

ALG 08

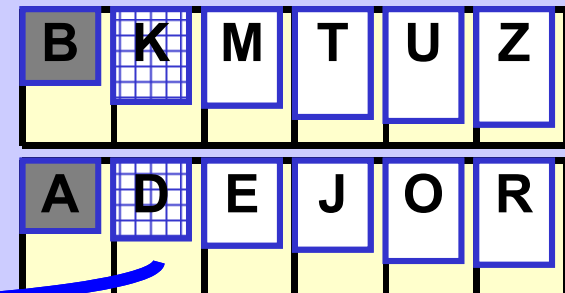
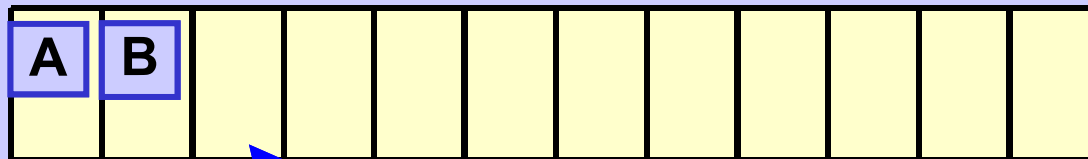
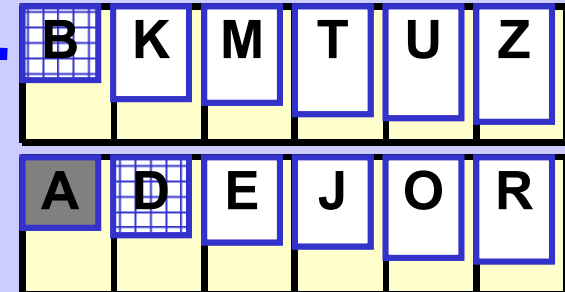
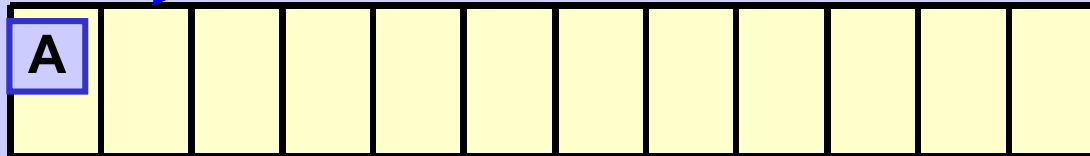
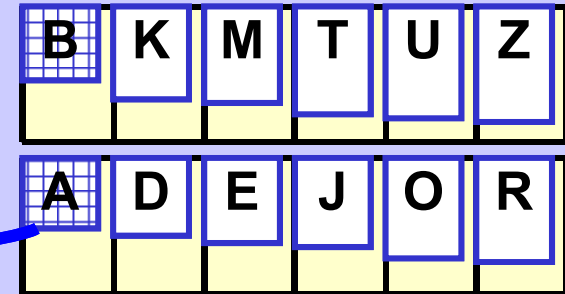
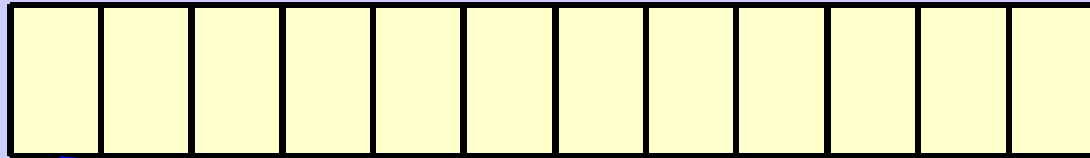
Merge sort -- řazení sléváním

Heap Sort -- řazení binární haldou

Prioritní fronta implementovaná binární haldou

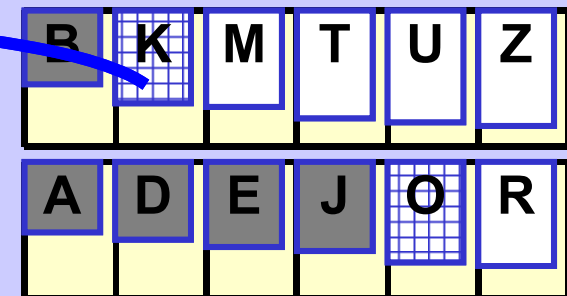
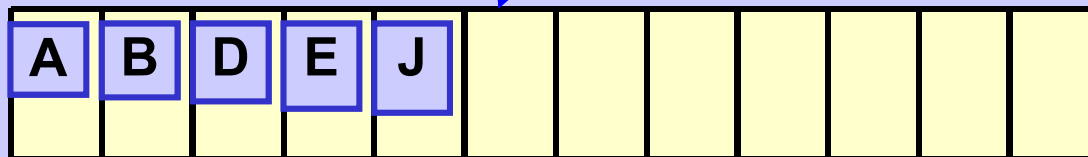
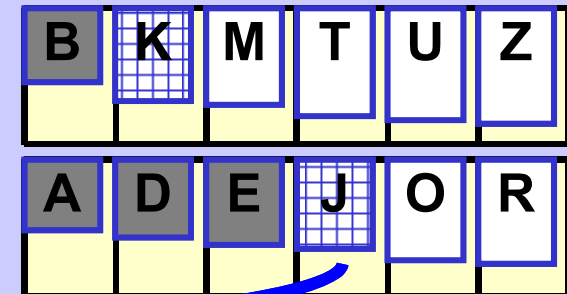
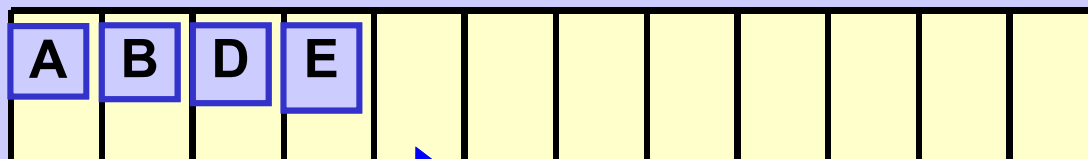
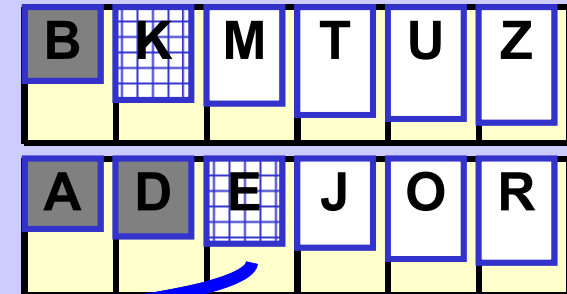
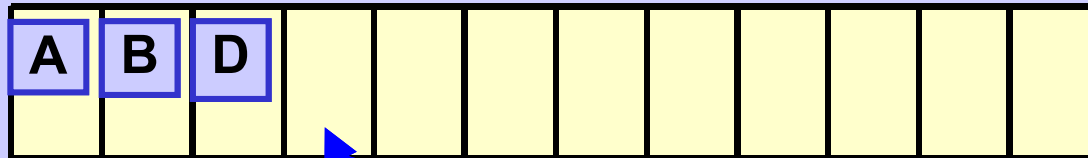
Merge sort

Sluč (slij?) dvě seřazená pole

Porovnávání prvky 

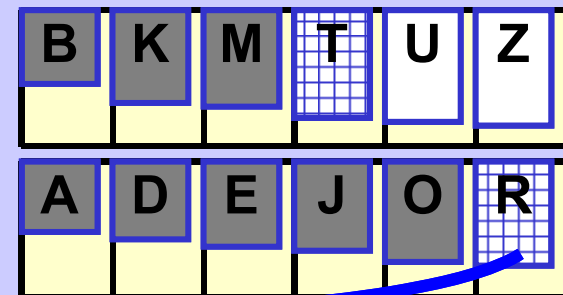
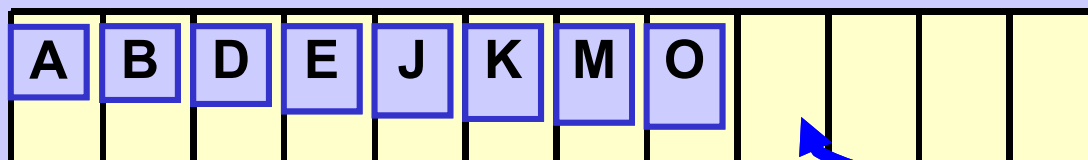
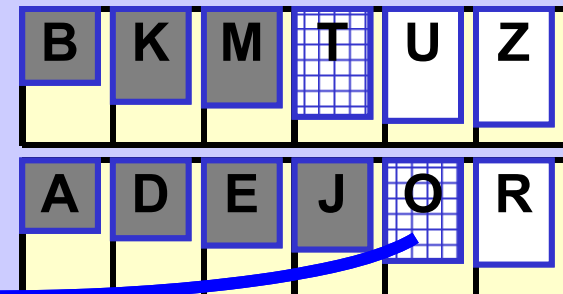
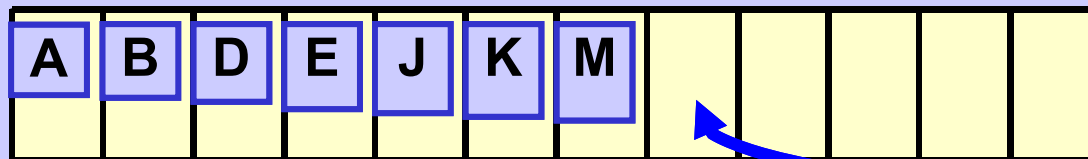
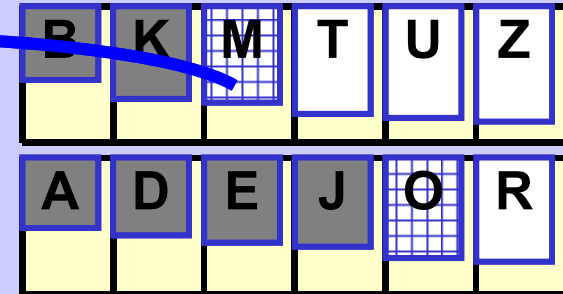
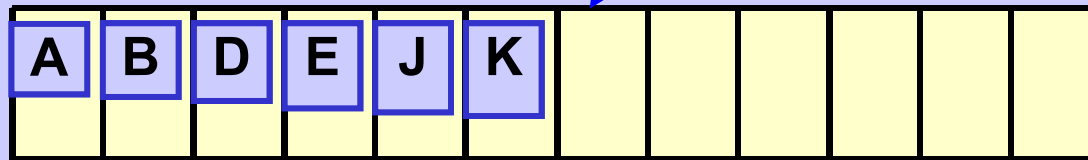
Merge sort

Sluč dvě seřazená pole - pokr.



Merge sort

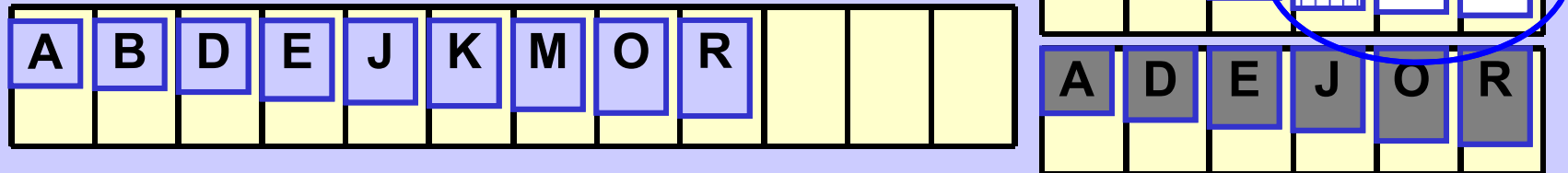
Sluč dvě seřazená pole - pokr.



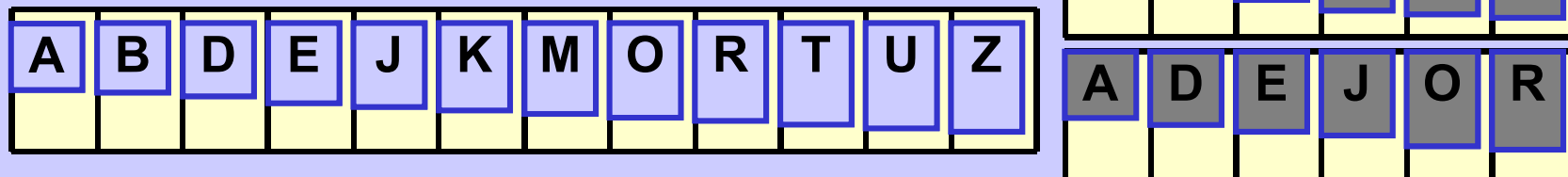
Merge sort

Sluč dvě seřazená pole - pokr.

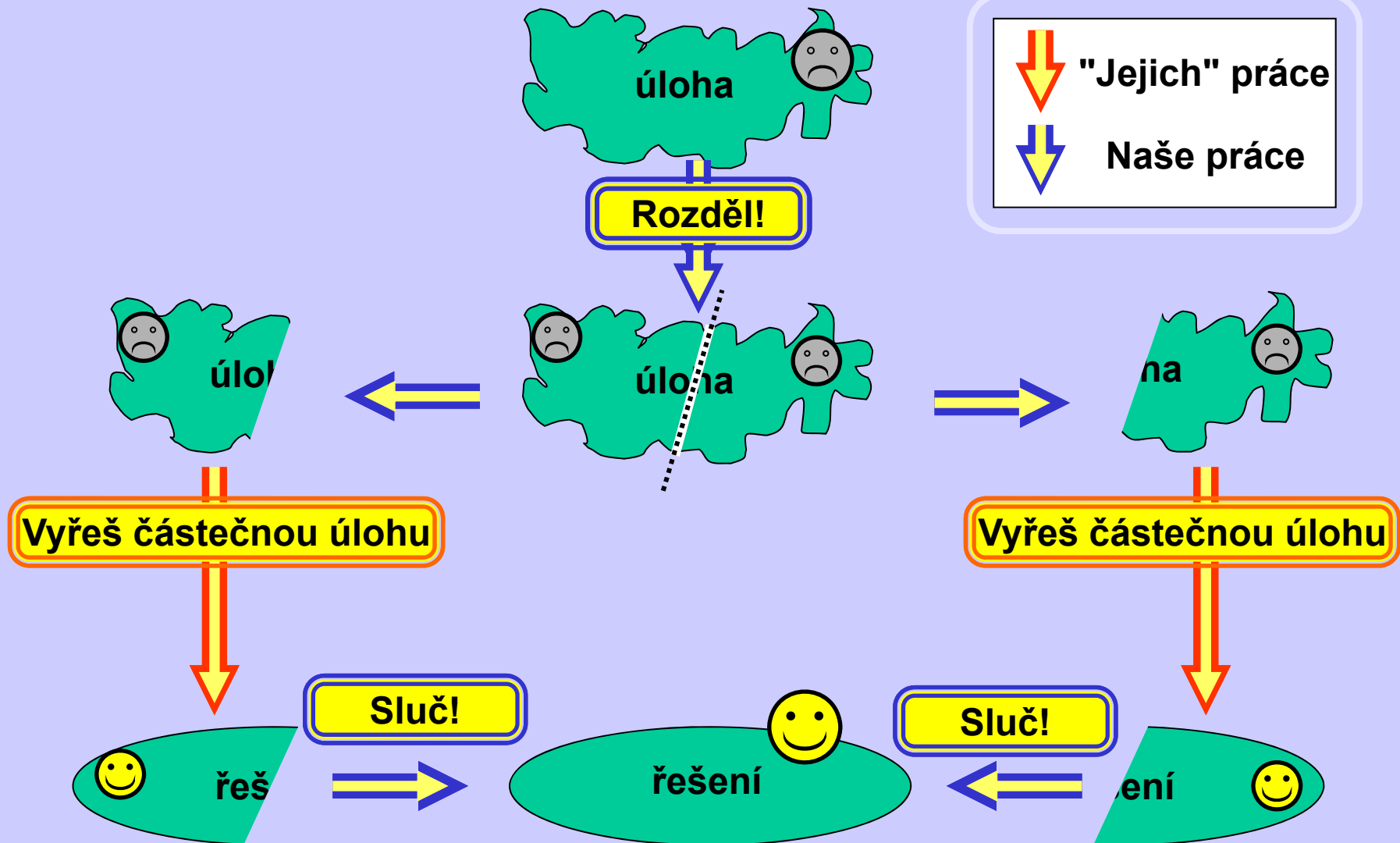
Kopíruj zbytek



Seřazeno



Rozděl a panuj! Divide and conquer! Divide et impera!



Merge sort

Neseřazeno

K	Z	U	B	M	T	E	A	J	R	D	O

Rozděľ!

Zpracuj
odděľně

K	Z	U	B	M	T

seřad'!

B	K	M	T	U	Z

E	A	J	R	D	O

seřad'!

A	D	E	J	O	R

Panuj!

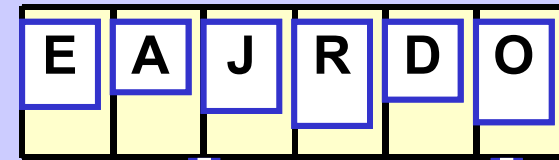
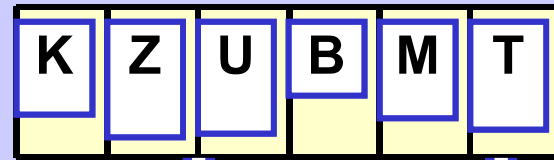
Sluč!

Seřazeno

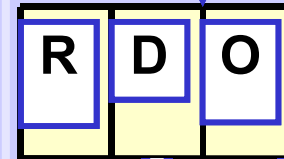
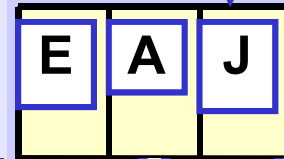
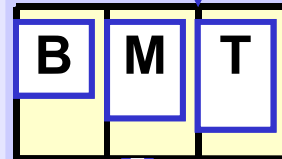
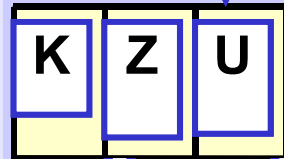
A	B	D	E	J	K	M	O	R	T	U	Z

Merge sort

Neseřazeno



Rozdě!



Rozdě!

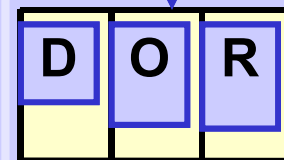
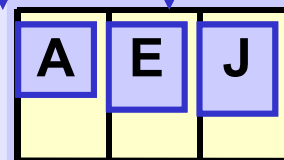
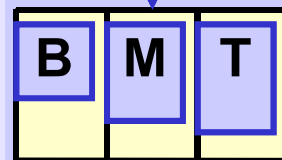
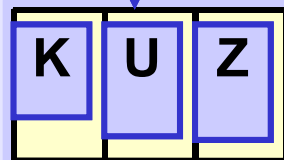
Rozdě!



... ..

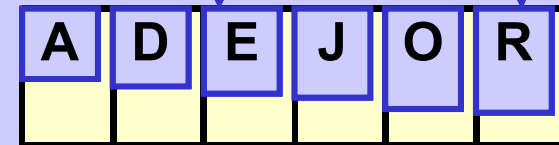
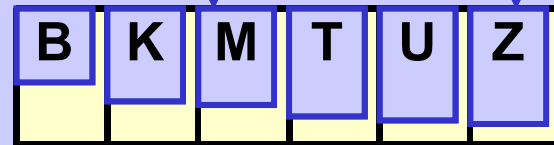
Sluč!

Sluč!



Sluč!

Seřazeno



Merge sort

```
void mergeSort (int [] a, int [] aux,
                int low, int high) {
    int half = (low+high)/2;
    if (low >= high) return;           // too small!

                                     // sort
    mergeSort(a, aux, low, half);     // left half
    mergeSort(a, aux, half+1, high); // right half
    merge(a, aux, low, high);        // merge halves

                                     // put result back to a -- clumsy method!
    for (int i = low; i <= high; i++) a[i] = aux[i];

    // optimization idea:
    /* swapArray(a, aux) */ // better to swap
                           // references to a & aux!
}
```

Merge sort

```
void merge(int []in, int []out, int low, int high) {  
    int half = (low+high)/2;  
    int i1 = low;  
    int i2 = half+1;  
    int j = low;  
  
                                // compare and merge  
    while ((i1 <= half) && (i2 <= high)) {  
        if (in[i1] <= in[i2]) { out[j] = in[i1]; i1++; }  
        else { out[j] = in[i2]; i2++; }  
        j++;  
    }  
  
                                // copy the rest  
    while (i1 <= half) { out[j] = in[i1]; i1++; j++; }  
    while (i2 <= high) { out[j] = in[i2]; i2++; j++; }  
}
```

Merge sort

Asymptotická sožitost

Rozdě! $\log_2(n)$ krát \Rightarrow

\Rightarrow Sluč! $\log_2(n)$ krát

Rozdě! $\Theta(1)$ operací

Sluč! $\Theta(n)$ operací

Celkem $\Theta(n) \cdot \Theta(\log_2(n)) = \Theta(n \cdot \log_2(n))$ operací

Asymptotická složitost Merge sortu je $\Theta(n \cdot \log_2(n))$

Merge sort

Stabilita

Rozděl! Nepohybuje prvky

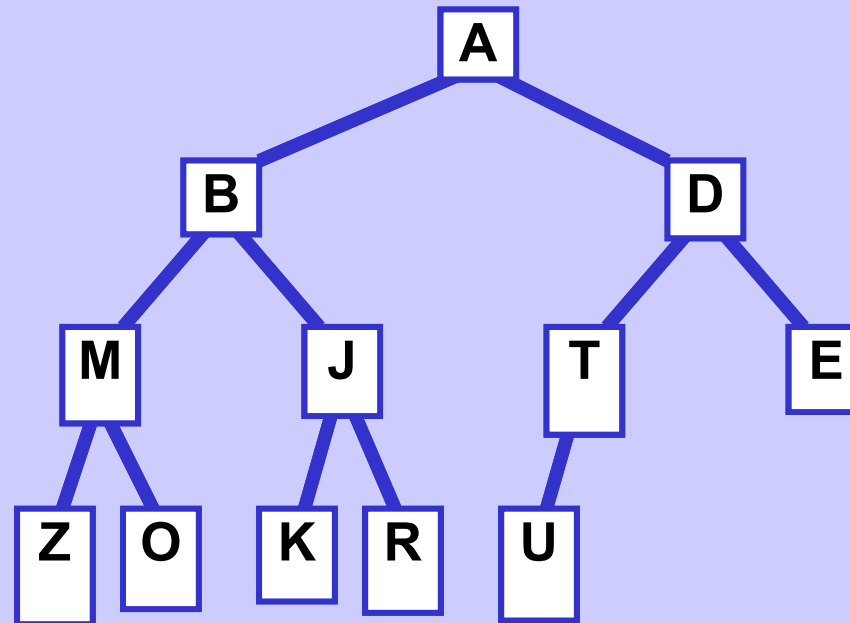
Sluč! “ if (in[i1] <= in[i2]) { out[j] = in[i1]; ...”

**Zařad' nejprve levý prvek,
když slučuješ stejné hodnoty**

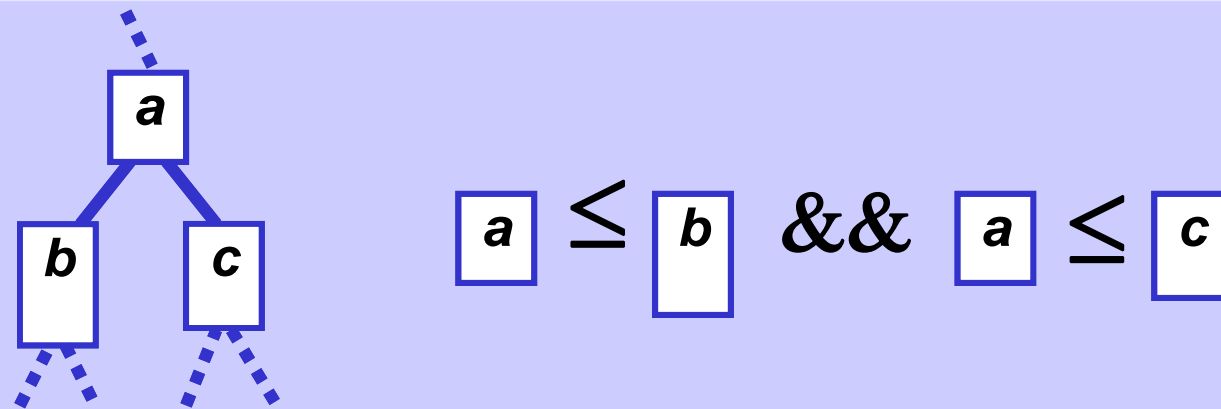
MergeSort je stabilní

Heap sort

Halda

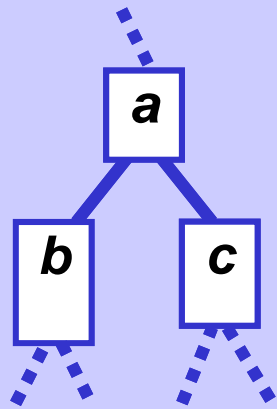


Pravidlo
haldy



Heap sort

Terminologie



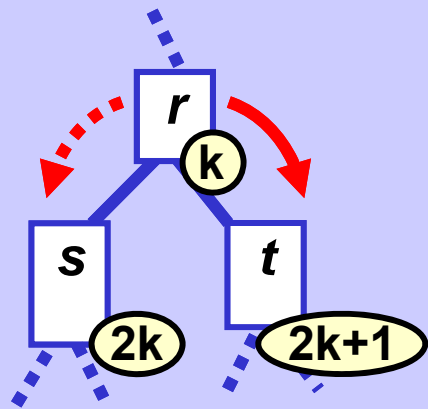
a predecessor, parent of **b** **c**
 předchůdce, rodič

b, **c** successor, child of **a**
 následník, potomek

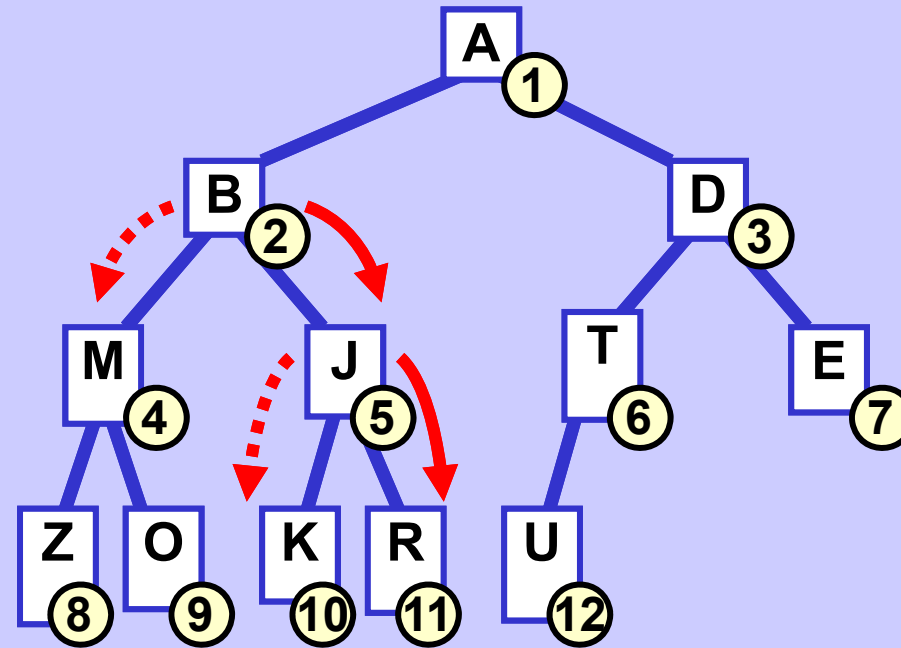
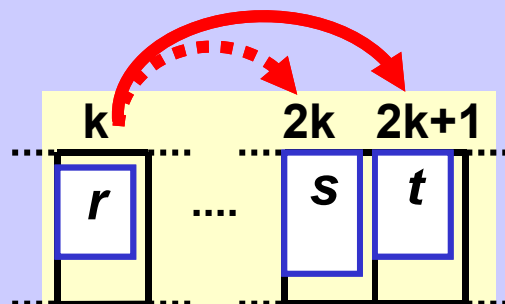


Heap sort

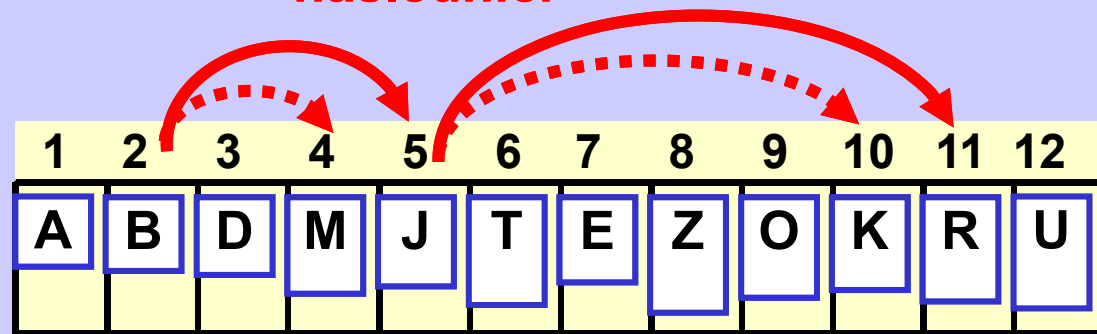
Halda uložená v poli



následníci



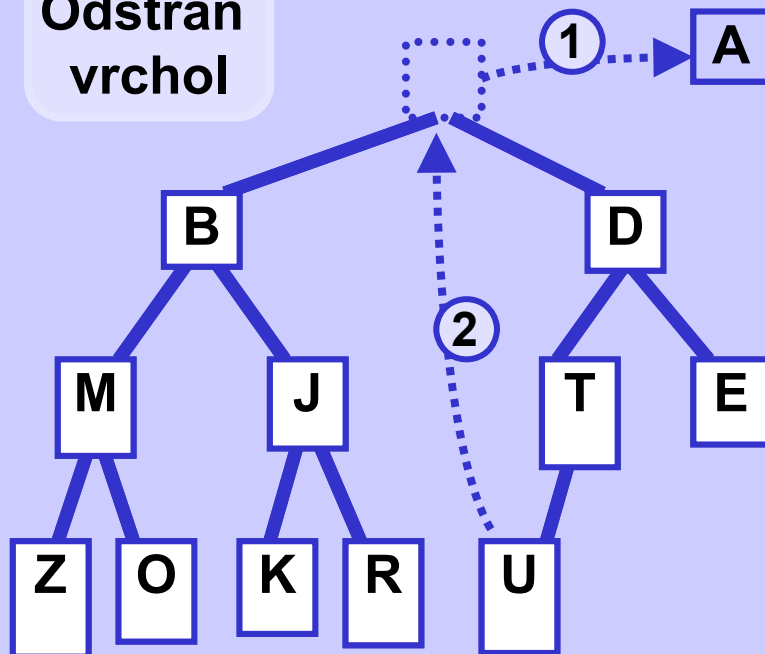
následníci



Oprava haldy

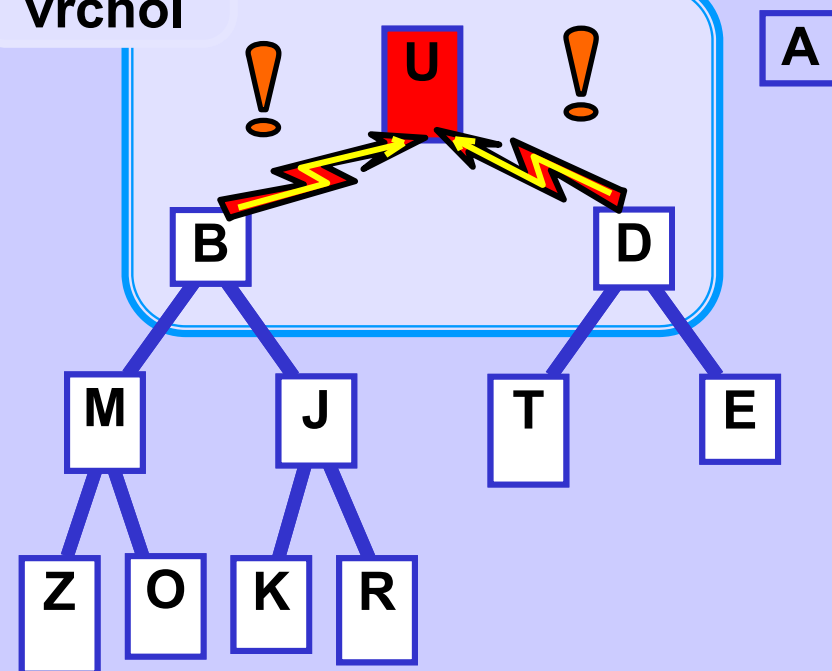
Vrchol odstraněn (1)

①
Odstraň
vrchol



②
poslední \longrightarrow první

③
Vlož na
vrchol

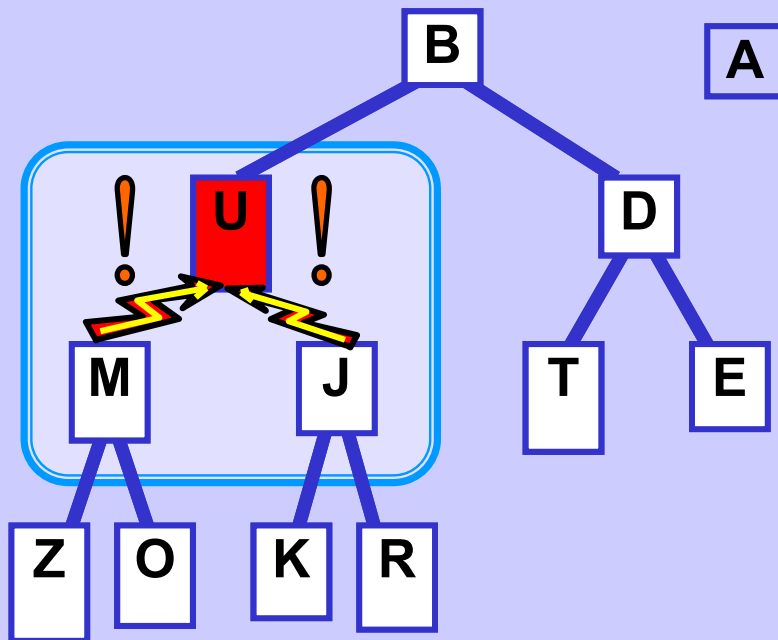


$U > B, U > D, \underline{B < D}$
 \Rightarrow prohod' $B \leftrightarrow U$

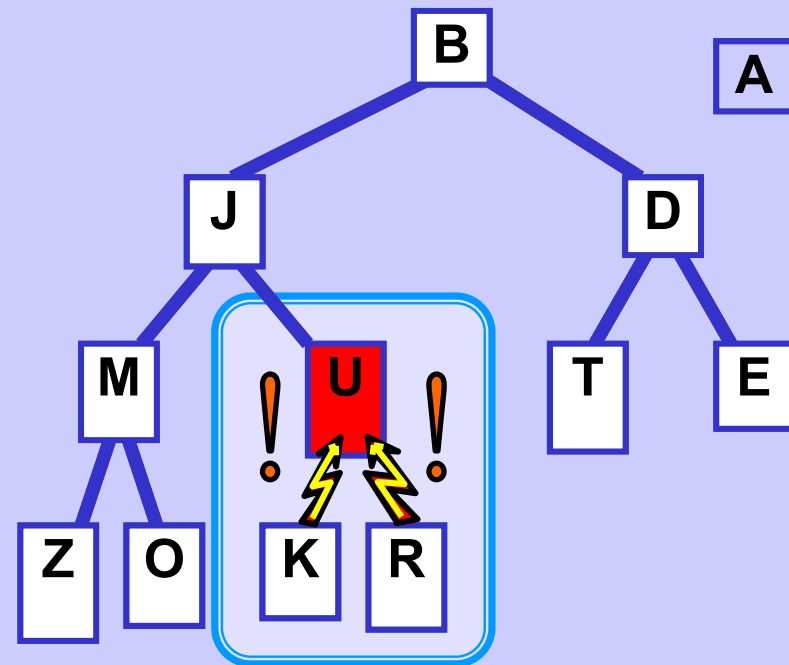
Oprava haldy

Vrchol odstraněn (2)

③ Vlož na vrchol - pokračování



$U > M, U > J, \underline{J < M}$
 \Rightarrow prohod' $J \leftrightarrow U$

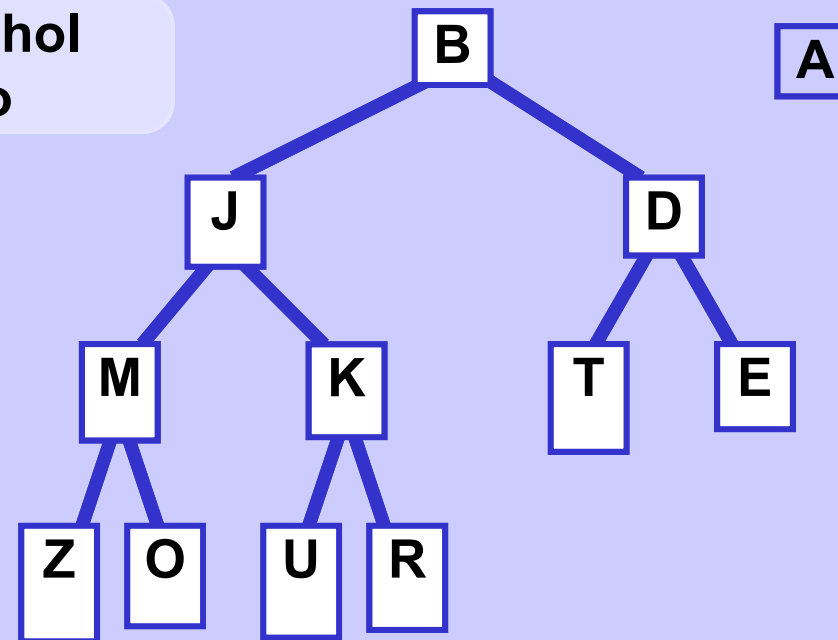


$U > K, U > R, \underline{K < R}$
 \Rightarrow prohod' $K \leftrightarrow U$

Oprava haldy

Vrchol odstraněn (3)

- ③ Vlož na vrchol
- hotovo

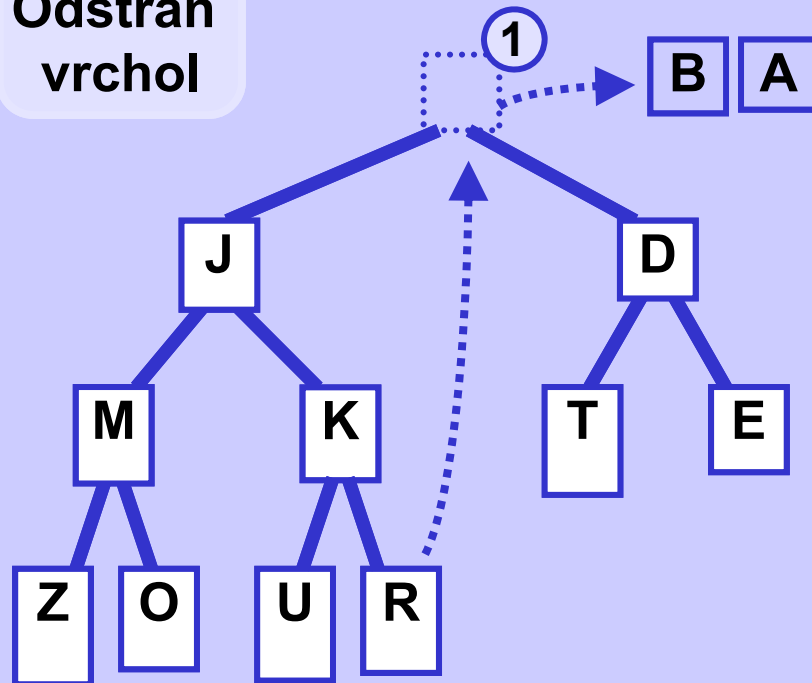


Nová halda

Oprava haldy

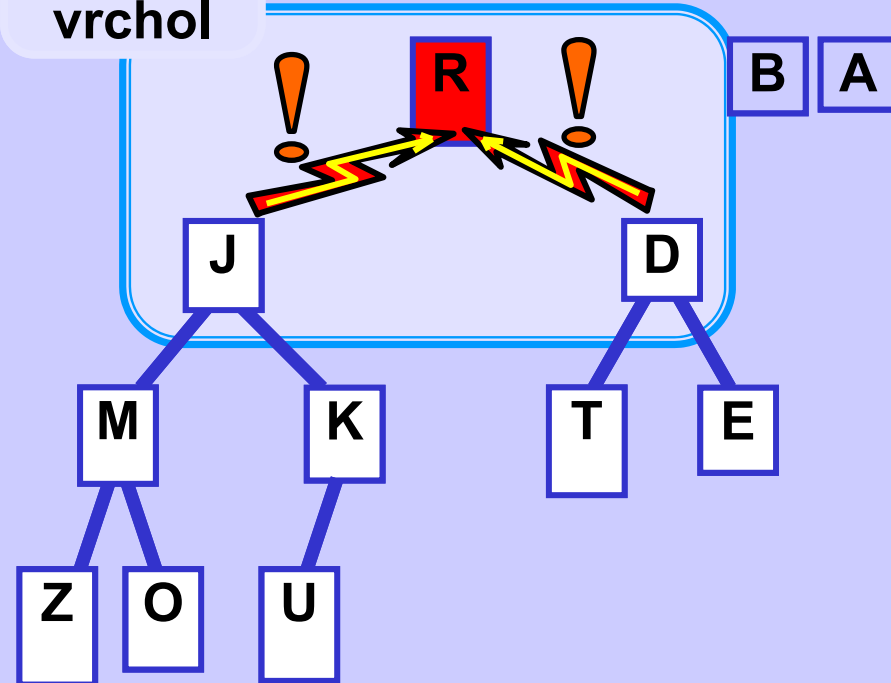
Vrchol odstraněn II (1)

①
Odstraň
vrchol



②
poslední → první

③
Vlož na
vrchol

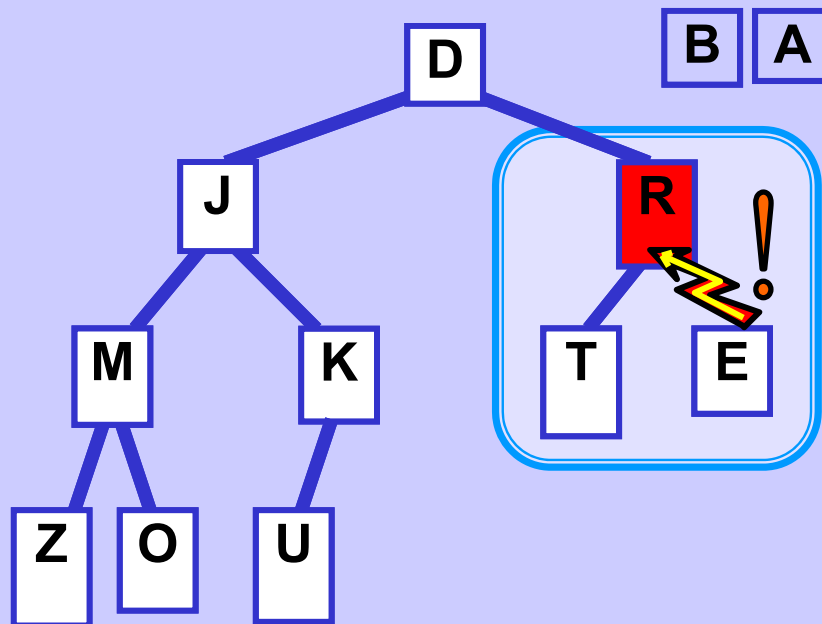


$R > J, R > D, \underline{D < J}$
 \Rightarrow prohod' $D \leftrightarrow R$

Oprava haldy

Vrchol odstraněn II (2)

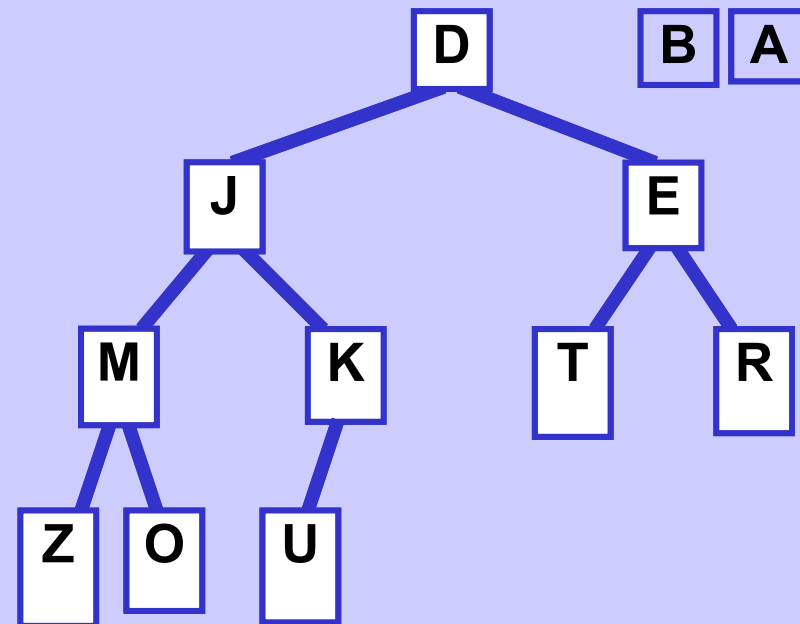
③ Vlož na vrchol - pokračování



$R < T, R > E$
 \Rightarrow prohod' $E \leftrightarrow R$

Vrchol odstraněn II (3)

③ Vlož na vrchol - hotovo

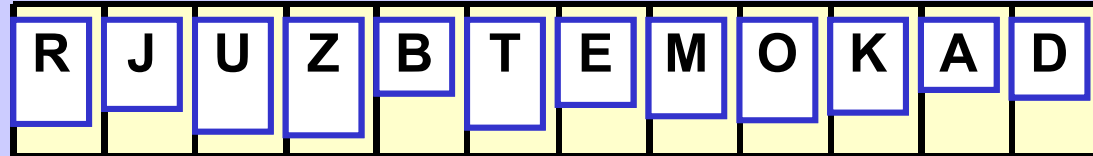


Nová halda

Heap sort

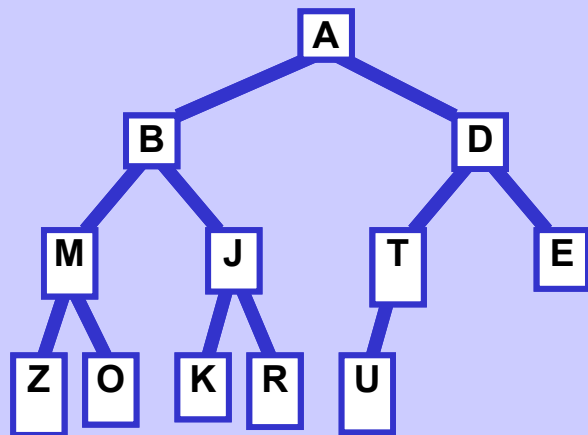
I

Neseřazeno



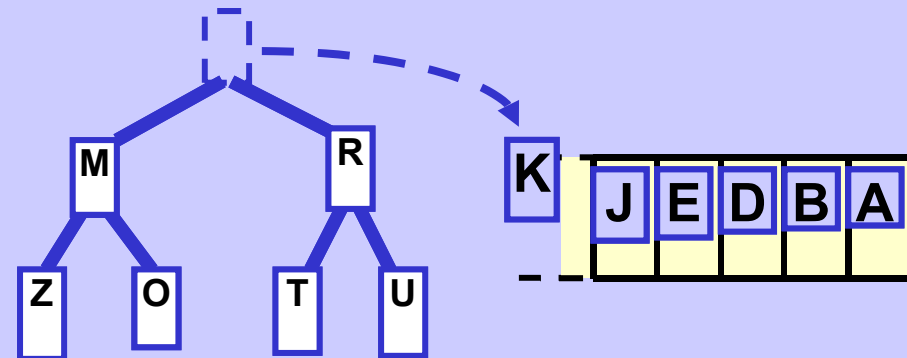
II

Vytvoř haldu



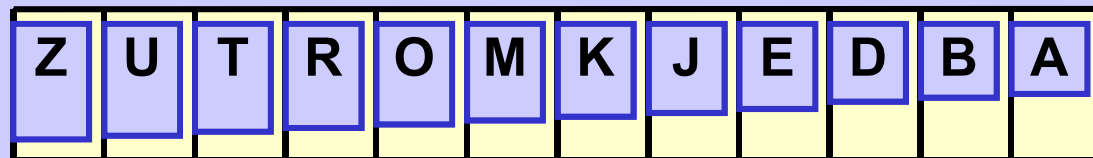
III

for ($i = 0$; $i < n$; $i++$)
 $a[i] =$ "odstraň vrchol";

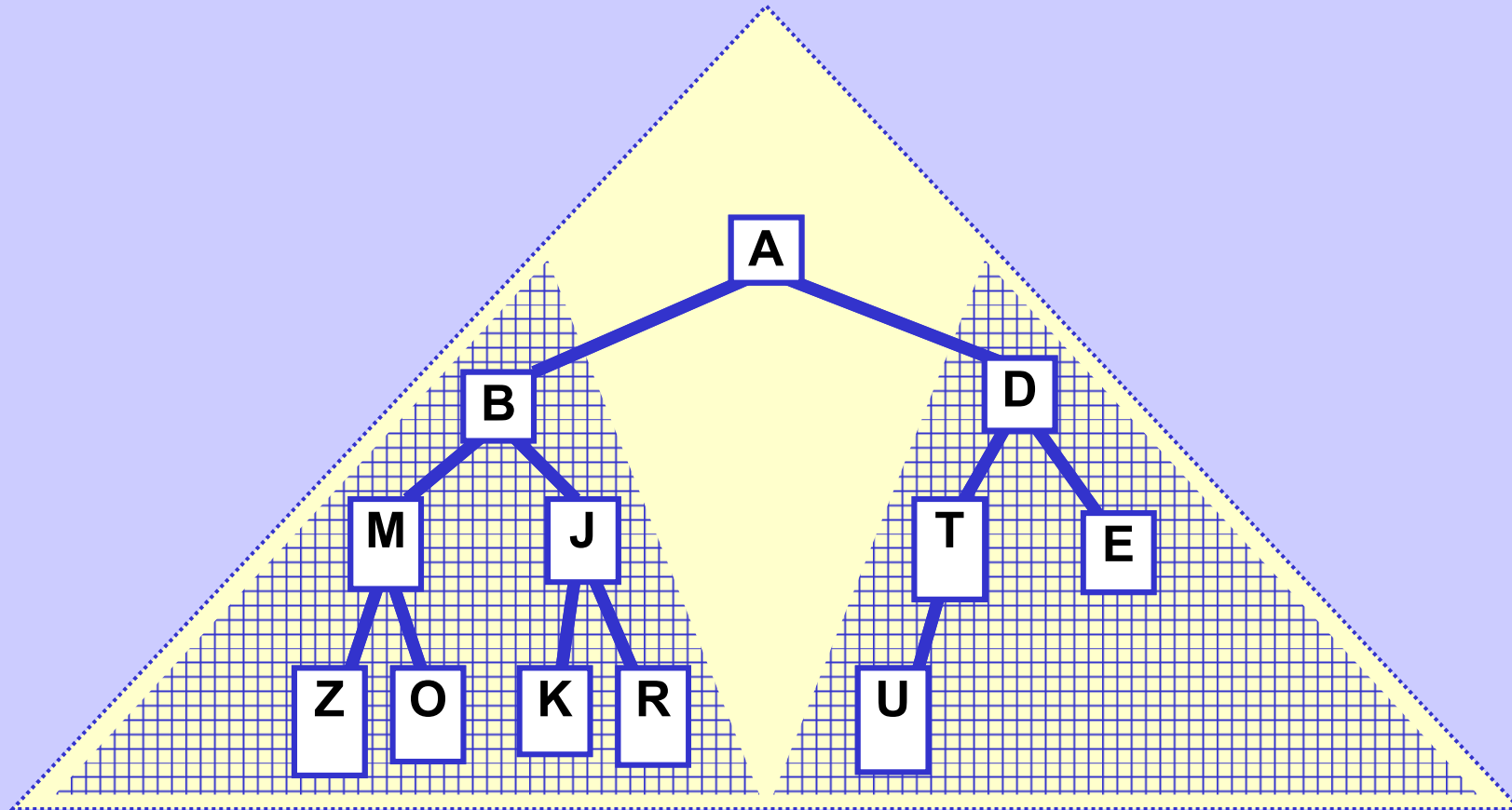


IV

Seřazeno

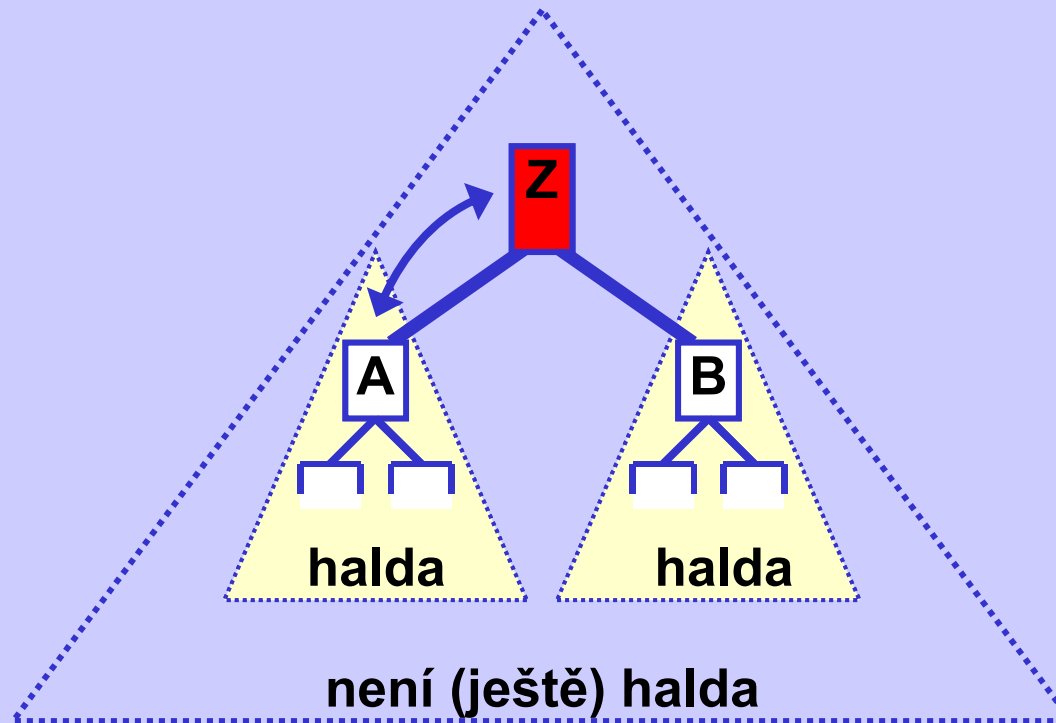


Rekurzivní vlastnost "býti haldou"



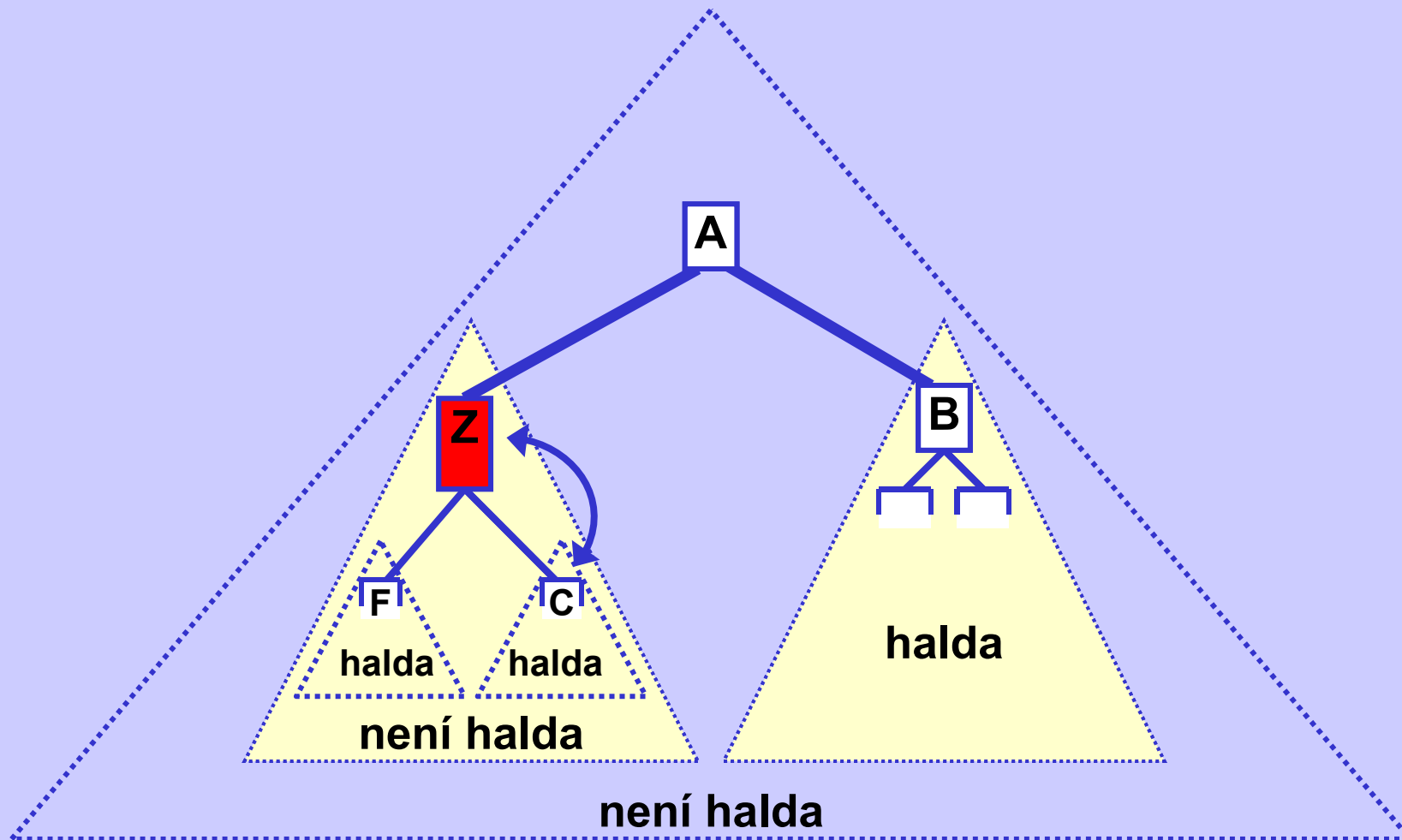
 je halda \Rightarrow  je halda a  je halda

Vytvoř jednu haldu ze dvou menších



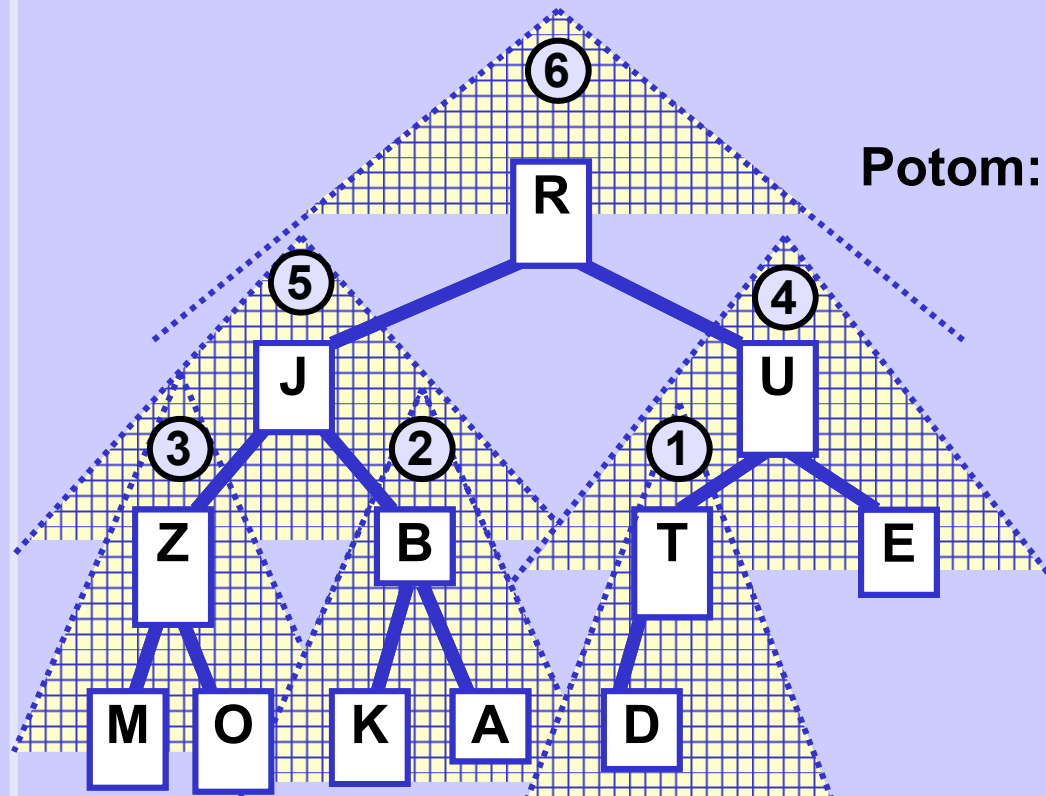
$Z > A$ nebo $Z > B$
 \Rightarrow prohod': $Z \leftrightarrow \min(A, B)$

Vytvoř jednu haldu ze dvou menších



Vytvoř haldu

Stavba haldy zdola



Uvaž: Každý list je samostanou (malinkou) haldou

Potom: Vytvoř haldu v ① ...

... a vytvoř haldu v ② ...

... a vytvoř haldu v ③ ...

... a vytvoř haldu v ④ ...

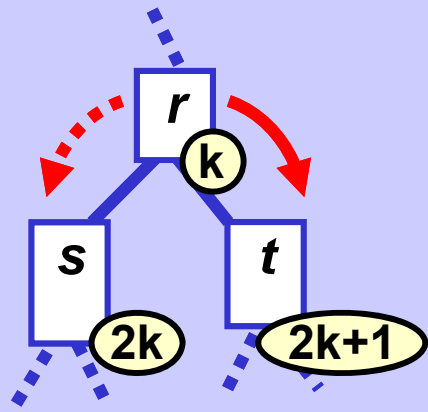
... a vytvoř haldu v ⑤ ...

... a vytvoř haldu v ⑥ ...

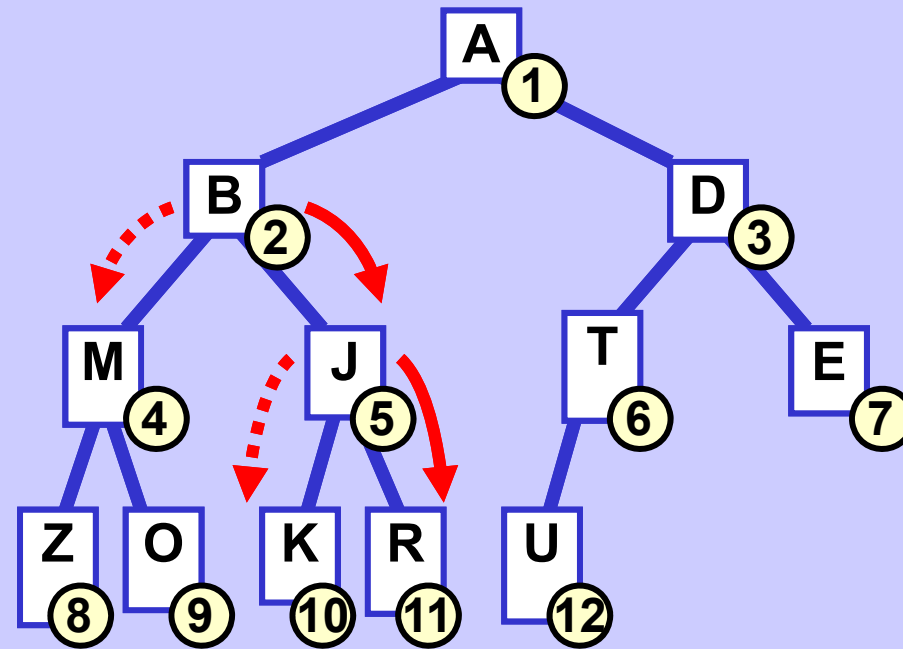
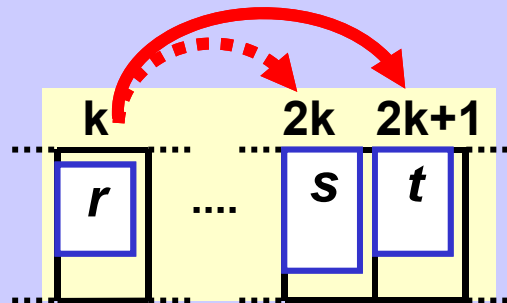
... a celá halda je hotova.

Halda v poli

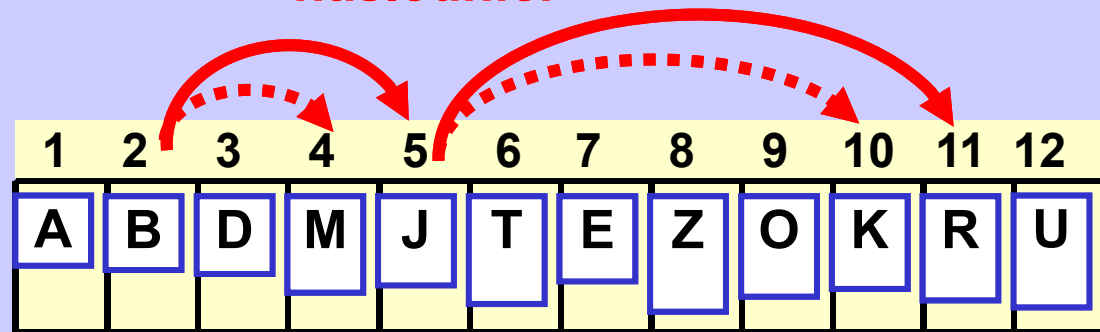
Halda uložená v poli



následníci



následníci



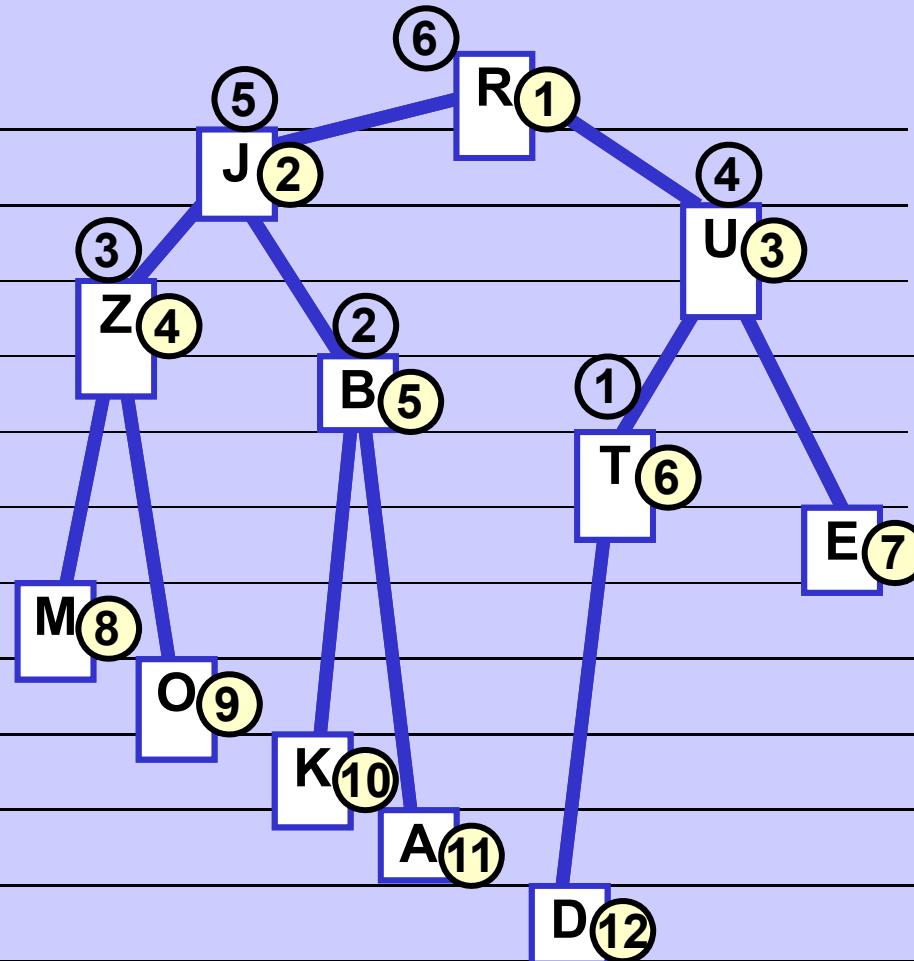
Tvorba haldy v poli

Pole

⑥	1	R
⑤	2	J
④	3	U
③	4	Z
②	5	B
①	6	T
	7	E
	8	M
	9	O
	10	K
	11	A
	12	D

Prvky náhodně uspořádaný

Není halda

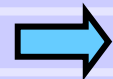
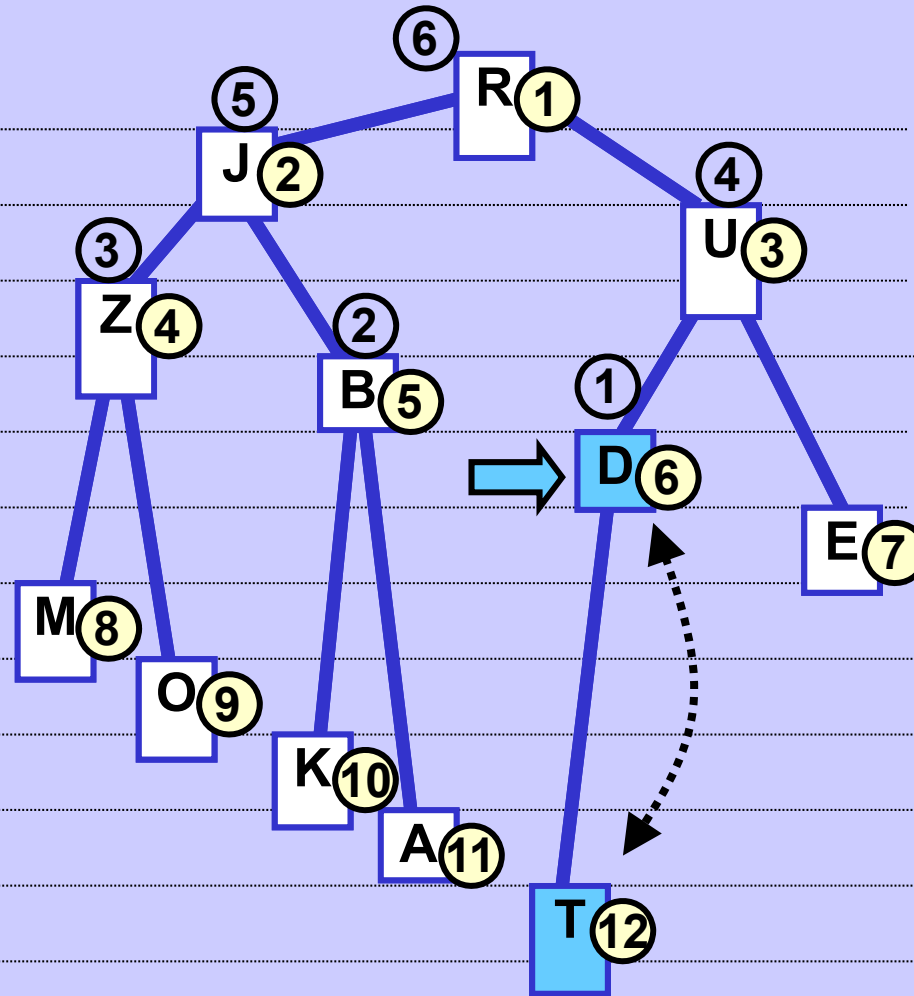


Tvorba haldy v poli

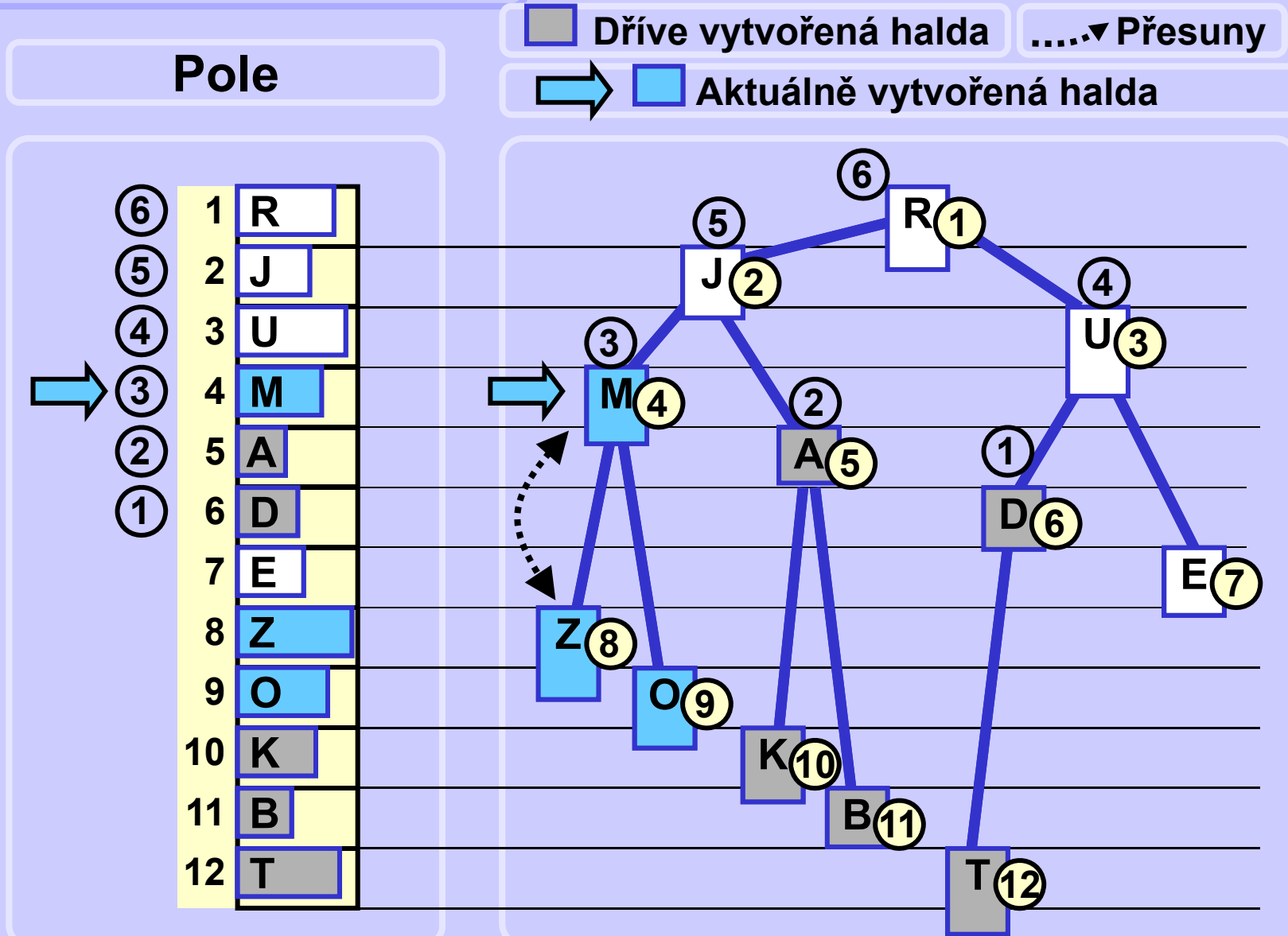
Pole

⑥	1	R
⑤	2	J
④	3	U
③	4	Z
②	5	B
①	6	D
	7	E
	8	M
	9	O
	10	K
	11	A
	12	T

.....▼ Přesuny


Aktuálně vytvořená halda


Tvorba haldy v poli



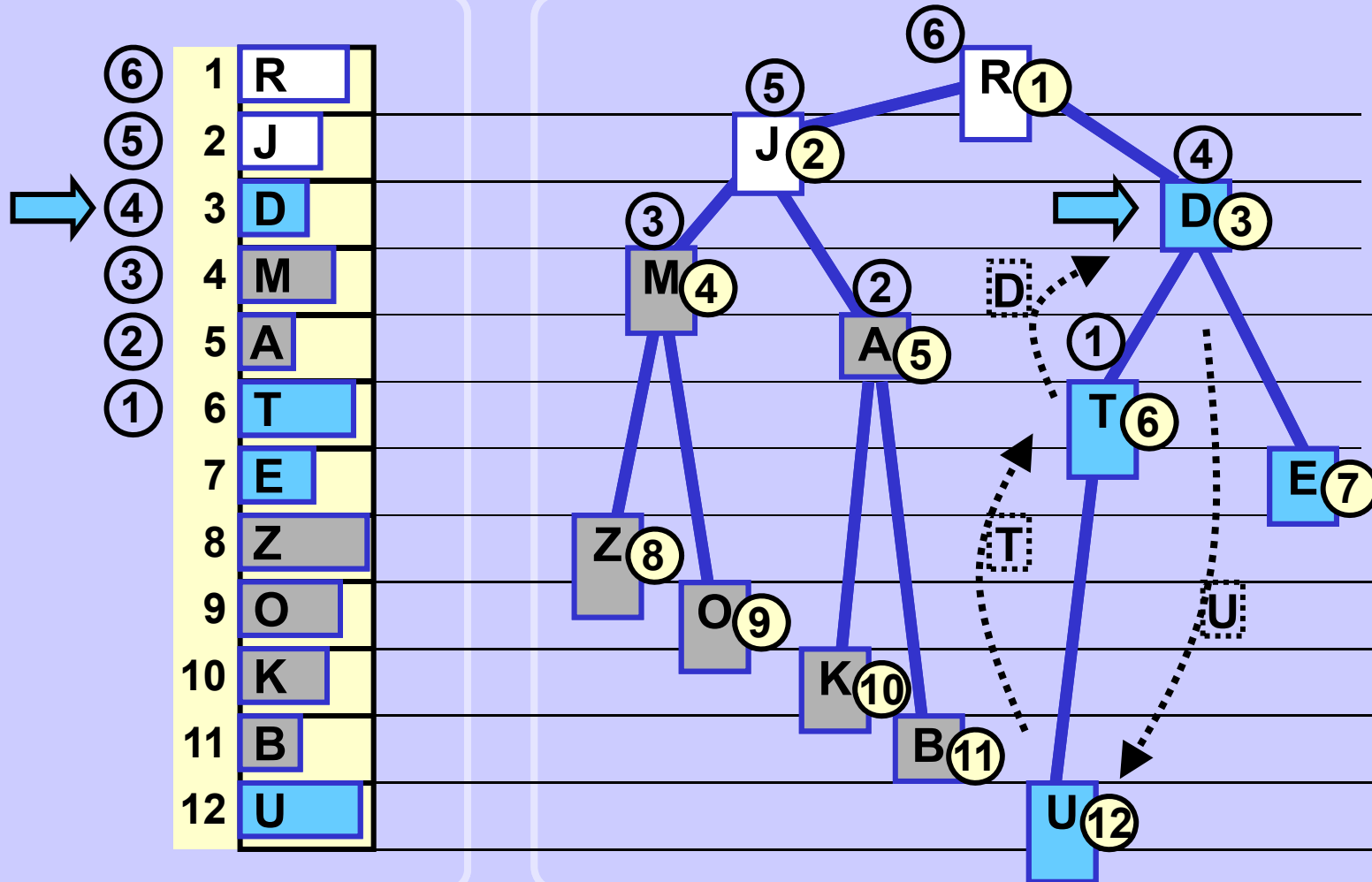
Tvorba haldy v poli

Pole

⑥	1	R
⑤	2	J
④	3	D
③	4	M
②	5	A
①	6	T
	7	E
	8	Z
	9	O
	10	K
	11	B
	12	U

■ Dříve vytvořená halda ▼ Přesuny

➔ ■ Aktuálně vytvořená halda



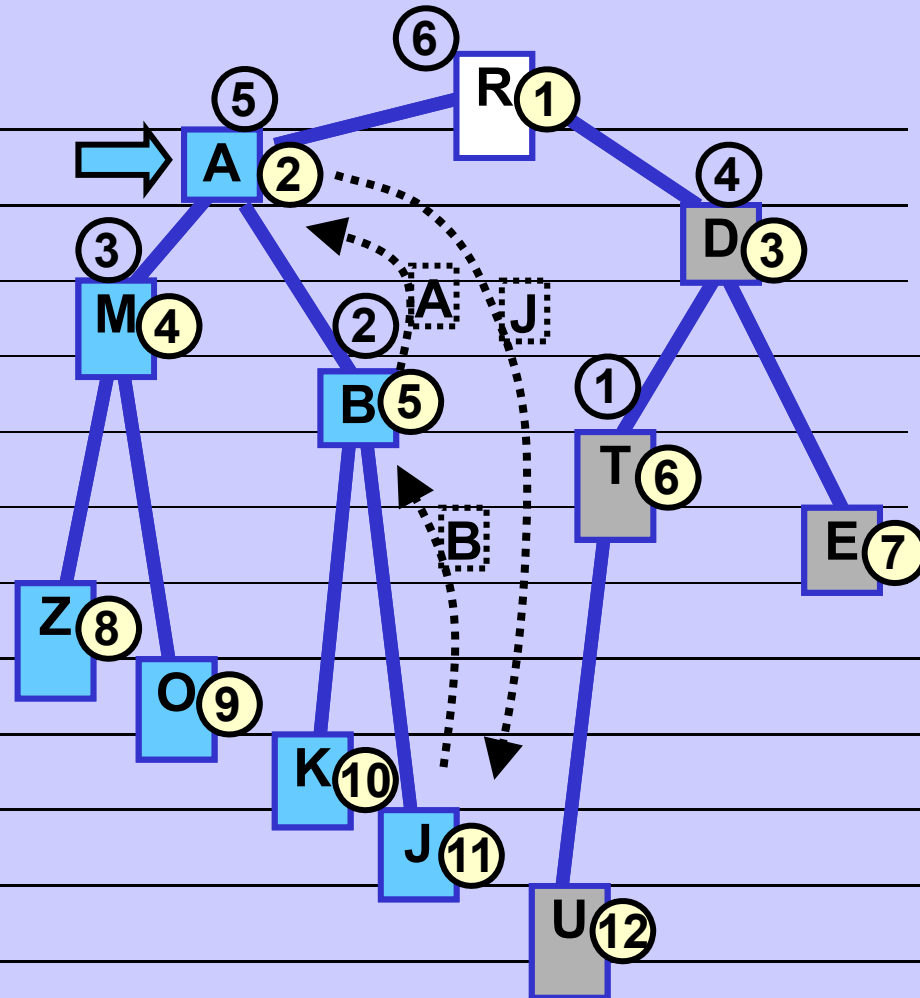
Tvorba haldy v poli

Pole

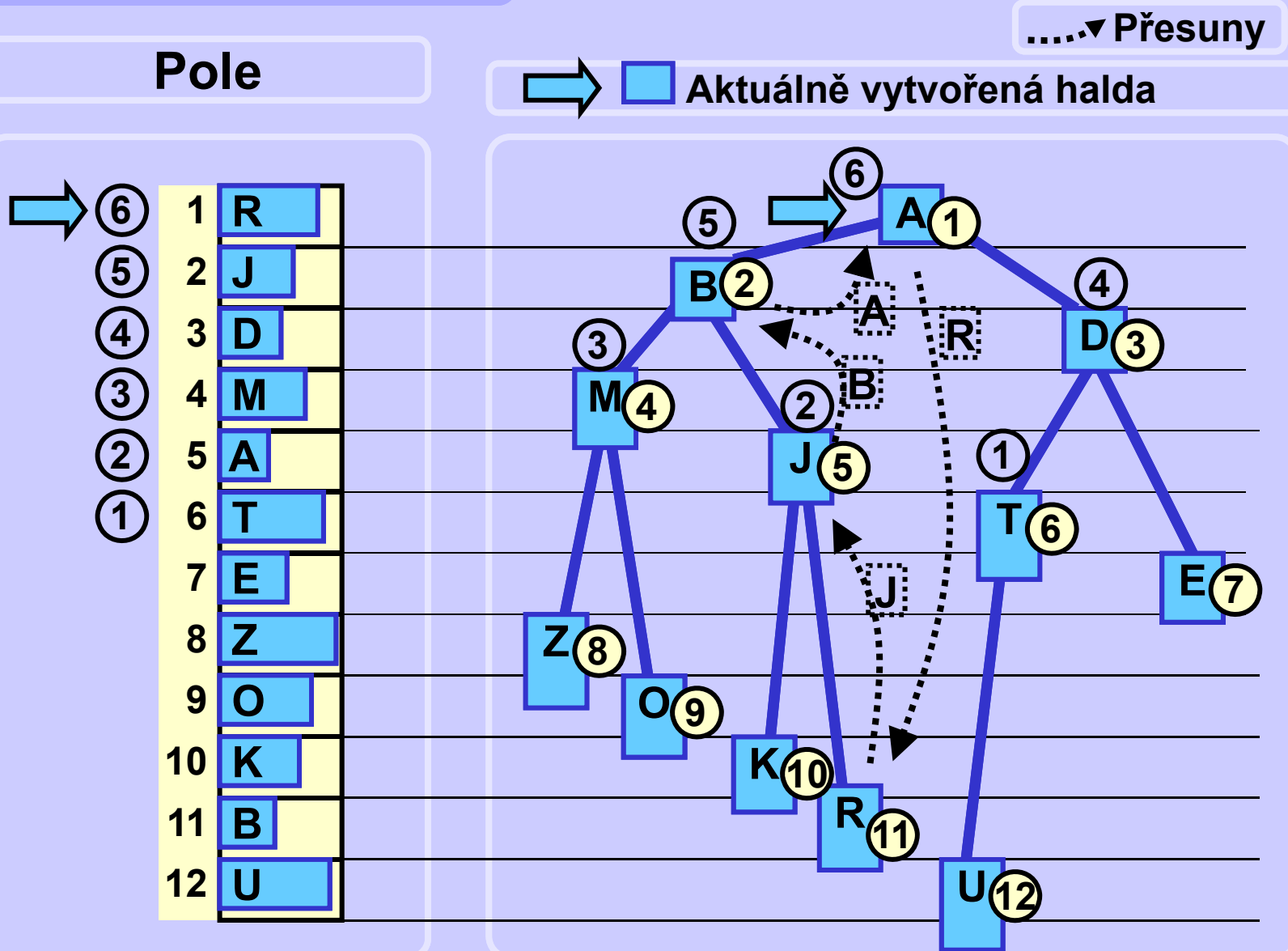
⑥	1	R
→ ⑤	2	J
④	3	D
③	4	M
②	5	A
①	6	T
	7	E
	8	Z
	9	O
	10	K
	11	B
	12	U

■ Dříve vytvořená halda ▼ Přesuny

→ ■ Aktuálně vytvořená halda

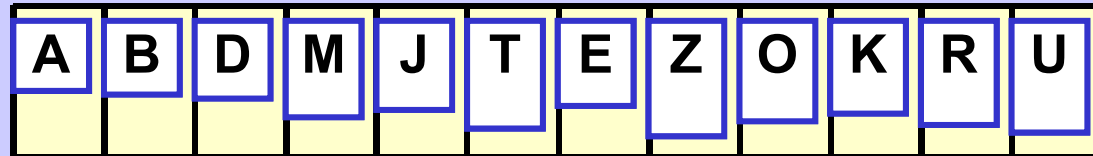


Tvorba haldy v poli

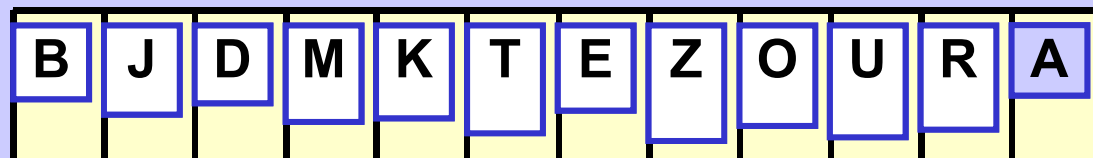
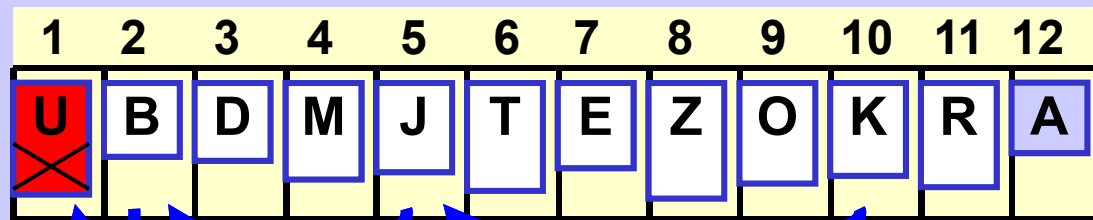
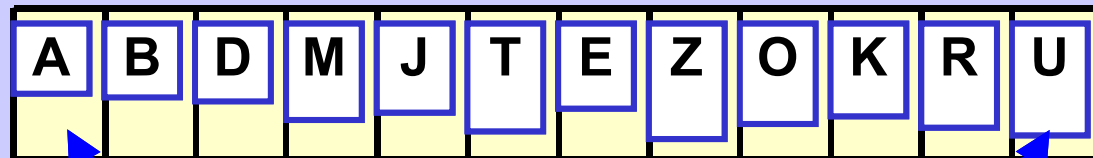


Heap sort

Halda



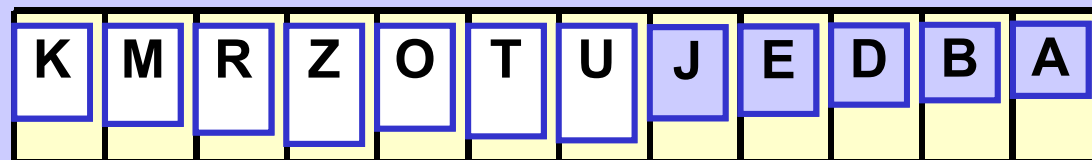
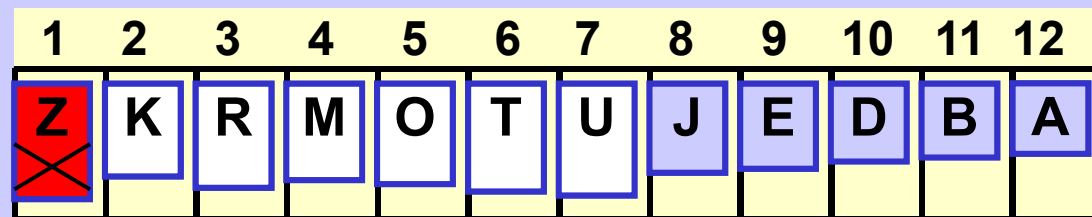
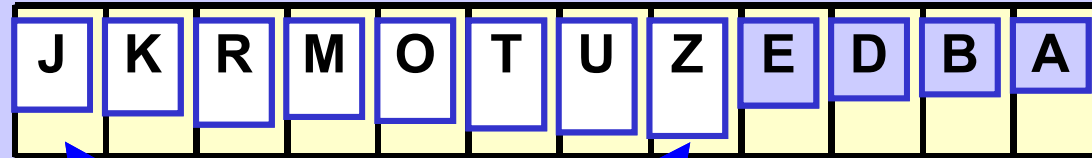
Krok 1



Halda

Heap sort

Krok k



Halda

k

Heap sort

```
                                // array: a[1]...a[n] !!!!  
  
void heapSort(Item [] a, int n) {  
  
                                // create a heap  
    for (int i = n/2; i > 0; i--)  
        repairTop(a, i, n);  
  
                                // sort  
    for (int i = n; i > 1; i--) {  
        swap(a, 1, i);  
        repairTop(a, 1, i-1);  
    }  
}
```

Heap sort

```

// array: a[1]...a[n] !!!!!
void repairTop(Item [] a, int top, int bottom) {
    int i = top;           // a[2*i] and a[2*i+1]
    int j = i*2;          // are children of a[i]

    Item topVal = a[top];

    // try to find a child < topVal
    if ((j < bottom) && (a[j] > a[j+1])) j++;

    // while (children < topVal)
    //     move children up
    while ((j <= bottom) && (topVal > a[j])) {
        a[i] = a[j];
        i = j;  j = j*2; // skip to next child
        if ((j < bottom) && (a[j] > a[j+1])) j++;
    }
    a[i] = topVal;       // put topVal where it belongs
}

```

Heap sort

repairTop operace nejhorší případ ... $\log_2(n)$ (n =velikost haldy)

vytvoř haldu ... $n/2$ repairTop operací

$$\log_2(n/2) + \log_2(n/2+1) + \dots + \log_2(n) \leq (n/2)(\log_2(n)) = \underline{\underline{O(n \cdot \log_2(n))}}$$

seřad' haldy ... $n-1$ repairTop operací, nejhorší případ:

$$\log_2(n) + \log_2(n-1) + \dots + 1 \leq n \cdot \log_2(n) = O(n \cdot \log_2(n))$$

$$\text{ale i nejlepší případ} = \underline{\underline{\Theta(n \cdot \log_2(n))}}$$

$$\text{celkem ... vytvoř haldu + seřad' haldy} = \underline{\underline{\Theta(n \cdot \log_2(n))}}$$

Asymptotická složitost Heap sortu je $\Theta(n \cdot \log_2(n))$

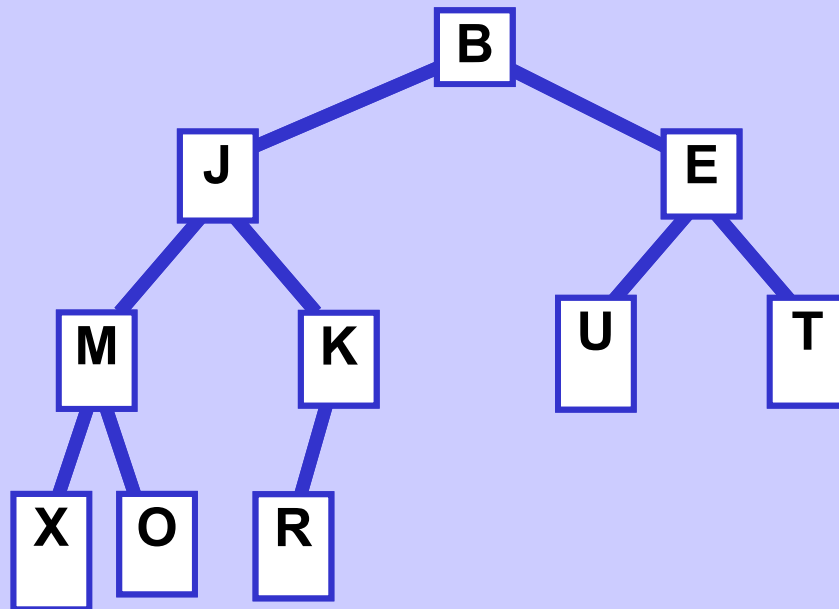
Heap sort není stabilní

Prioritní fronta

Prioritní fronta má stejné operace jako obyčejná fronta

- vlož do fronty (Insert, Enqueue, apod),
- čti první prvek (Front, Top, apod),
- smaž první prvek (Dequeue, Pop, apod).

Navíc interně zajišťuje, že na čele fronty je vždy prvek s minimální (maximální) hodnotou ze všech prvků ve frontě.



Prioritní frontu lze implementovat pomocí haldy.

Plným jménem: "Binární halda".

Prioritní fronta pomocí binární haldy -- operace

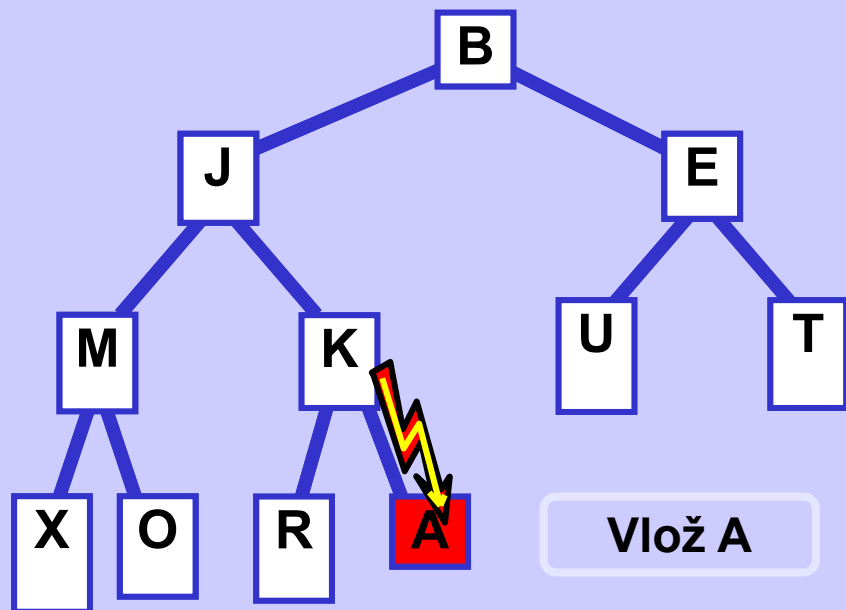
Čti první prvek (Front, Top, apod) .

Zřejmé.

Smaž první prvek (Dequeue, Pop, apod) = Odstraň vrchol a oprav haldu.

Viz výše.

Vlož do fronty (Insert, Enqueue, apod).



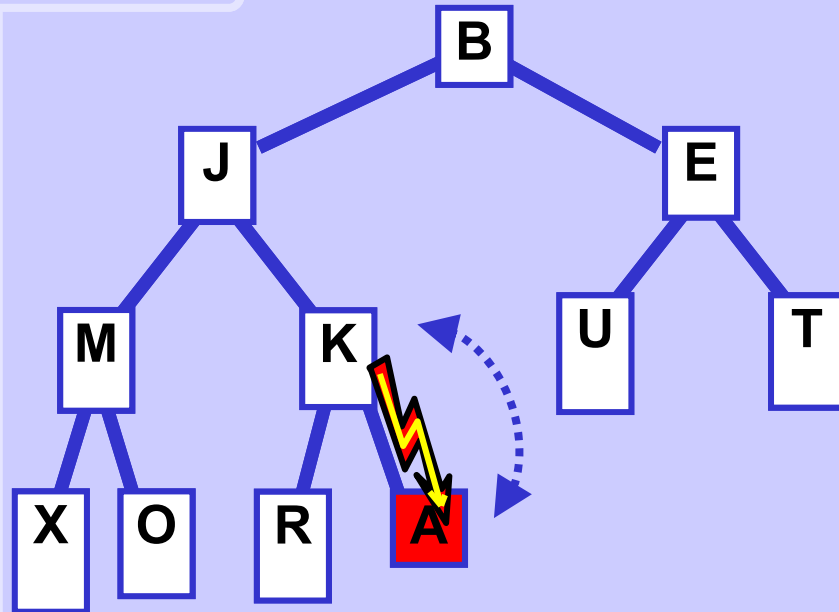
Vložíme prvek
na konec fronty (haldy).

Ve většině případů
se tím poruší pravidlo haldy

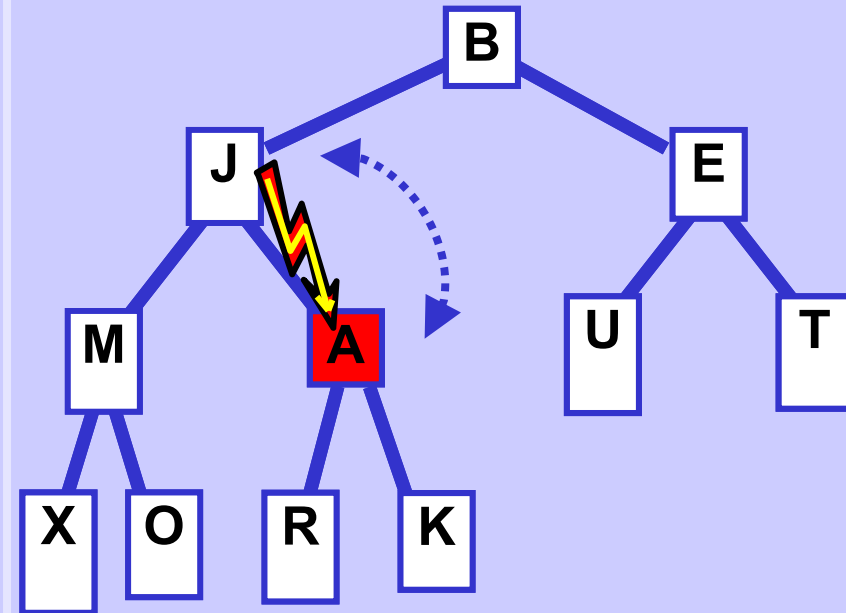
a je nutno haldu opravit.

Prioritní fronta pomocí binární haldy – vlož prvek

Vlož A



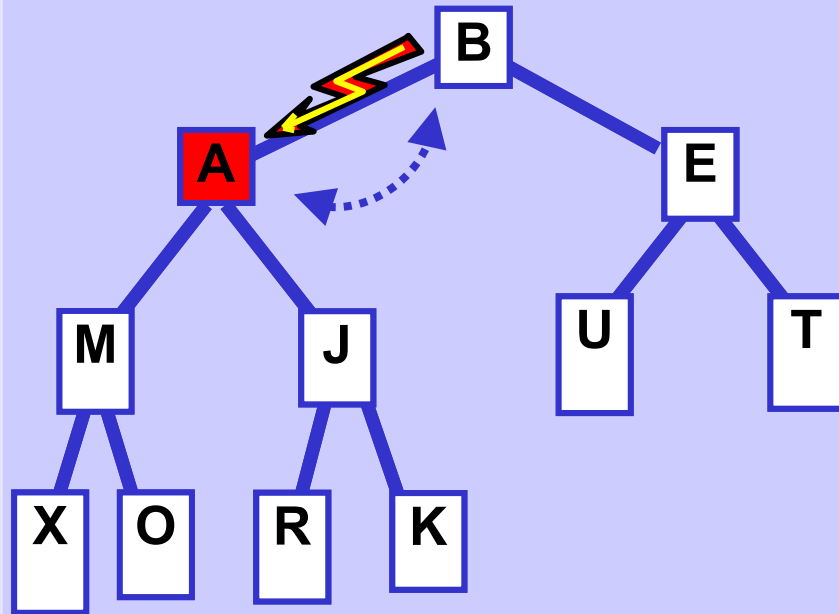
Pravidlo haldy je porušeno,
vyměň vkládaný prvek
s jeho rodičem.



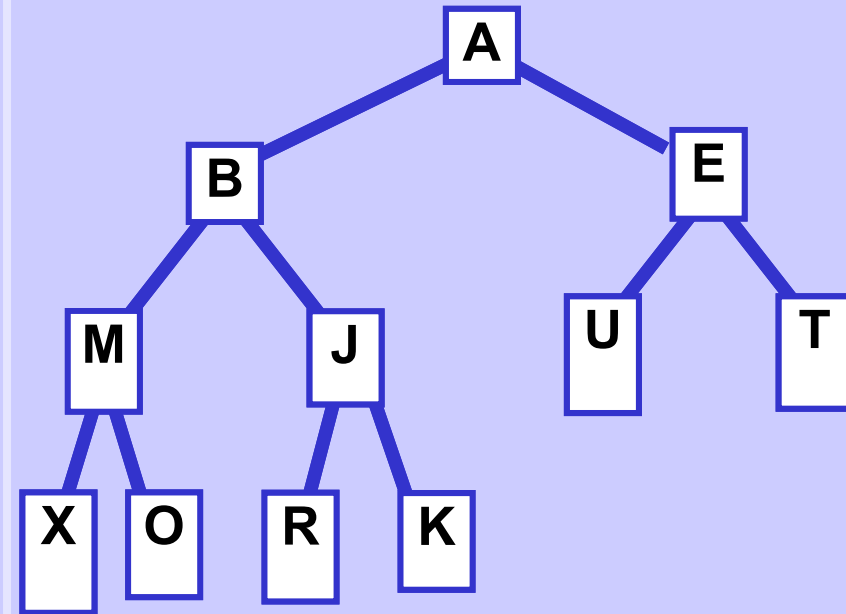
Pravidlo haldy je stále porušeno,
vyměň vkládaný prvek
s jeho rodičem.

Prioritní fronta pomocí binární haldy – vlož prvek

Vkládáme A



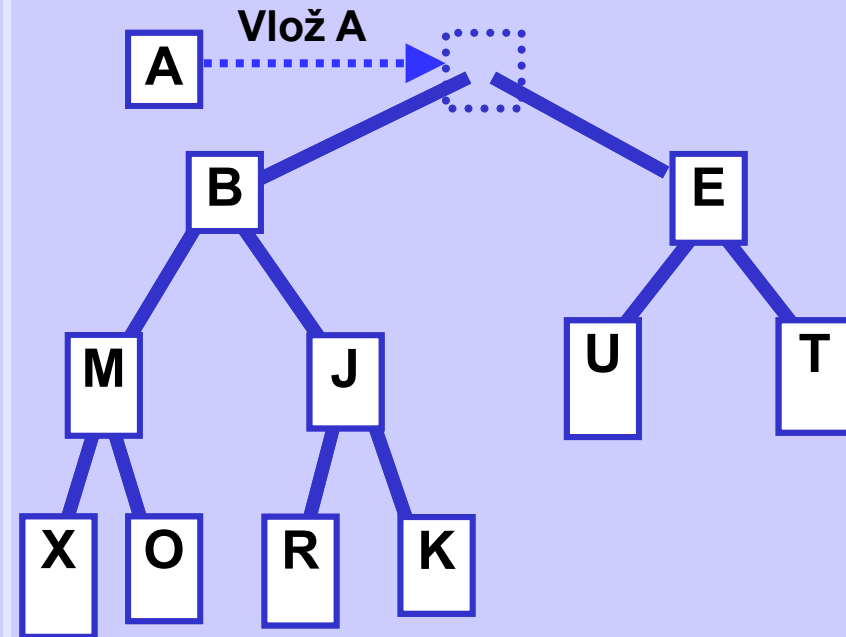
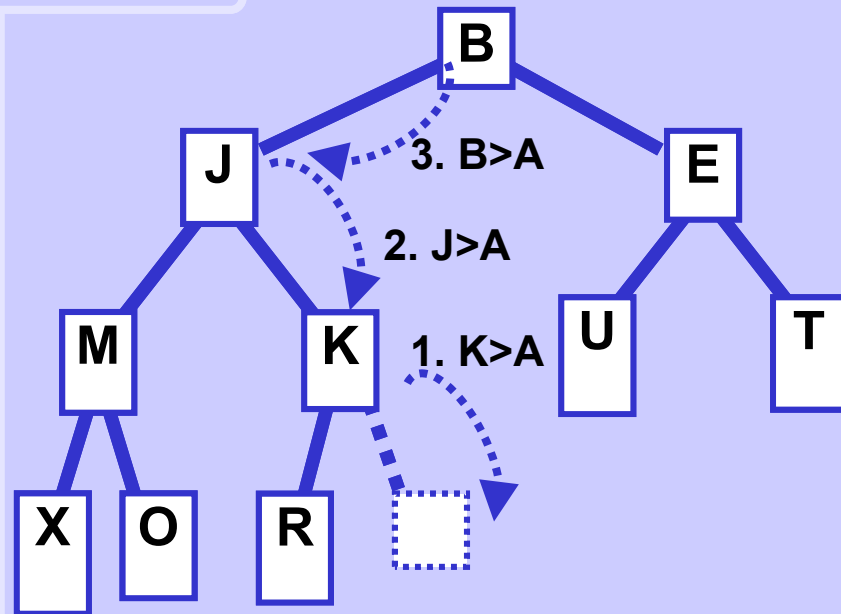
Pravidlo haldy je stále porušeno, vyměň vkládaný prvek s jeho rodičem.



Pravidlo haldy je zachováno, vkládaný prvek našel své místo v haldě.

Binární halda – vlož prvek, efektivněji

Vlož A



Vkládaný prvek na konec haldy
nevkládej.
Napřed zjisti, kam patří, ostatní
(větší) prvky posuň o patro dolů ...

... a teprve nakonec
vlož prvek na jeho místo.

Binární halda – vlož prvek

```

// array: a[1]...a[n] !!!!!
int insert(Item [] a, int x, int bottom) {
    int j = ++bottom;    // expand the heap
    int i = j/2;        // parent index

    while ((i > 0) && (a[i] > x)) {
        a[j] = a[i];    // move elem down the heap
        j = i; i /= 2;  // move indices up the heap
    }
    a[i] = x;          // put inserted elem to its place
    return bottom;
}

```

Binární halda – Složitost operace insert

Vkládání představuje průchod vyváženým stromem s n prvky od listu nejvýše ke kořeni, složitost operace Insert je tedy $O(\log_2(n))$.