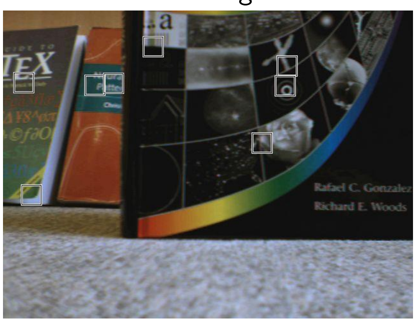
video: CTU campus, door of G building

Tracking of dense sequences — camera motion

T - Template

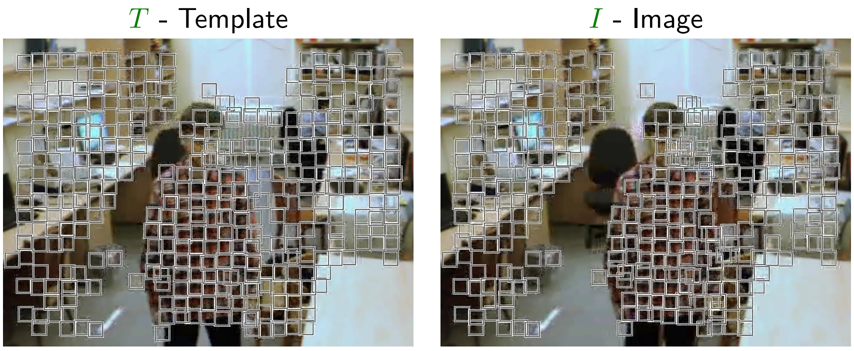


I - Image



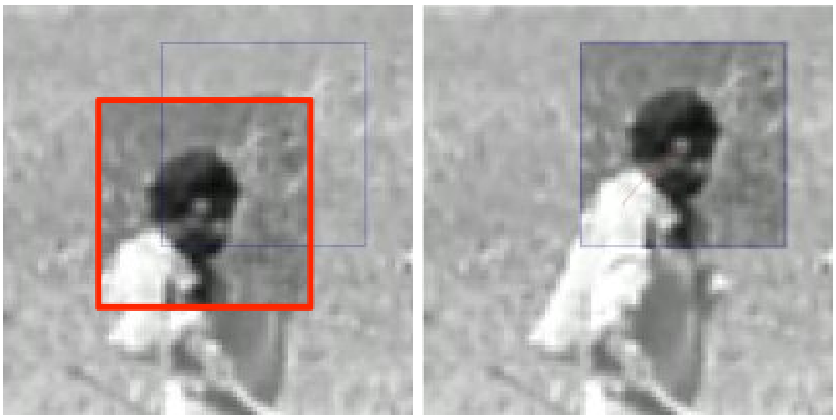
Scene static, camera moves.

Tracking of dense sequences — object motion



Camera static, object moves.

Alignment of an image (patch)



Goal is to align a template image $T(\mathbf{x})$ to an input image $I(\mathbf{x})$. $\mathbf{x}$ column vector containing image coordinates $[x, y]^\top$. The $I(\mathbf{x})$ could be also a small subwindow within an image.

How to measure the alignment?

- What is the best criterial function?
- How to find the best match, in other words, how to find extremum of the criterial function?

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Criterial function

What are the desired properties (on a certain domain)?

How to measure the alignment?

6/38

- What is the best criterial function?
- How to find the best match, in other words, how to find extremum of the criterial function?

Criterial function

What are the desired properties (on a certain domain)?

- convex (remember the optimization course?)
- discriminative
- . . .

Normalized cross-correlation

You may know it as correlation coefficient (from statistics)

$$\rho_{X,Y} = \frac{\mathrm{cov}(X,Y)}{\sigma_X \sigma_Y} = \frac{E[(X-\mu_X)(Y-\mu_Y)]}{\sigma_X \sigma_Y}$$

where σ means standard deviation.

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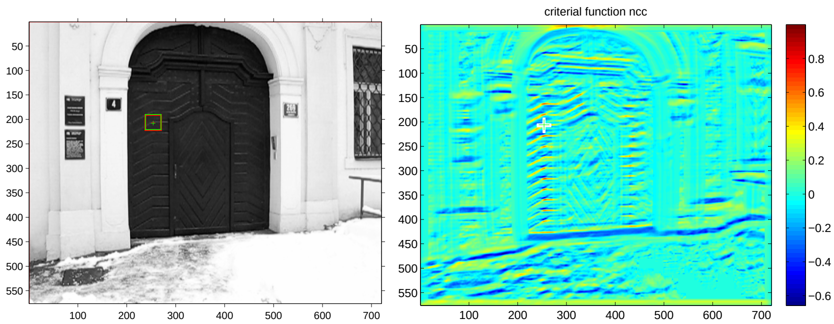
where σ means standard deviation.

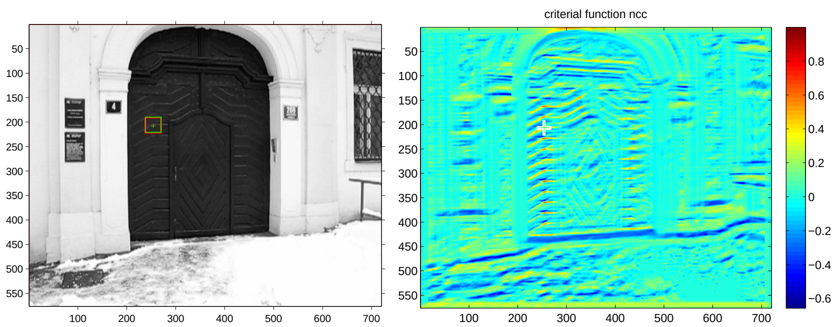
Having template $T(k,l)$ and image $I(x,y)$,

$$r(x,y) = \frac{\sum_k \sum_l \left(T(k,l) - \overline{T}\right)\left(I(x+k,y+l) - \overline{I(x,y)}\right)}{\sqrt{\sum_k \sum_l \left(T(k,l) - \overline{T}\right)^2} \sqrt{\sum_k \sum_l \left(I(x+k,y+l) - \overline{I(x,y)}\right)^2}}$$

Normalized cross-correlation – in picture



Normalized cross-correlation – in picture

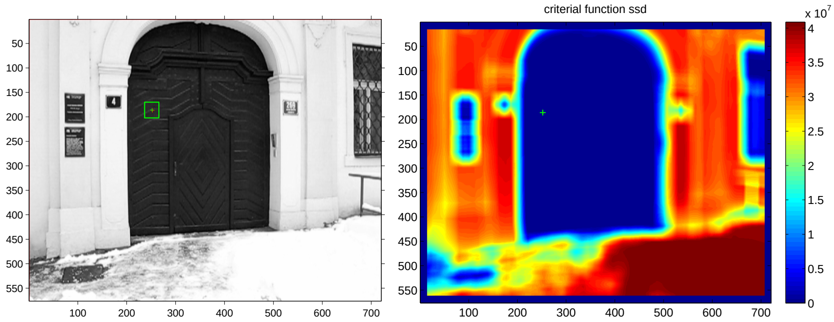


- well, definitely not convex
- but the discriminability looks promising
- very efficient in computation, see [3][2].

[2] check also `normxcorr2` in Matlab

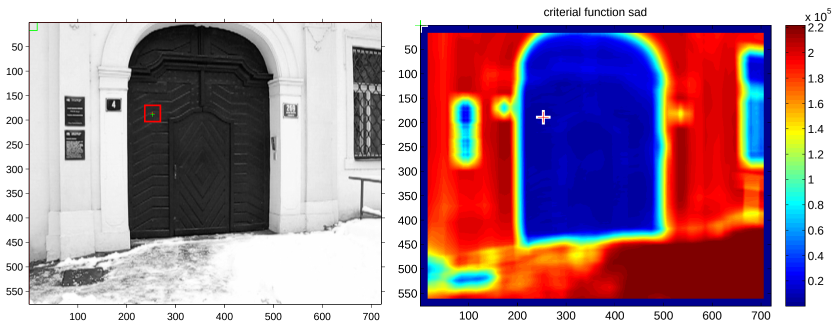
Sum of squared differences

$$ssd(x,y) = \sum_k \sum_l (T(k,l) - I(x+k, y+l))^2$$

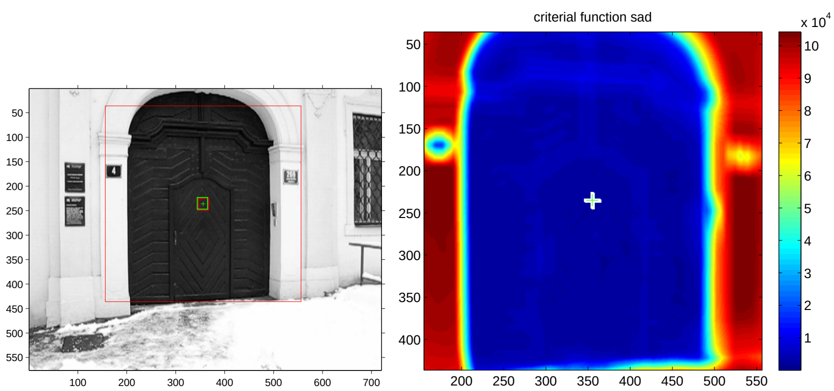


Sum of absolute differences

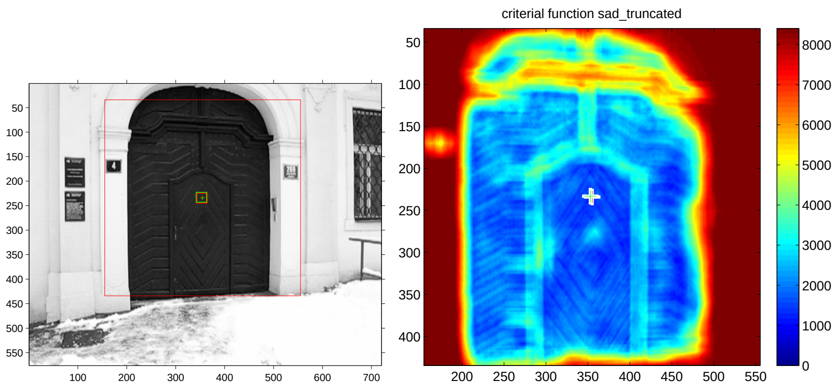
$$sad(x,y) = \sum_k \sum_l |T(k,l) - I(x+k, y+l)|$$



SAD for the door part



SAD for the door part – truncated

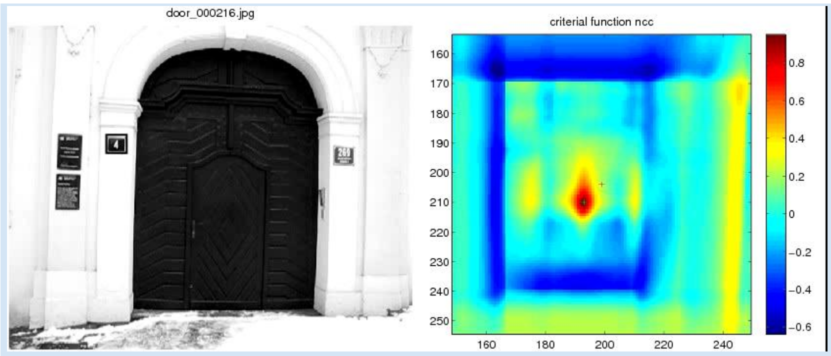


Differences greater than 20 intensity levels are counted as 20.

Normalized cross-correlation: how it works

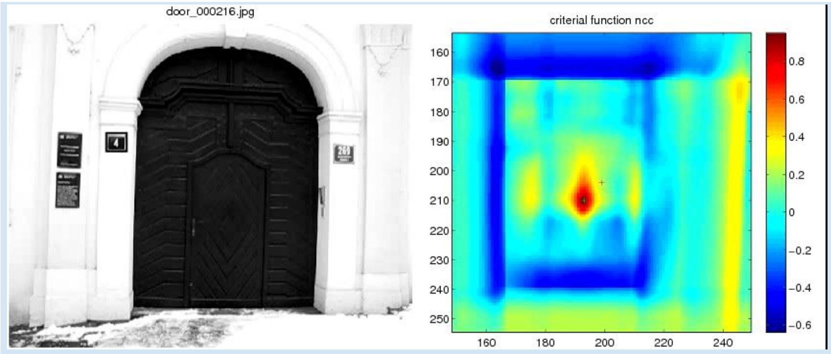
live demo for various patches

Normalized cross-correlation: tracking



video

Normalized cross-correlation: tracking



video

- What went wrong?
- Why did it failed?

Suggestions for improvement?

Tracking as an optimization problem

- finding extrema of a criterial function . . .

Tracking as an optimization problem

- finding extrema of a criterial function . . .
- . . . sounds like an optimization problem

Kanade–Lucas–Tomasi (KLT) tracker

- Iteratively minimizes sum of square differences.
- It is a Gauss-Newton gradient algorithm.

Importance in Computer Vision

- Firstly published in 1981 as an image registration method [4].
- Improved many times, most importantly by Carlo Tomasi [5, 6]
- Free implementation(s) available[3]. Also part of the OpenCV library[4].
- After more than two decades, a project[5] at CMU dedicated to this single algorithm and results published in a premium journal [1].
- Part of plethora computer vision algorithms.

Our explanation follows mainly the paper [1]. It is a good reading for those who are also interested in alternative solutions.

[3] http://www.ces.clemson.edu/~stb/klt/
[4] http://opencv.willowgarage.com/wiki/
[5] http://www.ri.cmu.edu/projects/project_515.html

Original Lucas-Kanade algorithm I

Goal is to align a template image $T(\mathbf{x})$ to an input image $I(\mathbf{x})$. $\mathbf{x}$ column vector containing image coordinates $[x, y]^\top$. The $I(\mathbf{x})$ could be also a small subwindow withing an image.

Set of allowable warps $\mathbf{W}(\mathbf{x}; \mathbf{p})$, where $\mathbf{p}$ is a vector of parameters. For translations

$$\mathbf{W}(\mathbf{x}; \mathbf{p}) = \begin{bmatrix} x + p_1 \\ y + p_2 \end{bmatrix}$$

$\mathbf{W}(\mathbf{x}; \mathbf{p})$ can be arbitrarily complex

The best alignment, $\mathbf{p}^*$, minimizes image dissimilarity

$$\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x}; \mathbf{p})) - T(\mathbf{x})]^2$$

Original Lucas-Kanade algorithm II

$$\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x}; \mathbf{p})) - T(\mathbf{x})]^2$$

$I(\mathbf{W}(\mathbf{x}; \mathbf{p})$ is nonlinear! The warp $\mathbf{W}(\mathbf{x}; \mathbf{p})$ may be linear but the pixels value are, in general, non-linear. In fact, they are essentially unrelated to $\mathbf{x}$.

Linearization of the image: It is assumed that some $\mathbf{p}$ is known and best increment $\Delta\mathbf{p}$ is sought. The modified problem

$$\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x}; \mathbf{p} + \Delta\mathbf{p})) - T(\mathbf{x})]^2$$

is solved with respect to $\Delta\mathbf{p}$. When found then $\mathbf{p}$ gets updated

$$\mathbf{p} \leftarrow \mathbf{p} + \Delta\mathbf{p}$$

. . .

Original Lucas-Kanade algorithm III

$$\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x};\mathbf{p}+\Delta\mathbf{p})) - T(\mathbf{x})]^2$$

linearization by performing first order Taylor expansion[6]

$$\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x})]^2$$

$\nabla I = [\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}]$ is the gradient image[7] computed at $\mathbf{W}(\mathbf{x};\mathbf{p})$. The term $\frac{\partial \mathbf{W}}{\partial \mathbf{p}}$ is the Jacobian of the warp.

[6]Detailed explanation on the blackboard.

[7]As a vector it should have been a column wise oriented. However, for sake of clarity of equations row vector is exceptionally considered here.

Original Lucas-Kanade algorithm IV

Derive $\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x})]^2$ with respect to $\Delta\mathbf{p}$

Original Lucas-Kanade algorithm IV

20/38

Derive $\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x})]^2$ with respect to $\Delta\mathbf{p}$

$$2\sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}}\right]^\top \left[I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x})\right]$$

setting equality to zero yields

Original Lucas-Kanade algorithm IV

Derive $\sum_{\mathbf{x}} [I(\mathbf{W}(\mathbf{x};\mathbf{p})) + \nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \Delta\mathbf{p} - T(\mathbf{x})]^2$ with respect to $\Delta\mathbf{p}$

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setting equality to zero yields

$$\Delta\mathbf{p} = \mathtt{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^\top [T(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p}))]$$

where $\mathtt{H}$ is (Gauss-Newton) approximation of Hessian matrix.

$$\mathtt{H} = \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^\top \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]$$

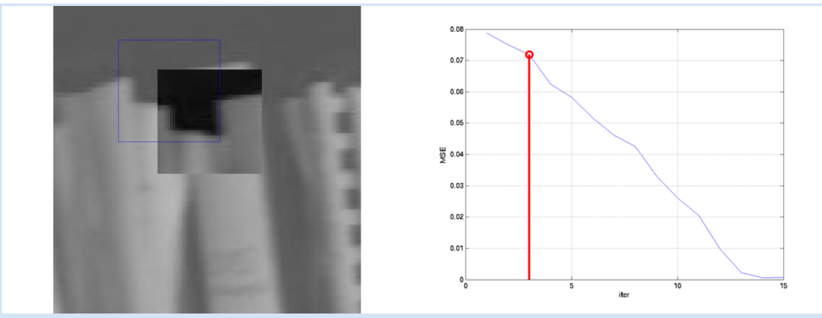
The Lucas-Kanade algorithm—Summary

Iterate:

1. Warp I with $\mathbf{W}(\mathbf{x};\mathbf{p})$
2. Warp the gradient ∇I with $\mathbf{W}(\mathbf{x};\mathbf{p})$
3. Evaluate the Jacobian $\frac{\partial \mathbf{W}}{\partial \mathbf{p}}$ at $(\mathbf{x};\mathbf{p})$ and compute the steepest descent image $\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}}$
4. Compute the $\mathtt{H} = \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^\top \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]$
5. Compute $\Delta\mathbf{p} = \mathtt{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^\top [T(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p}))]$
6. Update the parameters $\mathbf{p} \leftarrow \mathbf{p} + \Delta\mathbf{p}$

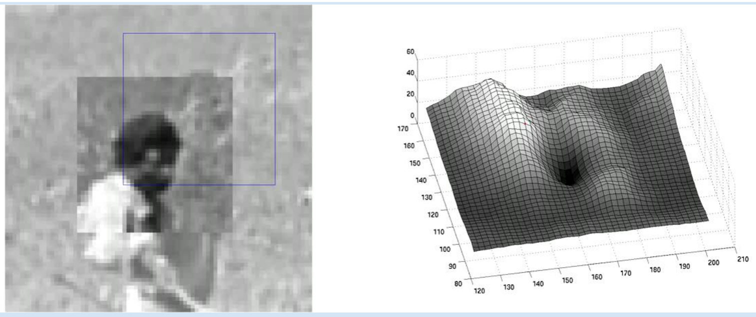
until $\|\Delta\mathbf{p}\| \leq \epsilon$

Example of convergence



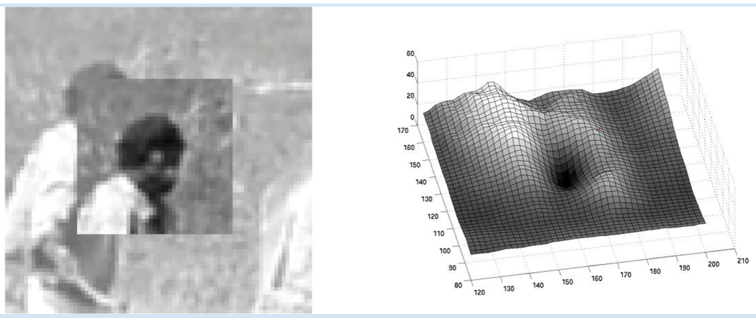
video

Example of convergence



Convergence video: Initial state is within the basin of attraction

Example of divergence

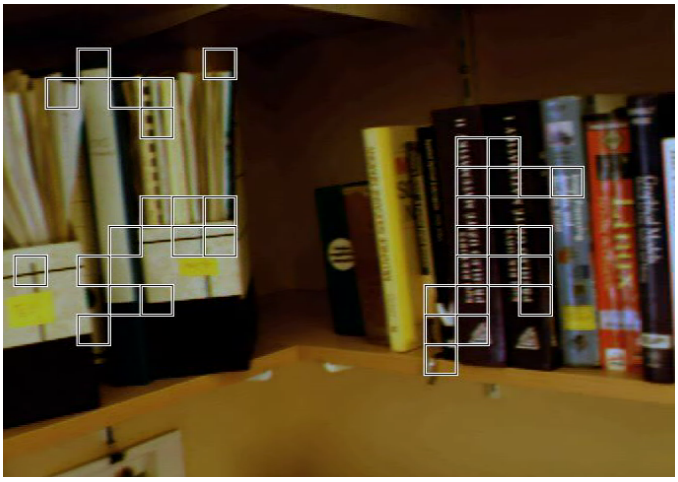


Divergence video: Initial state is outside the basin of attraction

Example – on-line demo

Let play and see . . .

What are good features (windows) to track?



What are good features (windows) to track?

How to select good templates $T(\mathbf{x})$ for image registration, object tracking.

$$\Delta\mathbf{p} = \mathtt{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^{\top} [T(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p}))]$$

What are good features (windows) to track?

27/38

How to select good templates $T(\mathbf{x})$ for image registration, object tracking.

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where $\mathtt{H}$ is the matrix

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What are good features (windows) to track?

How to select good templates $T(\mathbf{x})$ for image registration, object tracking.

$$\Delta\mathbf{p} = \mathtt{H}^{-1} \sum_{\mathbf{x}} \left[\nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^\top [T(\mathbf{x}) - I(\mathbf{W}(\mathbf{x};\mathbf{p}))]$$

where $\mathtt{H}$ is the matrix

$$\mathtt{H} = \sum_{\mathbf{x}} \left[\nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^\top \left[\nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]$$

The stability of the iteration is mainly influenced by the inverse of Hessian. We can study its eigenvalues. Consequently, the criterion of a good feature window is $\min(\lambda_1, \lambda_2) > \lambda_{min}$ (texturedness).

What are good features for translations?

Consider translation $\mathbf{W}(\mathbf{x};\mathbf{p}) = \begin{bmatrix} x + p_1 \\ y + p_2 \end{bmatrix}$. The Jacobian is then $\frac{\partial \mathbf{W}}{\partial \mathbf{p}} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\begin{aligned}
\mathtt{H} &= \textstyle\sum_{\mathbf{x}} \left[\nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right]^\top \left[\nabla I \, \frac{\partial \mathbf{W}}{\partial \mathbf{p}} \right] \\
&= \textstyle\sum_{\mathbf{x}} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{\partial I}{\partial x} \\ \frac{\partial I}{\partial y} \end{bmatrix} [\frac{\partial I}{\partial x}, \frac{\partial I}{\partial x}] \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
&= \textstyle\sum_{\mathbf{x}} \begin{bmatrix} \left(\frac{\partial I}{\partial x}\right)^2 & \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} \\ \frac{\partial I}{\partial x}\frac{\partial I}{\partial y} & \left(\frac{\partial I}{\partial y}\right)^2 \end{bmatrix}
\end{aligned}$$

The image windows with varying derivatives in both directions. Homeogeneous areas are clearly not suitable. Texture oriented mostly in one direction only would cause instability for this translation.