Solved Problems
(related to Lecture 1)

Author: Ondřej Drbohlav, comments/suggestions to drbohlav@fel.cvut.cz.
Version 2/Oct/2017.

Problem 1. Consider the same example as in the lecture slides, with joint probability p_{XK} as follows:

	cloudiness			
	1	2	3	4
rain	0.02	0.12	0.09	0.04
no rain	0.38	0.28	0.06	0.01

Compute marginal probabilities $p_K(k)$ for $k = \{\text{rain, no rain}\}$ and $p_X(x)$ for $x = \{1, 2, 3, 4\}$.

Solution. $p_K(\text{rain}) = 0.27$, $p_K(\text{no rain}) = 0.73$. $p_X(1) = 0.4$, $p_X(2) = 0.4$, $p_X(3) = 0.15$, $p_X(4) = 0.05$.

Problem 2. Consider the setup as in the previous problem. You have three possible decisions $D = \{\text{umbrella}, \text{no umbrella}, 100\}$ to make on a given day:

- umbrella : you take an umbrella with you,
- no umbrella: you do not take an umbrella with you and if it rains, you will get wet,
- 100: you do not take an umbrella with you but you make a fixed decision that if it rains, you will buy a new umbrella for 100 CZK.

Let the loss matrix $W(k, d)$ be as follows:

	umbrella	no umbrella	100
rain	0	10	5
no rain	5	-2	0

Compute the optimal strategy $q^*(x)$.

Solution. Evaluate the partial risk

$$R(x, d) = \sum_{k \in K} p_{Kx}(k \,|\, x)\, W(k, d) \tag{1}$$

	umbrella	no umbrella	100
1	$\frac{0.02}{0.4} \cdot 0 + \frac{0.38}{0.4} \cdot 5 = 4.75$	$\frac{0.02}{0.4} \cdot 10 - \frac{0.38}{0.4} \cdot 2 = \mathbf{-1.4}$	$\frac{0.02}{0.4} \cdot 5 + \frac{0.38}{0.4} \cdot 0 = 0.25$
2	$\frac{0.12}{0.4} \cdot 0 + \frac{0.28}{0.4} \cdot 5 = 3.5$	$\frac{0.12}{0.4} \cdot 10 - \frac{0.28}{0.4} \cdot 2 = 1.6$	$\frac{0.12}{0.4} \cdot 5 + \frac{0.28}{0.4} \cdot 0 = \mathbf{1.5}$
3	$\frac{0.09}{0.15} \cdot 0 + \frac{0.06}{0.15} \cdot 5 = \mathbf{2}$	$\frac{0.09}{0.15} \cdot 10 - \frac{0.06}{0.15} \cdot 2 = 5.2$	$\frac{0.09}{0.15} \cdot 5 + \frac{0.06}{0.15} \cdot 0 = 3$
4	$\frac{0.04}{0.05} \cdot 0 + \frac{0.01}{0.05} \cdot 5 = \mathbf{1}$	$\frac{0.04}{0.05} \cdot 10 - \frac{0.01}{0.05} \cdot 2 = 7.6$	$\frac{0.04}{0.05} \cdot 5 + \frac{0.01}{0.05} \cdot 0 = 4$

and for each observation x, select the decision d which produces the lowest partial risk (indicated by bold numbers in each column). Thus,

$$q^*(1) = \text{no umbrella},\ q^*(2) = 100,\ q^*(3) = \text{umbrella},\ q^*(4) = \text{umbrella}\,. \quad (2)$$