Instructions for seminars from Numerical Analysis:
Interpolation

All methods of appoximation studied in this course are implemented in the Maple document `aproximE.mw`. You can choose the method, number and distribution of nodes and function values. Additionally, the plain interpolation and the least squares method admit the choice of approximating functions (incl. their number). When considering the numerical errors, various choices of the form of output and precision of arithmetic can be of interest. Before trying to solve the first task for assessment, you should pay attention to the following topics:

A. The influence of the distribution of nodes on the error of the method of approximation
Interpolate the following functions

1. $f_1(t) = \sin t$,
2. $f_2(t) = \arctan(10 * (t - 5))$

(by a polynomial or spline) at interval $\langle 0, 10\rangle$ for the following choices of nodes:

a. 6 equidistant nodes;
b. 11 equidistant nodes;
c. 11 nodes with the cosine distribution,
$$x_i = 5 - 5 * \cos\frac{(i + \frac{1}{2})\pi}{n};$$
d. 11 nodes with another distribution, e.g., $0, 0.5, ..., 4.5, 10$.

Compare the graphs and the maxima of errors (also for the derivatives and intergrals of the resulting approximations).

B. Round-off errors
Try different forms of the interpolating polynomial (Lagrange, Newton, and their expanded form) and splines, e.g., for 5 equidistant nodes and functions

1. $f_1(t) = \sin t$ at interval $\langle 0, 1\rangle$,
2. $f_2(t) = f_1(t - p)$ at interval $\langle p, p + 1\rangle$,
3. $f_3(t) = f_1(t) + p$ at interval $\langle 0, 1\rangle$,

where p is a big constant. (The error of the method is the same in all the above cases.)

C. Aproximation by Taylor polynomial
For some function and an interval of your choice, compare the graphs and the maxima of errors of the interpolating polynomial and the Taylor polynomial of the same degree.