

Mean shift ¹

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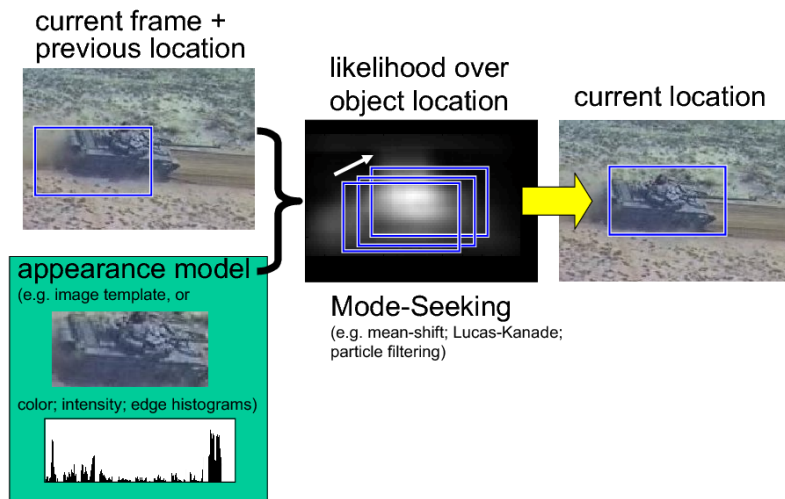
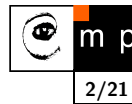
Last update: April 8, 2013

Talk Outline

- ◆ appearance based tracking
- ◆ patch similarity using histogram
- ◆ tracking by mean shift
- ◆ experiments, discussion

¹Please note that the lecture will be accompanied by several sketches and derivations on the blackboard and few live-interactive demos in Matlab

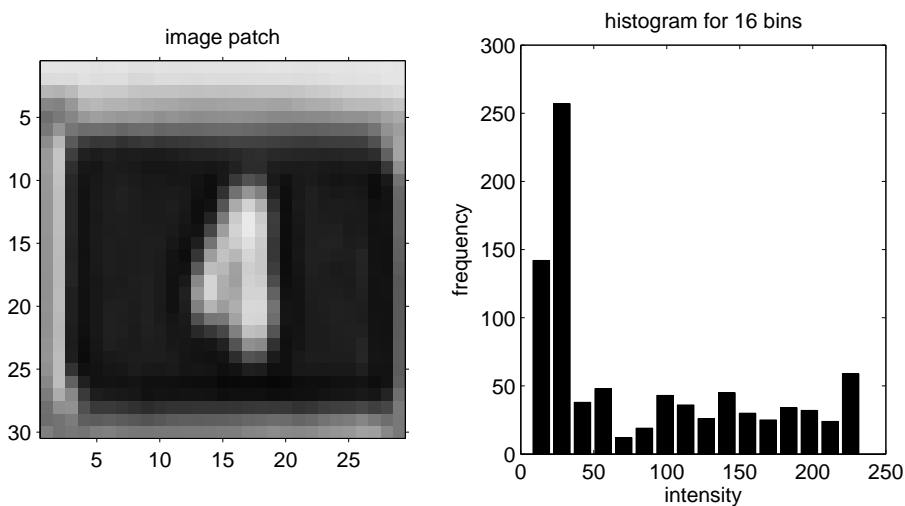
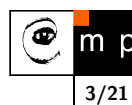
Appearance based tracking



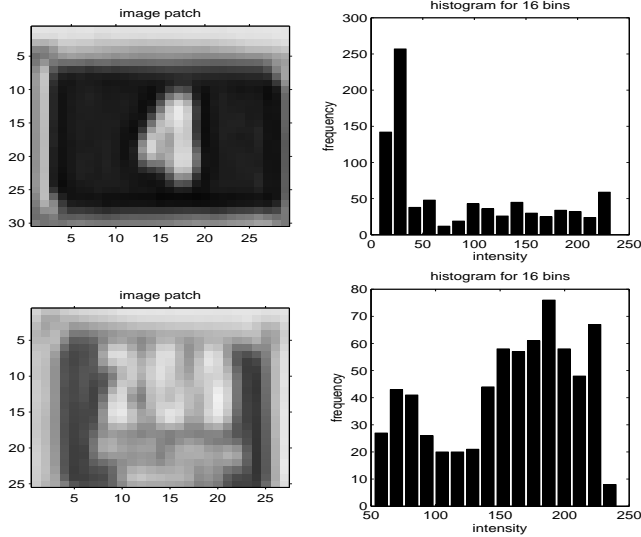
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²illustration from [1]

Histogram based representation



Patch comparison



histogram difference

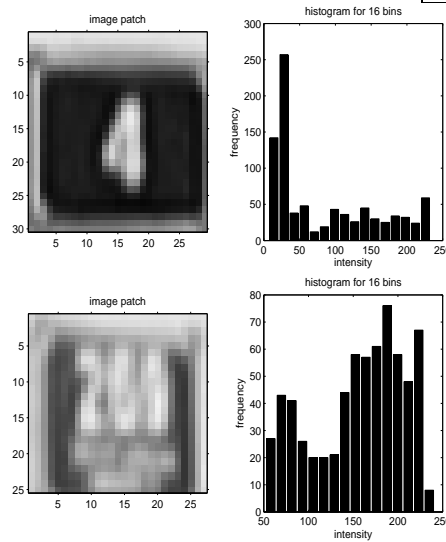
assume **normalized** histograms, i.e

$$\sum_{u=1}^m p_u = 1$$

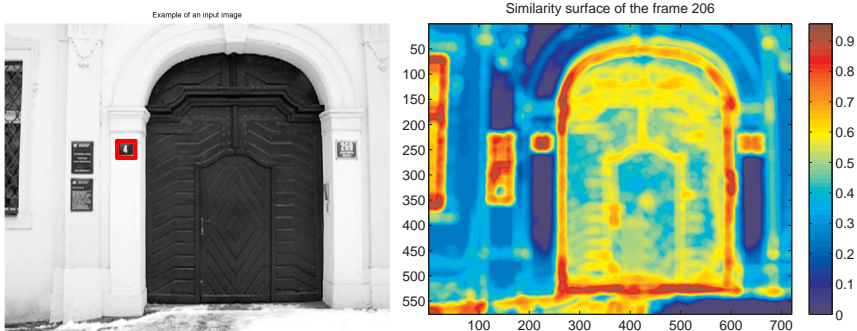
$$d = \sqrt{1 - \rho[p, q]}$$

where $\rho[p, q]$ is the **Bhattacharyya coefficient**

$$\rho[p, q] = \sum_{u=1}^m \sqrt{p_u q_u}$$



Similarity measured by the Bhattacharyya coefficient

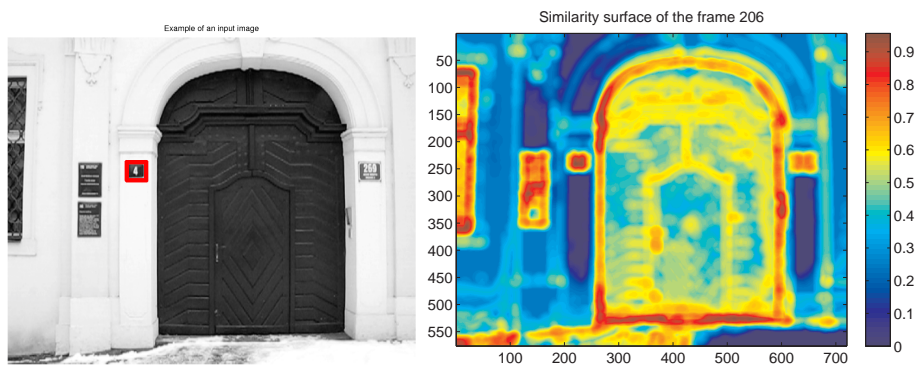


The object is the "4" plate and the model is histogram of image intensities.

$$s(\mathbf{y}) = \sum_{u=1}^m \sqrt{p_u(\mathbf{y}) q_u}$$

where $p(\mathbf{y})$ is the histogram of image patch at position \mathbf{y} and q is the histogram of the template.

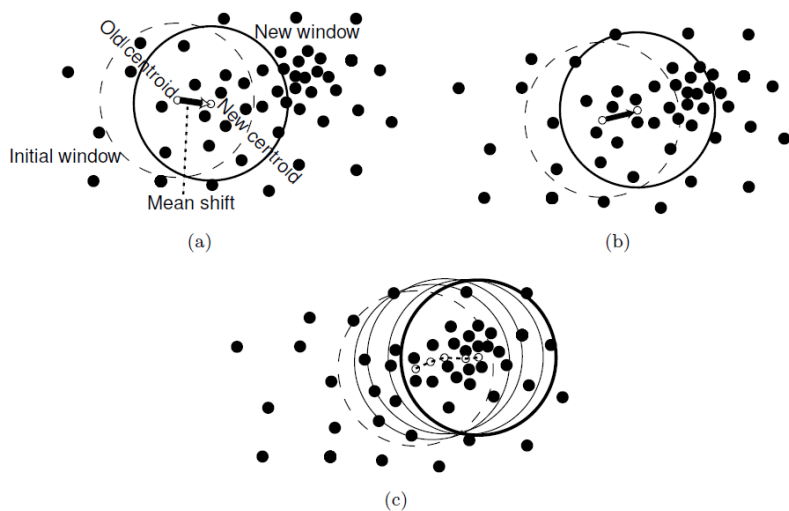
Problem: finding modes in probability density



- ◆ the complete enumeration of similarity surface can be costly,
- ◆ can we do it faster and more elegantly?

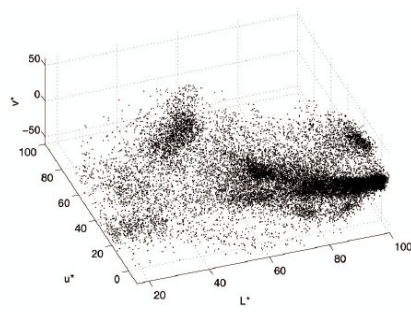
Density Gradient Estimation

Mean shift procedure



³Figure borrowed from [4]

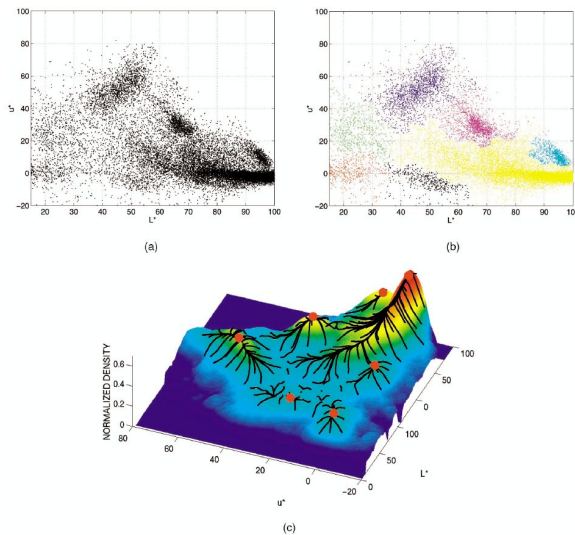
Meanshift segmentation of colours - color distribution



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⁴Figure from [2]

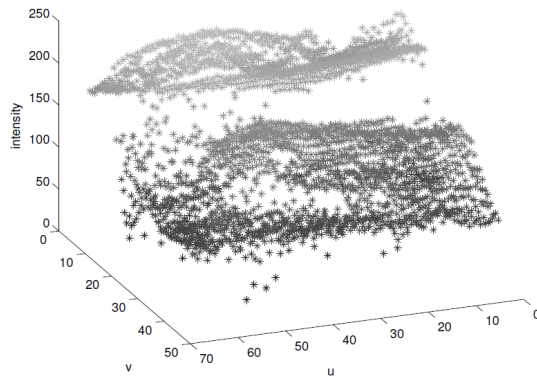
Meanshift segmentation of colours - color modes seeking



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⁵Figure from [2]

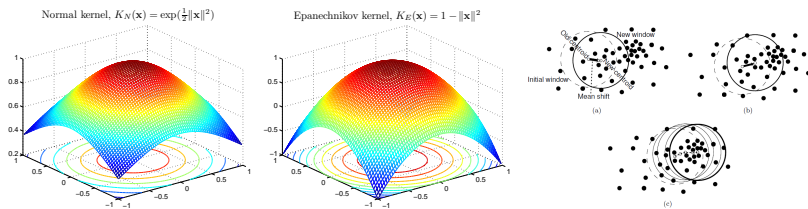
Mean shift segmentation - intensity and space



u, v are here spatial pixel coordinates

different normalization for intensity and spatial coordinates

Multivariate kernel density estimator

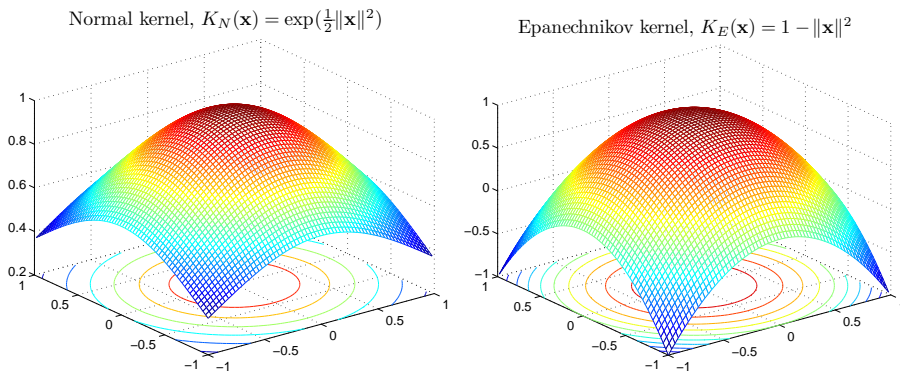


Given n data points \mathbf{x}_i in d -dimensional space R^d .

$$\tilde{f}_{h,K}(\mathbf{x}) = \frac{1}{nh^d} \sum_{i=1}^n K\left(\frac{\mathbf{x} - \mathbf{x}_i}{h}\right)$$

- ◆ looking for extremum of $f_{h,K}(\mathbf{x})$
- ◆ gradient $\nabla f_{h,K}(\mathbf{x}) = \mathbf{0}$

Kernels

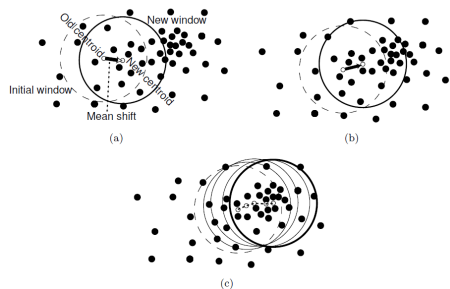


Can be seen as membership function.
Remind **Kernel density estimation** (Parzen method).

Remember convolution? 6

⁶Taken from http://en.wikipedia.org/wiki/Kernel_density_estimation

Mean-shift iterations

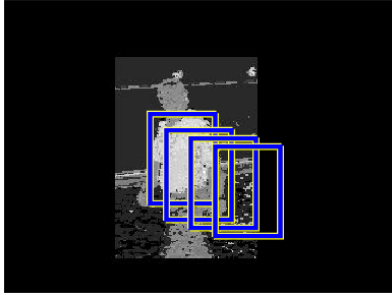
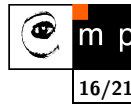


Assuming a reasonable differentiable kernel K , iterate till convergence:

$$\mathbf{y}_{k+1} = \frac{\sum_{i=1}^n \mathbf{x}_i g(\|\mathbf{y}_k - \mathbf{x}_i\|^2)}{\sum_{i=1}^n g(\|\mathbf{y}_k - \mathbf{x}_i\|^2)}$$

g is the derivative of **kernel profile**.

Mean-shift tracking - ratio histogram



Ratio histogram:

$$r_u = \min\left(\frac{q_u}{p_u}, 1\right)$$

where q is the histogram of the target and p is the histogram of the **current** frame. $w_i = r_{b(x_i)}$ (just binning)

Image intensities (or colors) are transformed into **weights**, w_i , by back projection of the ratio histogram. Mean-shift iterations:

$$\mathbf{y}_{k+1} = \frac{\sum_{i=1}^n w_i \mathbf{x}_i g(\|\mathbf{y}_k - \mathbf{x}_i\|^2)}{\sum_{i=1}^n w_i g(\|\mathbf{y}_k - \mathbf{x}_i\|^2)}$$

Mean-shift tracking - Bhattacharya coefficient



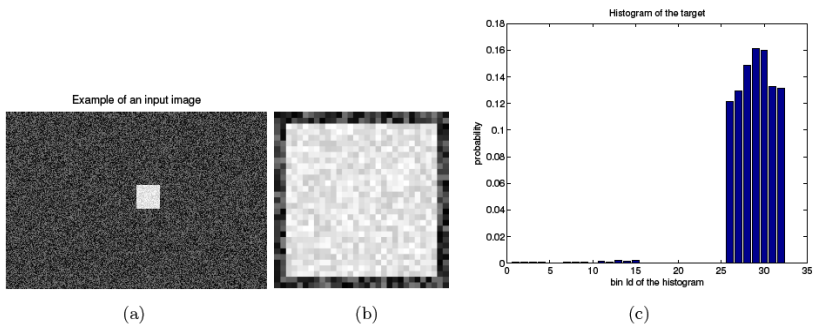
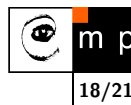
model, coordinates \mathbf{x}_i^* centered at $\mathbf{0}$:

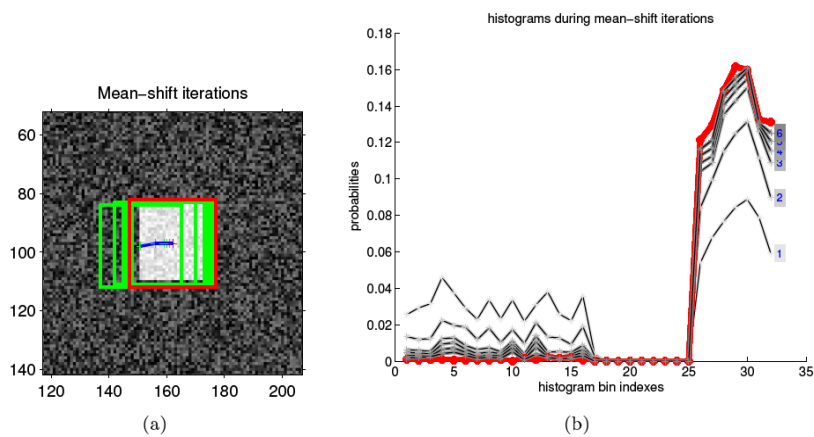
$$q_u = C \sum_{i=1}^n k(\|\mathbf{x}_i^*\|^2) \delta(b(\mathbf{x}_i^*) - u)$$

target candidate centered at \mathbf{y} :

$$p_u(\mathbf{y}) = C_h \sum_{i=1}^{n_h} k\left(\left\|\frac{\mathbf{y} - \mathbf{x}_i}{h}\right\|^2\right) \delta(b(\mathbf{x}_i) - u)$$

ms tracking - object and its model





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⁸Figure from [5]

References

Mean-shift originally from [3].

- [1] Robert Collins. CSE/EE486 Computer Vision I. slides, web page. <http://www.cse.psu.edu/~rcollins/CSE486/>. Robert kindly gave general permission to reuse the material.
- [2] Dorin Comaniciu and Peter Meer. Mean shift: A robust approach toward feature space analysis. *IEEE Transactions on Pattern Analysis and Machine Analysis*, 24(5):603–619, May 2002.
- [3] Keinosuke Fukunaga and Larry D. Hostetler. The estimation of the gradient of a density function, with applications in pattern recognition. *IEEE Transactions on Information Theory*, 21(1):32–40, January 1975.
- [4] Milan Šonka, Václav Hlaváč, and Roger Boyle. *Image Processing, Analysis and Machine Vision*. Thomson, 3rd edition, 2007.
- [5] Tomáš Svoboda, Jan Kybic, and Václav Hlaváč. *Image Processing, Analysis and Machine Vision. A MATLAB Companion*. Thomson, 2007. Accompanying www site <http://visionbook.felk.cvut.cz>.

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