Evolutionary Algorithms: Real-Parameter Evolutionary Algorithms

Jiří Kubalík
Department of Cybernetics, CTU Prague

http://cw.felk.cvut.cz/doku.php/courses/a4m33bia/start

Evolutionary Algorithms for Real-Parameter Optimizations

Real Coding Advantages

- The use of real parameters makes it possible to use **large domains** (even unknown domains!) for the variables.
- The capacity to exploit the **graduality** of the functions with continuous variables. The concept of graduality refers to the fact that slight changes in the variables correspond to slight changes in the function.
- The capacity for the **local tuning** of the solutions.
- The coding and decoding processes are avoided, thus **increasing the EA's speed**.

Evolutionary Algorithms for Real-Parameter Optimizations

Real Coding Advantages

- The use of real parameters makes it possible to use **large domains** (even unknown domains!) for the variables.
- The capacity to exploit the **graduality** of the functions with continuous variables. The concept of graduality refers to the fact that slight changes in the variables correspond to slight changes in the function.
- The capacity for the **local tuning** of the solutions.
- The coding and decoding processes are avoided, thus **increasing the EA's speed**.

Evolutionary Algorithms

- Standard selecto-recombinative genetic algorithms with real representation.
- Evolution strategies.
- Differential Evolution.

Recombination Operators for Real-Parameter GAs

Standard recombination operators

- Simple Crossover (1-point)
- Average Crossover
- Arithmetic Crossover
- Flat Crossover
- Blend Crossover BLX-(α)

Specialized recombination operators

- Unimodal Normal Distribution Crossover (UNDX)
- Simplex Crossover (SPX)
- Parent-Centric (PCX)

Standard Recombination Operators for Real-Parameter GAs

Let us assume that $P_1 = (p_1^1 \dots p_n^1)$ and $P_2 = (p_1^2 \dots p_n^2)$ are two parental chromosomes.

- Simple Crossover – a position $i \in 1, 2, \dots, n-1$ is randomly chosen and the two offspring chromosomes O_1 and O_2 are built as follows

  $O_1 = (p_1^1, p_2^1, \dots, p_i^1, p_{i+1}^2, \dots, p_n^2)$

  $O_2 = (p_1^2, p_2^2, \dots, p_i^2, p_{i+1}^1, \dots, p_n^1)$

- Average Crossover – an offspring $O = (o_1, \dots, o_i, \dots, o_n)$ is created such that

  $o_i = (p_i^1 + p_i^2)/2$, for $i = 1 \dots n$

- Arithmetical Crossover – two offspring $O_k = (h_1^k, \dots, h_n^k)$, where $k = 1, 2$, are built as follows

  $o_i^1 = \lambda p_i^1 + (1 - \lambda) p_i^2$, for $i = 1 \dots n$

  $o_i^2 = \lambda p_i^2 + (1 - \lambda) p_i^1$

  where λ is a constant or varies with regard to the number of generations made.

Standard Recombination Operators for Real-Parameter GAs

Let us assume that $P_1 = (p_1^1 \dots p_n^1)$ and $P_2 = (p_1^2 \dots p_n^2)$ are two parental chromosomes.

- Simple Crossover – a position $i \in 1, 2, \dots, n-1$ is randomly chosen and the two offspring chromosomes O_1 and O_2 are built as follows

  $O_1 = (p_1^1, p_2^1, \dots, p_i^1, p_{i+1}^2, \dots, p_n^2)$

  $O_2 = (p_1^2, p_2^2, \dots, p_i^2, p_{i+1}^1, \dots, p_n^1)$

- Average Crossover – an offspring $O = (o_1, \dots, o_i, \dots, o_n)$ is created such that

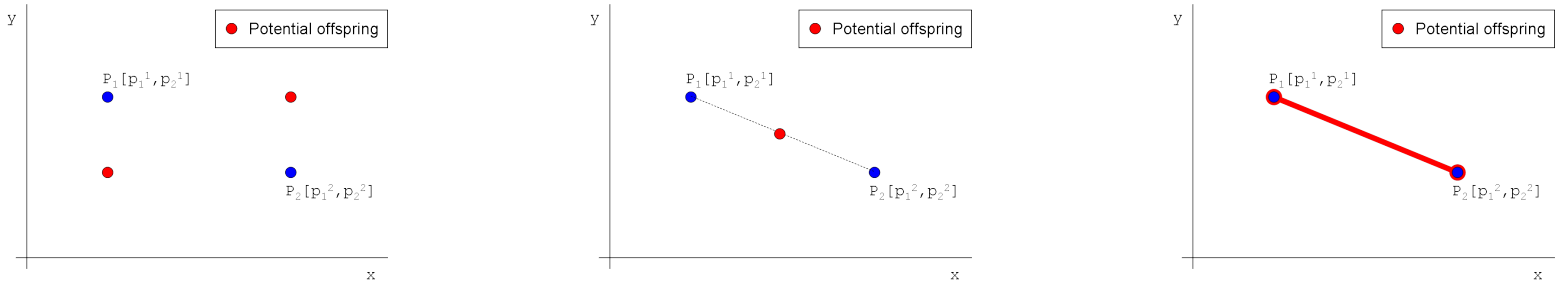
  $o_i = (p_i^1 + p_i^2)/2$, for $i = 1 \dots n$

- Arithmetical Crossover – two offspring $O_k = (h_1^k, \dots, h_n^k)$, where $k = 1, 2$, are built as follows

  $o_i^1 = \lambda p_i^1 + (1 - \lambda) p_i^2$, for $i = 1 \dots n$

  $o_i^2 = \lambda p_i^2 + (1 - \lambda) p_i^1$

  where λ is a constant or varies with regard to the number of generations made.



Standard Recombination Operators for Real-Parameter GAs

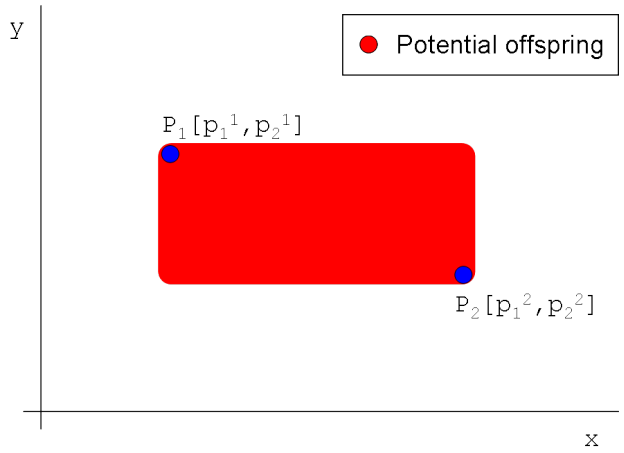
Let us assume that $P_1 = (p_1^1 \dots p_n^1)$ and $P_2 = (p_1^2 \dots p_n^2)$ are two parental chromosomes.

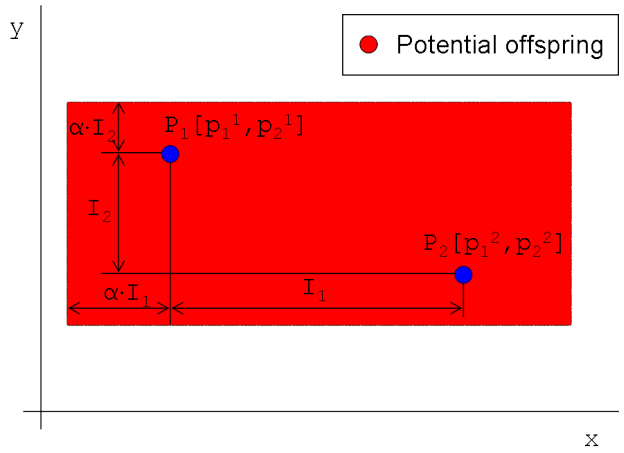
- Flat Crossover – an offspring $O = (o_1, \dots, o_i, \dots, o_n)$ is created such that
  o_i is randomly (uniformly) chosen value from interval $[min(p_i^1, p_i^2), max(p_i^1, p_i^2)]$.
- Blend Crossover – an offspring $O = (o_1, \dots, o_i, \dots, o_n)$ is created such that o_i is randomly (uniformly) chosen number from interval
  $c_{min} - I \cdot \alpha, c_{max} + I \cdot \alpha$, for $i = 1 \dots n$,
  where $c_{min} = min(p_i^1, p_i^2)$, $c_{max} = max(p_i^1, p_i^2)$, $I = c_{max} - cmin$

Standard Recombination Operators for Real-Parameter GAs

Let us assume that $P_1 = (p_1^1 \dots p_n^1)$ and $P_2 = (p_1^2 \dots p_n^2)$ are two parental chromosomes.

- Flat Crossover – an offspring $O = (o_1, \dots, o_i, \dots, o_n)$ is created such that
  o_i is randomly (uniformly) chosen value from interval $[min(p_i^1, p_i^2), max(p_i^1, p_i^2)]$.
- Blend Crossover – an offspring $O = (o_1, \dots, o_i, \dots, o_n)$ is created such that o_i is randomly (uniformly) chosen number from interval
  $c_{min} - I \cdot \alpha, c_{max} + I \cdot \alpha$, for $i = 1 \dots n$,
  where $c_{min} = min(p_i^1, p_i^2)$, $c_{max} = max(p_i^1, p_i^2)$, $I = c_{max} - cmin$





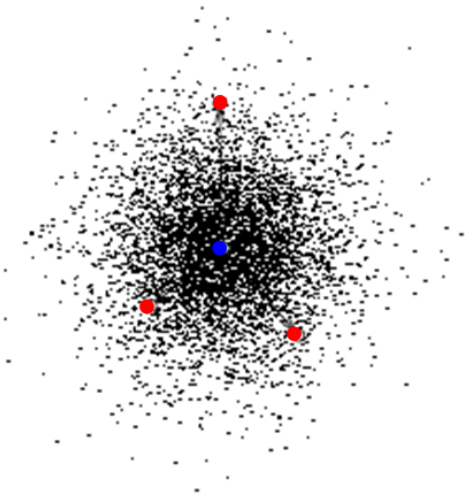
Unimodal Normal Distribution Crossover

Unimodal Normal Distribution Crossover (UNDX) creates offspring solutions around the center of mass of μ parents as follows:

$$\vec{o} = \vec{m} + \sum_{i=1}^{\mu-1} w_i |\vec{d}^{(i)}| \vec{e}^{(i)} + \sum_{j=\mu}^{n} v_j D \vec{e}^{(j)}$$

where

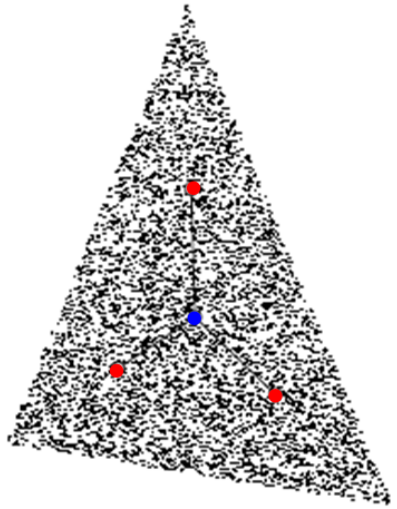
- μ is the number of parents involved in the process of generating the offspring
- $\vec{m}$ is the mean of the first $(\mu - 1)$ parents
- $\vec{d}^{(i)} = \vec{p}^{(i)} - \vec{m}$ are $(\mu - 1)$ direction vectors
- $\vec{e}^{(i)} = \frac{\vec{d}^{(i)}}{|\vec{d}^{(i)}|}$ are the orthogonal basis vectors spanning the sub-space orthogonal to the subspace spanned by all $\vec{e}^{(i)}$
- D is the length of the vector orthogonal to all $\vec{e}^{(i)}$
- w_i and v_j are zero-mean normally distributed variables



Simplex Crossover

Simplex Crossover (SPX)

- generates offspring around the mean of parents parents,
- offspring restricted within a region that is a simplex $\gamma = \sqrt{\mu + 1}$ times bigger than the parent simplex,
- assigns a uniform probability distribution for creating solutions in the restricted area,
- the computational complexity for creating one offspring is $O(\mu)$.



Parent-Centric Crossover

Parent-Centric Crossover (PCX)

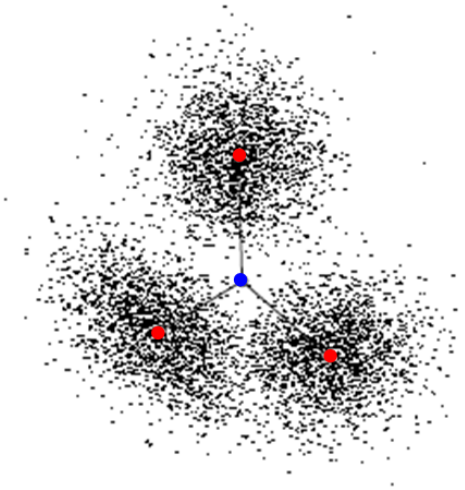
- offspring are centered around each parent; it assigns more probability for an offspring to remain closer to the parents than away from parents,
- the complexity for creating one offspring is $O(\mu)$.

PCX creates offspring as follows

$$\vec{o} = \vec{p} + w_1 |\vec{d}^{\,(p)}| + \sum_{i=1, i \neq p}^{\mu} w_2 \overline{D} \vec{e}^{\,(i)}$$

where

- $\vec{m}$ is the mean vector of μ parents
- $\vec{p}$ is a parent chosen as the basis for the offspring
- $\vec{d}^{\,(p)} = \vec{p} - \vec{m}$
- $\overline{D}$ is the average over all perpendicular distances calculated from each of the other $(\mu - 1)$ parents to the vector $\vec{d}^{\,(p)}$
- w_1 and w_2 are zero-mean normally distributed variables.

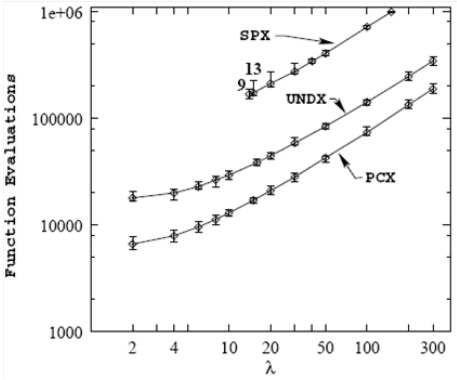


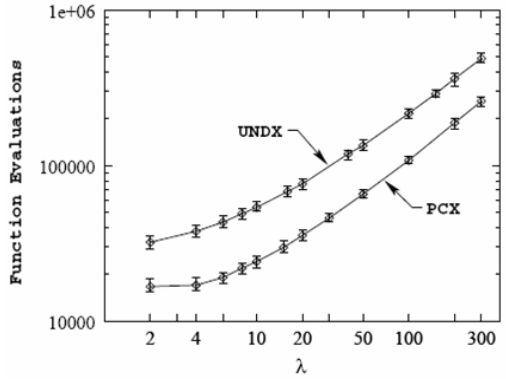
Generalized Generation Gap (G3) Evolutionary Algorithm Model

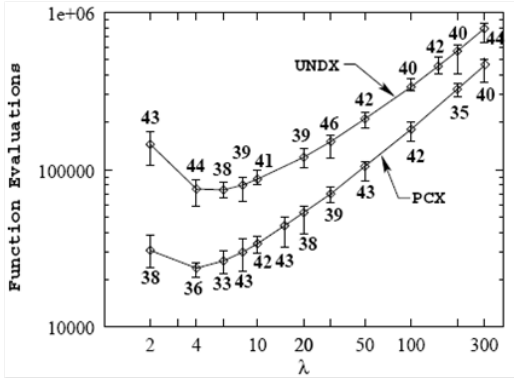
G3 – elite-preserving, steady-state, and computationally fast.

1. From the population $P(t)$, select the best parent and $(\mu - 1)$ other parents randomly.
2. Generate λ offspring from μ parents using a recombination scheme.
3. Choose two parents at random from μ parents.
4. Form a combined subpopulation of chosen two parents and λ offspring, choose the best two solutions and replace the chosen two parents with these solutions.

Comparisons of UNDX, SPX and PCX with the G3 model on Ellipsoidal, Schwefel's, and Generalized Rosenbrock's functions and $n = 20$.







Evolution Strategies

Evolution strategy (ES)

- ES is an optimization technique based on ideas of **adaptation and evolution**.
- ES uses natural problem-dependent representations, and primarily mutation and selection as search operators.
- For real-valued search spaces, mutation is performed by adding a normally distributed random value to each vector component.
- **The step size is often governed by self-adaptation**. Individual step sizes for each coordinate or correlations between coordinates are either governed by self-adaptation or by covariance matrix adaptation (CMA-ES).

Two Membered ES

The two membered ES – (1+1)-ES – is a simple mutation-selection scheme ES.

- Population consists of one parent individual and one descendant, created by means of adding normally distributed random numbers.
- The better of both individuals then serves as the ancestor of the following iteration.

Formal definition of (1+1)-ES:

$$(1+1) - ES = (P^0, m, s, c_d, c_i, f, g, t)$$

where

P^0	$=$	$(x^0, \sigma^0) \in I$	population x^0 is a **vector of solution parameters** σ^0 is **vector of standard deviations** $I = R^n \times R^n$
m	:	$I \to I$	mutation operator
s	:	$I \times I \to I$	selection operator
$c_d,\ c_i$	$\in$	R	**step-size control**
f	:	$R^n \to R$	objective function
g_j	:	$R^n \to R$	constraint functions
t	:	$I \times I \to \{0, 1\}$	termination criterion

Two Membered ES cnd.

P^0 denotes the initial population consisting of a single parent.

Mutation operator m is applied to all components of the object parameter x^t. It is realized by adding normally distributed random numbers

$$x'^t = x^t + \mathbf{N}_0(\sigma^t)$$

where $\mathbf{N_0}$ denotes a vector of independent Gaussian random numbers with zero mean and standard deviations $\sigma_i^t = (i = 1, \ldots, n)$.

Selection operator just determines the fitter individual of the parent p^t and offspring o^t to become the parent for the next generation.

- $o^t = m(p^t) = (x'^t, \sigma^t)$ survives if it is not worse than the parent p^t,
- $p^t = (x^t, \sigma^t)$ survives otherwise.

Termination criterion can be

- elapsed CPU time,
- elapsed number of generations,
- absolute or relative progress per generation, etc.

1/5 Success Rule

Progress rate (φ) – a quotient of the distance covered towards the optimum and the number of trials needed for this distance.

- Calculated for two model functions

  Corridor model: $f_1(x) = F(x_1) = c_0 + c_1 x_1$, $\forall i \in \{2, \ldots, n\} : -b/2 \leq x_i \leq b/2$

  Sphere model: $f_2(x) = \sum_{i=1}^{n} x_i^2$
- In both cases the step size σ_i^{opt} is inversely proportional to the number of object variables n.
- The maximum progress rate is also inversely proportional to n.

For the optimum step sizes Rechenberg obtained the following probabilities for a successful mutation,

- $p_1^{opt} = 1/(2e) \approx 0.184$
- $p_2^{opt} \approx 0.270$

1/5 success rule – *In order to obtain nearly optimal (local) performance of the (1+1)-ES in real-valued search spaces, tune the mutation strength in such a way that the (measured) success rate is about 1/5.*

- If it is greater than 1/5, increase the variance; if it is less, decrease the mutation variance.

Extended Mutation Operator

Mutation operator m modifies all components of the object parameter x^t according to

$$x'^t = x^t + \mathbf{N}_0(\sigma^t)$$

while the σ^t variances are dynamically adjusted (all at the same time) according to

$$\sigma^{t+n} = \begin{cases} c_d \cdot \sigma^t & \text{, if } p_s^t < 1/5 \\ c_i \cdot \sigma^t & \text{, if } p_s^t > 1/5 \\ \sigma^t & \text{, if } p_s^t = 1/5 \end{cases}$$

where p_s^t is the frequency of successful mutations, measured over $10n$ trials.

Recommended setup:

- increase step size: $c_i = 1/0.82$,
- decrease step size: $c_d = 0.82$,
- adjustment takes place every n mutations.

Multimembered EAs

Multimembered $(\mu+1)$-ES where $\mu > 1$ parents can participate in the generation of one offspring:

$$(\mu + 1) - ES = (P^0, \mu, r, m, s, c_d, c_i, f, g, t)$$

where

P^0	$=$	$(a_1^0, \ldots, a_\mu^0) \in I^\mu$	population $I = R^n \times R^n$
μ	$>$	1	number of parents
r	$:$	$I^\mu \to I$	**recombination operator**
m	$:$	$I \to I$	mutation operator
s	$:$	$I^{\mu+1} \to I^\mu$	selection operator
c_d, c_i	$\in$	R	step-size control
f	$:$	$R^n \to R$	objective function
g_j	$:$	$R^n \to R$	constraint functions
t	$:$	$I^\mu \to \{0, 1\}$	termination criterion

Multimembered EAs: Recombination

Recombination operator r:

$$r(P^t) = a^{'} = (x^{'}, \sigma^{'}) \in I, x^{'} \in R^n, \sigma^{'} \in R^n$$

$$\begin{aligned} x_i^{'} &= x_{a,i},\ \chi \leq 0.5 \quad \forall i \in 1, \ldots, n \\ &\quad\ x_{b,i},\ \chi > 0.5 \end{aligned}$$

$$\begin{aligned} \sigma_i^{'} &= \sigma_{a,i},\ \chi \leq 0.5 \quad \forall i \in 1, \ldots, n \\ &\quad\ \sigma_{b,i},\ \chi > 0.5 \end{aligned}$$

where $a = (x_a, \sigma_a)$ and $b = (x_b, \sigma_b)$ are two parents.

Remarks:

- Parents a and b are determined randomly (uniform).

  All individuals in the population have the same mating probabilities.
- A uniform random variable χ is sampled anew for each component of the vectors $x^{'}$ and $\sigma^{'}$.
- The selection operator s removes the least fit individual out of μ parents plus 1 offspring from the population.

$(\mu + \lambda)$- and (μ, λ)- Multimembered EAs

Motivation to extend $(\mu + 1)$-ES to $(\mu + \lambda)$- and (μ, λ)-ES:

- To enable **self-adaptation of strategic parameters** standard deviations.

  The σ^t parameters are considered a part of the individual's genome that is subject to recombination and mutation.

  Those individuals with better adjusted strategy parameters are expected to perform better.

- To make use of parallel computers, where several newly **generated individuals can be processed (evaluated) simultaneously**.

$(\mu + \lambda)$-EA

Model:

- μ parents produce λ offspring which are reduced again to the μ parents for the next generation.
- Selection s operates on joined set of parents and offspring.

  Some very well adapted individuals can survive in the population for ever.

Deficiencies:

- Can get stuck at some good location if the internal parameter setting becomes unsuitable to jump to the new field of possible improvements (dynamic opt. problems, problems with noisy objective function).

(μ, λ)-EA

Model:

- μ parents produce λ offspring, only the offspring undergo selection;
- Forgetting principle – the lifetime of every individual is limited to one generation if $\mu < \lambda$.

Remarks:

- Possible short phases of recession.
- It avoids long stagnation phases due to misadapted strategy parameters.

Handling the Internal Strategy Parameters

Internal strategy parameters σ^t are part of the genetic information of an individual

$$a_i^t = (x^t, \sigma^t) \in I$$

which are not controlled by the 1/5 *success rule* anymore.

Mutation operator works on both the x^t and σ^t:

$$\begin{array}{lcll} a_i^{'t} & = & r(P^t) & // \text{ recombination} \\ m(a_i^{'t}) & = & a_i^{''t} = (x^{''t}, \sigma^{''t}) & \\ \sigma^{''t} & = & \sigma^{'t} exp(\mathbf{N}_0(\triangle\ \sigma)) & // \text{ strategy mutation} \\ x^{''t} & = & x^{'t} + \mathbf{N}_0(\sigma^{''t}) & // \text{ mutation of solution params } x \end{array}$$

Remarks:

- Unsuitable $\sigma^{''t}$ are removed by means of selection.
- EAs which are operated with an optimum ratio of μ/λ for a maximum rate of convergence, are biased towards local search.

  They tend to reduce the number of different parameters σ^t in a population.

ES with Correlated Mutations

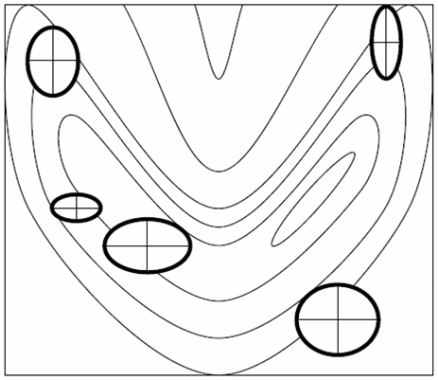
Motivation:

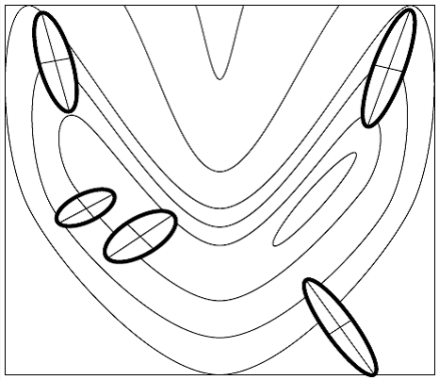
- With dedicated σ_i for each object variable x_i preferred directions of search can be established only along the axes of the coordinate system.
- The optimum rate of progress is achieved by chance when suitable mutations coincide.

  The trajectory of the population through the search space does not straightforwardly follow the gradient.

Correlated mutations – additional strategy vector θ is added to the genotype $a_i^t = (x^t, \sigma^t, \theta^t)$, where a set of inclination angles $\theta \in R^w$ define n-dim rotating hyperellipsoids, $w = n(n-1)/2$.

Ellipsoid contours of equal prob. density to place an offspring for simple and correlated mutations.





©B ack T. et al.: A Survey of Evolution Strategies.

Differential Evolution

Differential Evolution (DE) – an evolutionary algorithm named after a special kind of differential operator, which is used to create new offspring.

DE evolves a population of N D-dimensional variable vectors. The ith vector of the population at time t is represented as

$$\overrightarrow{X}_i(t) = [x_{i,1}(t), x_{i,2}(t), \ldots, x_{i,D}(t)]$$

Values $x_{i,j}$ can be restricted within certain range between x_j^L and x_j^U.

Differential Evolution: Differential Operator

Differential Operator – generates an offspring for each population member $\overrightarrow{X}_i(t)$ as follows

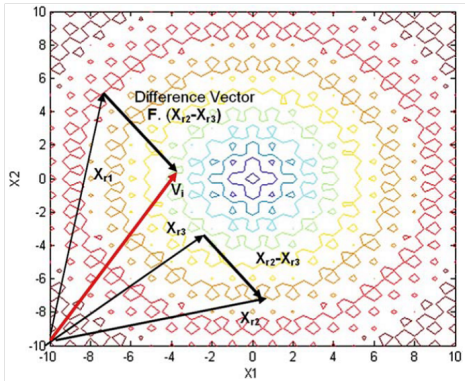
1. **Donor vector** $\overrightarrow{V}_i(t)$ is created.

   DE/rand/1 scheme:

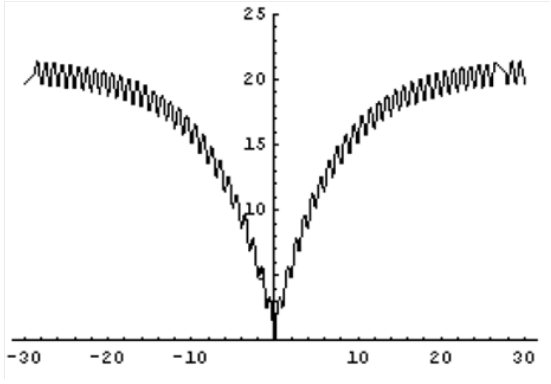
   - Three population members (indexed r_1, r_2 and r_3) are chosen at random.
   - A difference between any two of the three vectors scaled by value F is added to the third one.

     $v_{i,j}(t) = x_{r1,j}(t) + F \cdot (x_{r2,j}(t) - x_{r3,j}(t))$, for $j = 1 \ldots D$.

   Illustration of Creation of the Donor Vector in 2D



©Das S. et al.: Particle Swarm Optimization and Differential Evolution Algorithms.



Projection of the Ackley's function

Differential Evolution: Crossover

2. **Crossover** – to increase a diversity of the population.

   The donor vector exchanges its components with the target vector $\overrightarrow{X}_i(t)$ under one of the following schemes:

   - **Exponential scheme**

     L – the number of components the donor vector will contribute to the target.

     n – a starting point in the target vector $\overrightarrow{X}_i(t)$, from where the crossover with the donor vector $\overrightarrow{V}_i(t)$ starts.

     CR – crossover parameter determining $Prob(L \geq m) = (CR)^{m-1}$, for any $m > 0$.

     Trial vector $\overrightarrow{U}_i(t)$ (offspring) is formed as follows

     $$\begin{aligned} u_{i,j}(t) &= v_{i,j}(t) \quad \text{for } j = < n >_D, < n+1 >_D, \ldots, < n+L-1 >_D, \\ &= x_{i,j}(t) \quad \text{for remaining components} \end{aligned}$$

     where $<>_D$ denote a modulo function with modulus D.

   - **Binomial scheme**. The crossover is performed on each of the D variables whenever a randomly picked number between 0 and 1 is within CR value

     $$\begin{aligned} u_{i,j}(t) &= v_{i,j}(t) \quad \text{if rand(0,1)} < CR \\ &= x_{i,j}(t) \quad \text{otherwise} \end{aligned}$$

Differential Evolution: Selection

3. **Selection** – determines which one of the target vector and the trial vector will survive in the next generation.

$$\begin{aligned} \overrightarrow{X}_i(t+1) &= \overrightarrow{U}_i(t) \quad \text{if } f(\overrightarrow{U}_i(t)) \leq f(\overrightarrow{X}_i(t)) \\ &= \overrightarrow{X}_i(t) \quad \text{otherwise} \end{aligned}$$

The population either gets better (w.r.t. fitness function) or remains constant.

Differential Evolution: Pseudocode

```
Begin
    Initialize population;
    Evaluate fitness;
    For i = 0 to max-iteration do
    Begin
    Create Difference-Offspring;
    Evaluate fitness;
    If an offspring is better than its parent
    Then replace the parent by offspring in the next generation;
    End If;
  End For;
End.
```

Various DE schemes – DE/x/y

1. x – type of the vector to be perturbed by the differential operator (rand/best).
2. y – number of difference vectors considered for perturbation of x.

Reading

- Deb K. et al.: Real-Coded Evolutionary Algorithms with Parent-Centric Recombination, 2001

  http://www.iitk.ac.in/kangal/papers/tech_rep2001003.pdf

- Herrera F. et al.: Tackling Real-Coded Genetic Algorithms: Operators and Tools for Behavioural Analysis, 2004

  http://www.springerlink.com/content/nh44700284m80307/

- Ono I. et al.: A Robust Real-Coded Genetic Algorithm using Unimodal Normal Distribution Crossover Augmented by Uniform Crossover: Effects of Self-Adaptation of Crossover Probabilities, 1999

  http://www.cs.bham.ac.uk/w̃bl/biblio/gecco1999/Ono_gecco99.pdf

- Bäck T. et al.: A Survey of Evolution Strategies, 1991

  http://www6.uniovi.es/pub/EC/GA/papers/icga91.ps.gz

- Das S. et al.: Particle Swarm Optimization and Differential Evolution Algorithms: Technical Analysis, Applications and Hybridization Perspectives, 2008

  http://www.softcomputing.net/aciis.pdf