Ant Colony Optimization Algorithms

- Construction heuristics
- How ants find shortest route
  - Stigmergy
- General ACO metaheuristic
- Ant System for TSP

Motivation

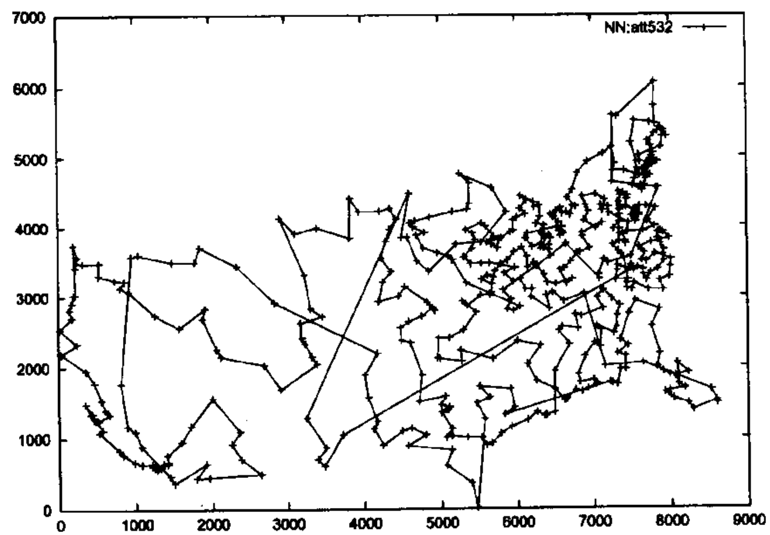
- **NP-hard problems** – no algorithms that could solve large instances of these problems to optimality
  - Discrete combinatory problems
- **Approximate metods** – can find solutions of good quality in reasonable time
- **Approximate metods**
  - **Local search/optimization**
    - Iteratively improves a complete solution (typically initialized at random) till it reaches some local optimum.
  - **Construction algorithms**
    - Build a solution making use of some problem-specific heuristic information
- **Ant Colony Optimization** (ACO) algorithms – extend traditional construction heuristics with an ability to exploit experience gathered during the optimization process.

Construction Algorithms

- Build solutions to a problem under consideration in an incremental way starting with an empty initial solution and iteratively adding opportunely defined solution components without backtracking until a complete solution is obtained.

Procedure *GreedyConstructionHeur*
    s_p = *empty_solution*
    while not complete(s_p) **do**
        e = *GreedyComponent*(s_p)
        $s_p = s_p \otimes$ e
    end
    return s_p
end

TSP: nearest neighbor heuristic



- **Pros/Cons**

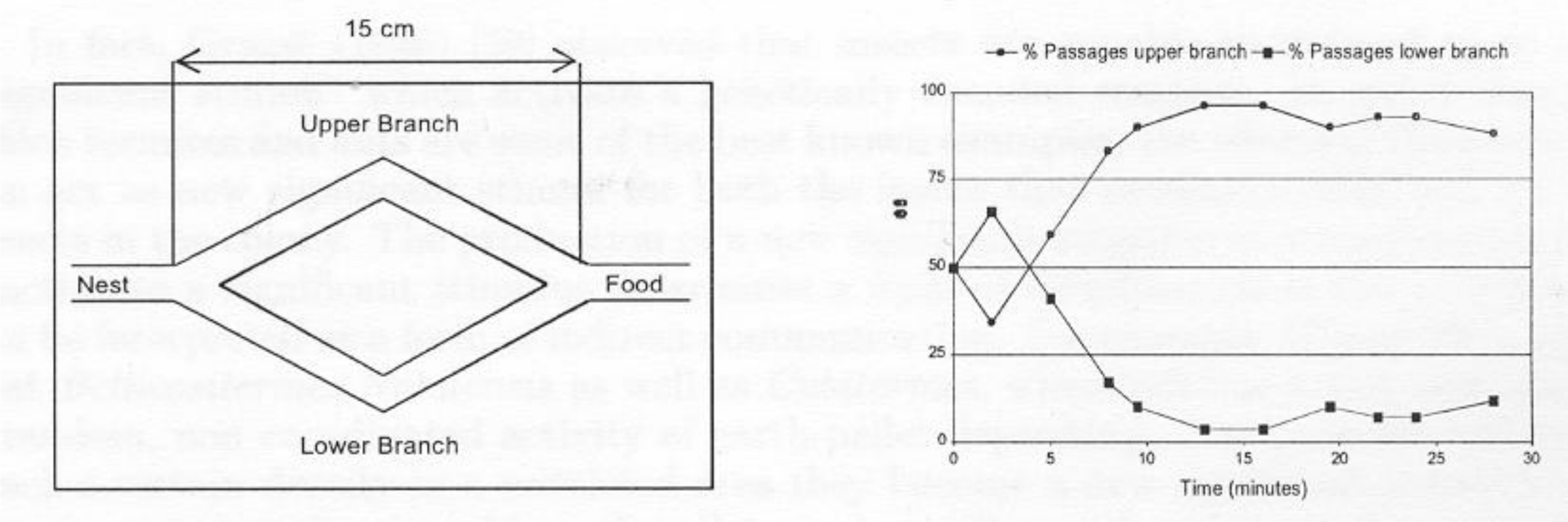
  + fast, solutions of reasonable quality
  - Solution may be far from optimum
  - Generate only limited number of different solutions
  - Decisions made at early stages reduce a set of possible steps at latter stages

Ant Algorithms: Biological Inspiration

- **Inspired by behavior of an ant colony**
  - Social insects – behave towards survival of the colony
  - Simple individual behavior × complex behavior of a colony
- Ability to find the shortest path from the colony to the source of food and back using an **indirect communication via pheromone**
  - **Write** – ants lay down pheromone on their way to food
  - **Read** – ant detects pheromone (can sense different intensity) laid down by other ants and can choose a direction of the highest concentration of pheromone.
  - **Emergence** – this simple behavior applied by the whole colony can lead to emergence of the shortest path.

Experiments with Real Ants

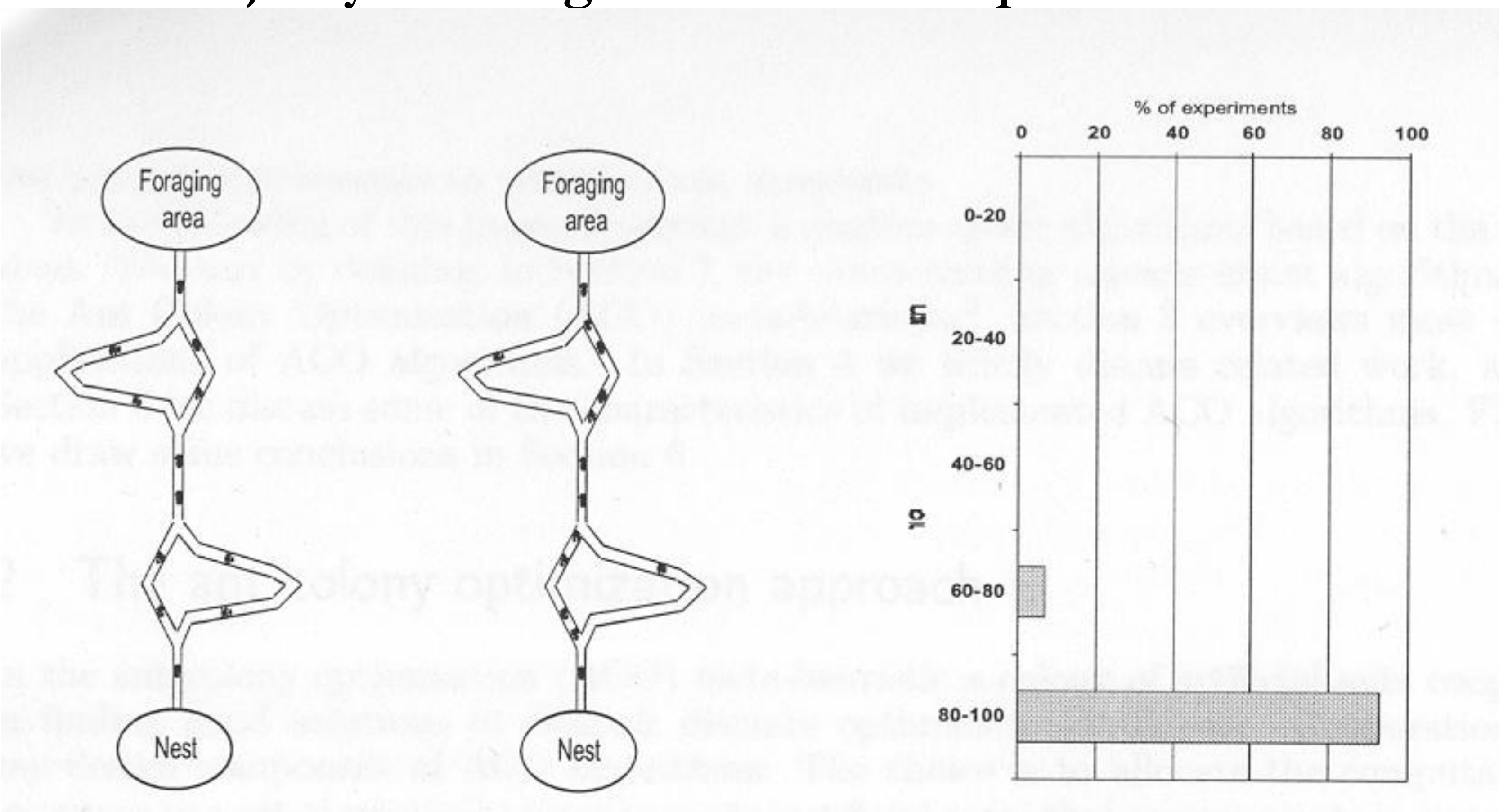
- **Deneuborg et al.** (ants *Linepithema humile*)
- **Nest separated from food with a double-bridge**
  - Both path of the same length
  - At the beginning there is no pheromone
  - After some time one of the alternatives gets dominant due to random fluctuations



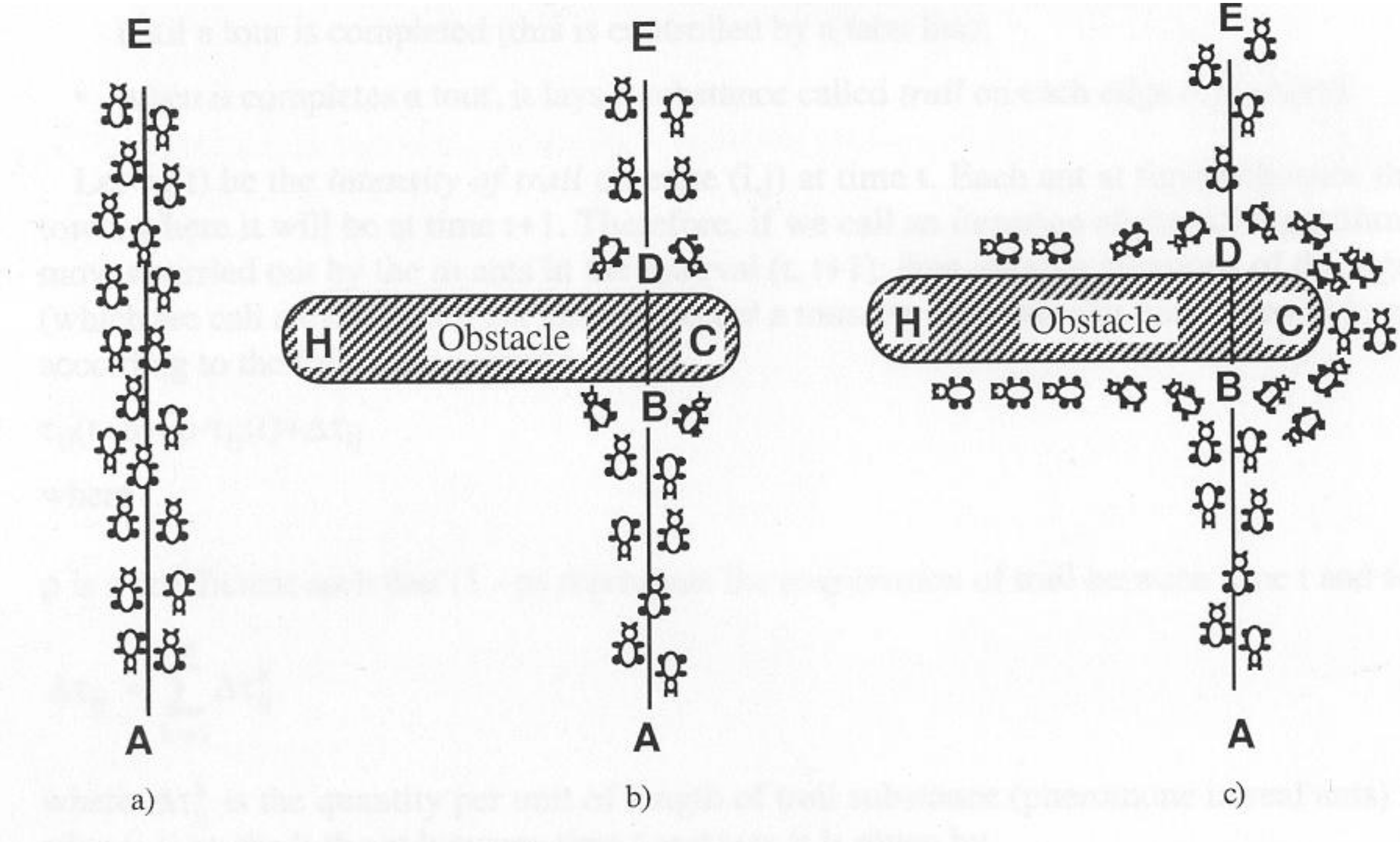
(a) (b)

Bridges with Different Branches

- **Influence of random fluctuations is significantly reduced and majority of ants go for the shorter path in the end.**

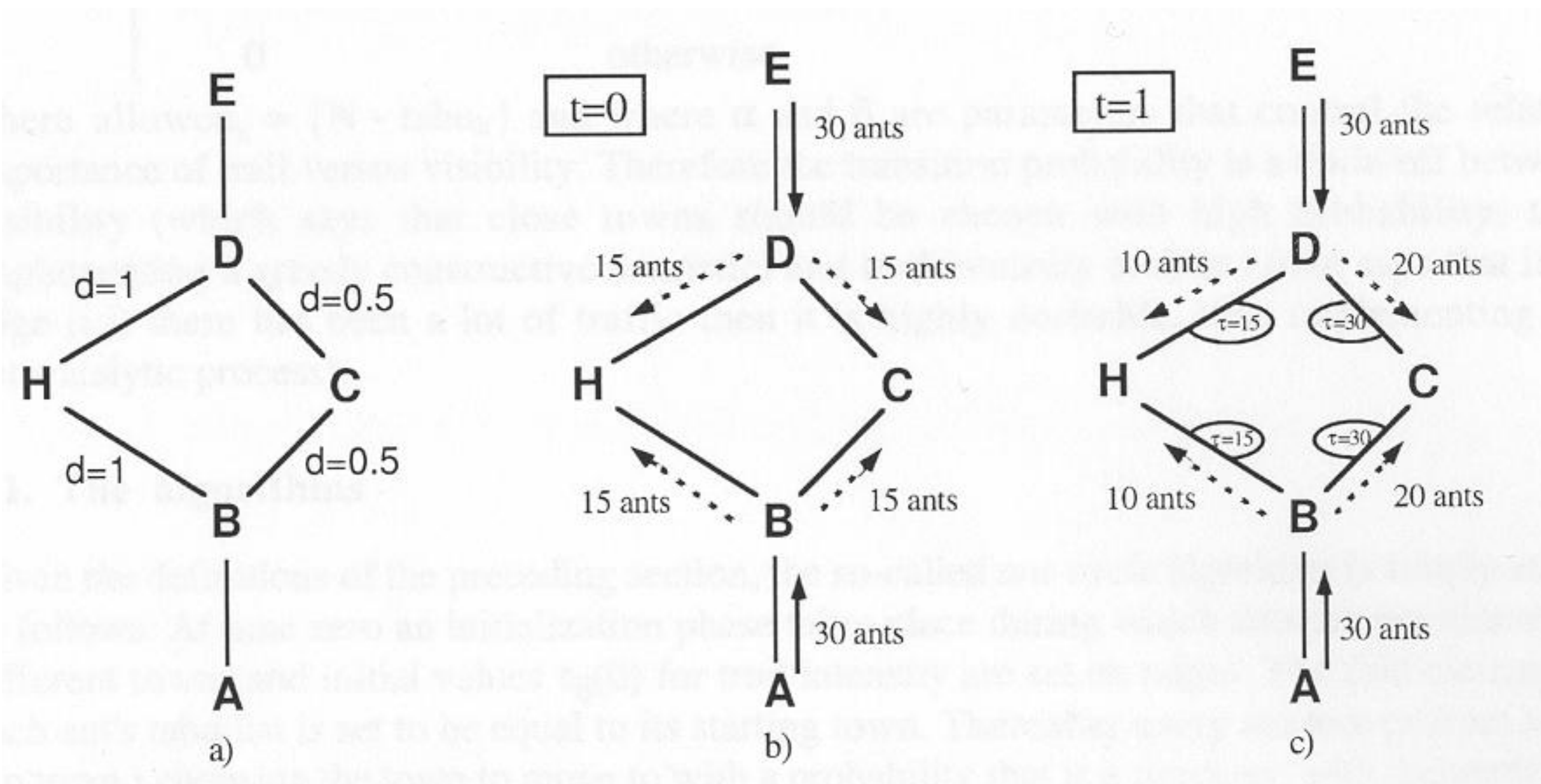


Example



Example

- In each step 30 new ants go from A to B, and 30 ants from E to D
  - All ants go with the same speed 1 s^{-1}
  - Each ant deposits down 1 unit of pheromone per 1 time unit



Stigmergy

- **Stigmergie** - two individuals interact indirectly when one of them modifies the environment and the other responds to the new environment at a later time.
  - **Physically** – by depositing a pheromone the ants modify the place they have visited**.**
  - **Locality of information** – pheromone is "visible" only to ants that are in its close vicinity.
  - **Autocatalytic behavior** – the more ants follow a trail, the more attractive that trail becomes for being followed.
    The process is thus characterized by a **positive feedback loop**, where the probability of a discrete path choice increases with the number of times the same path was chosen before
- **Pheromone evaporation** – realizes forgetting, which prevents premature convergence to suboptimal solutions.

Real Ants Resume

- Almost blind
- Incapable of achieving complex tasks alone
- Capable of establishing shortest-route paths from their colony to feeding sources and back
- Use *stigmergic* communication via pheromone trails
- Follow existing pheromone trails with high probability

Artificial Ants

- **Similarity with real ants**:
  - Colony of cooperating ants
  - Pheromone trail and stigmergy
  - Probabilistic decision making, locality of the strategy
    - Prior information given by the problem specification
    - Local modification of states, induced by preceding ants
- **Differences from real ants**:
  - Discrete world
  - Inner states – personal memory with already performed actions
  - Ants are not completely blind
  - Amount of deposited pheromone is a function of the quality of the solution
  - Problem dependent timing of depositing the pheromone
  - Extras – local optimization, backtracking

Ant Colony Optimization Metaheuristic

- ACO can be applied to any discrete optimization problem for which some solution construction mechanism can be conceived.
- **Artificial ants are stochastic solution construction heuristics** that probabilistically build a solution by iteratively adding solution components to partial solutions by taking into account
  - heuristic information on the problem instance being solved, if available,
  - (artificial) pheromone trails which change dynamically at run-time to reflect the agents' acquired search experience.
- **Stochastic component** allows generating a large number of different solutions.

General ACO metaheuristic

```
procedure ACO metaheuristics
    ScheduleActivities
        ManageAntActivity()
        EvaporatePheromone()          // forgetting
        DaemonActions() {optional}    // centralized actions
                                         local search, elitism
    end ScheduleActivities
end ACO metaheuristics
```

Steps for implementing ACO

- Choose appropriate graph representation
- Define positive feedback
- Choose constructive heuristic
- Choose a model for constraint handling (*tabu* list at TSP)

Ant System (AS) for TSP

- **Problem:** Given $\boldsymbol{n}$ cities, the goal is to find the shortest path going through all cities and visiting each exactly once.
  - Consider complete graph.
  - $\boldsymbol{d_{ij}}$ is Euclidean distance from city i to city j
- **Definition**
  - $\boldsymbol{m}$ is the number of ants
  - $\boldsymbol{\tau_{ij}(t)}$ is the intensity of pheromone on the link (i, j) in time t
  - $\boldsymbol{\eta_{ij}}$ is visibility (heuristic information) expressed by $1/\boldsymbol{d_{ij}}$
  - $\boldsymbol{(1\text{-}\rho)}$ evaporation factor, ρ is constant for the whole opt. process
  - $\boldsymbol{tabu_k}$ is dynamically growing vector of cities that have already been visited by k-th ant
  - **AS iteration** - each ant adds one city to the built route
  - **AS cycle** – composed of n iterations during which all ants complete their routes

AS: Pheromone Deposition

- $\tau_{ij}(t+n) = \rho \cdot \tau_{ij}(t) + \Delta\tau_{ij}$
- $\Delta\tau_{ij} = \sum_k \Delta\tau_{ij}^k$
- $\Delta\tau_{ij}^k = \begin{cases} Q/L_k, & \text{if } k\text{-th ant used the edge } (i, j) \\ 0, & \text{otherwise.} \end{cases}$

where

$\Delta\tau_{ij}^k$ is the amount of pheromone deposited on the edge (i, j) by k-th ant within a time interval $(t, t+n)$

Q is a constant

L_k is the length of the route constructed by k-th ant

ρ must be smaller than 1, otherwise the pheromone would accumulate unboundedly (recommended is 0.5)

$\tau_{ij}(0)$ is set to small positive values

AS: Probabilistic Decision Making

- Probability of adding a link i-j (where $j \in \{N - tabu_k\}$) into the route

$$p_{ij}^{k}(t) = \begin{cases} [\tau_{ij}(t)]^{\alpha} \cdot [\eta_{ij}]^{\beta} \,/\, \sum_l [\tau_{ij}(t)]^{\alpha} \cdot [\eta_{ij}]^{\beta}, & \text{if } j \in \{N - tabu_k\} \\ 0, & \text{otherwise.} \end{cases}$$

where

$l \in \{N - tabu_k\}$

α, β define relative importance of the pheromone and the visibility

- Probability is a compromise between
  - **visibility** that prefers closer cities to more distant ones and
  - **intensity of pheromone** that prefers more frequently used edges.

AS: Cycle

- ***Ant-cycle*:**
  1. Initialization
     - time: t=0
     - number of cycles: NC=0
     - pheromone: $\tau_{ij}(t)=c$
     - Initial positioning of m ants to n cities
  2. Initialization of *tabu* lists
  3. Ants' action
     - Each ant iteratively builds its route
     - Calculate length of the routes L_k for all ants $k \in (1,...,m)$
     - update the shortest route found
     - Calculate $\Delta\tau_{ij}^k$ and update $\tau_{ij}(t+n)$
  4. Increment discrete time
     - $t = t+n$, $NC = NC+1$
  5. If($NC < NC_{max}$) then goto step 2

     else stop

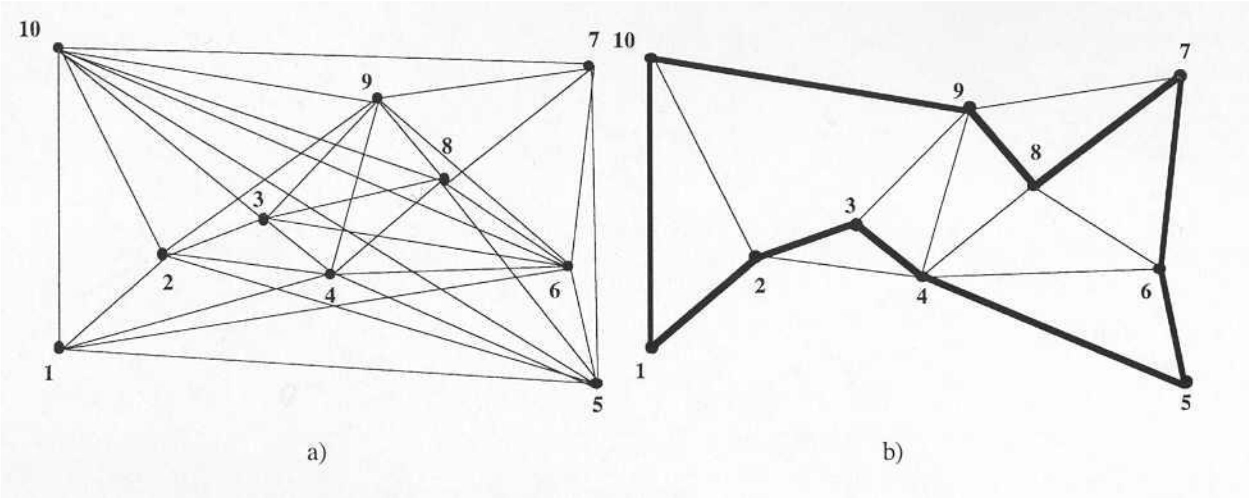
AS: Elitism

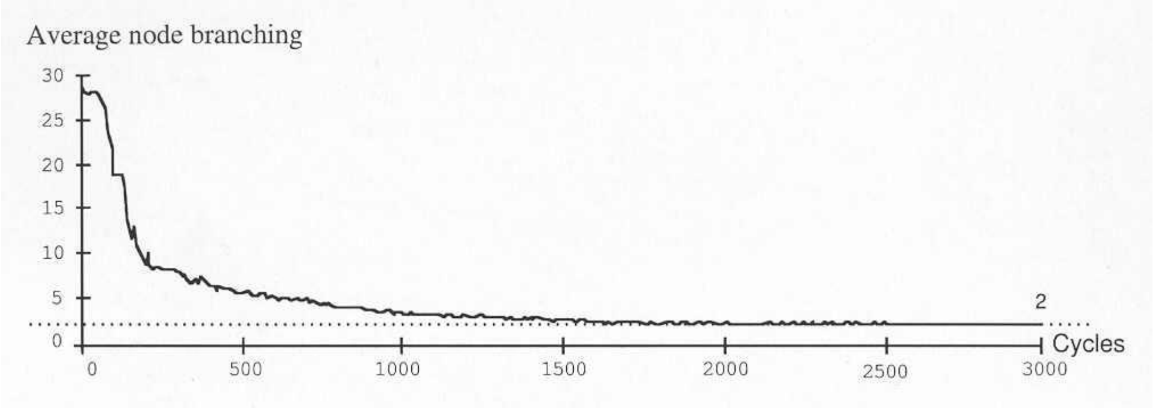
- Intensity of pheromone is strengthened on edges that lie on the shortest path out of all generated paths
  - Amount of added pheromone: $e \cdot Q/L^*$,

    where e is a number of „elite“ ants and L* is the shortest path
  - **Beware of premature convergence.**

AS: Evolution of Solution for 10 Cities

- After greedily searching the space it is desirable to adapt global information stored in $\tau_{ij}(t)$ (it is necessary to partially forget)





- **Stagnation** – branching factor is 2, all ants go the same way.

Applications of ACO algorithms

- **Static problems**
  - Traveling salesman
  - Quadratic assigment
  - Job-shop scheduling
  - Vehicle routing
  - Graph colouring
  - Shortest common supersequence
- **Dynamic problems**
  - Network routing

References

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http://iridia.ulb.ac.be/~mdorigo/ACO/ACO.html