# Localization in Mobile Robotics

## Part I.

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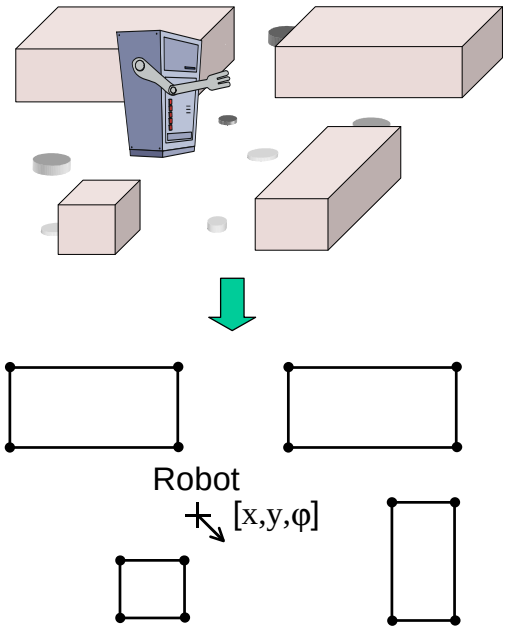
# Lecture outline

- Taxonomy
- Continuous localization
  - Iterative Closest Point
  - Iterative Matching Range Point
  - Iterative Dual Correspondence
  - Correlative Scan Matching
- Probabilistic methods
  - Bayes filter
  - Particle filter
  - Histogram filter
  - Kalman filter
  - Extended filter

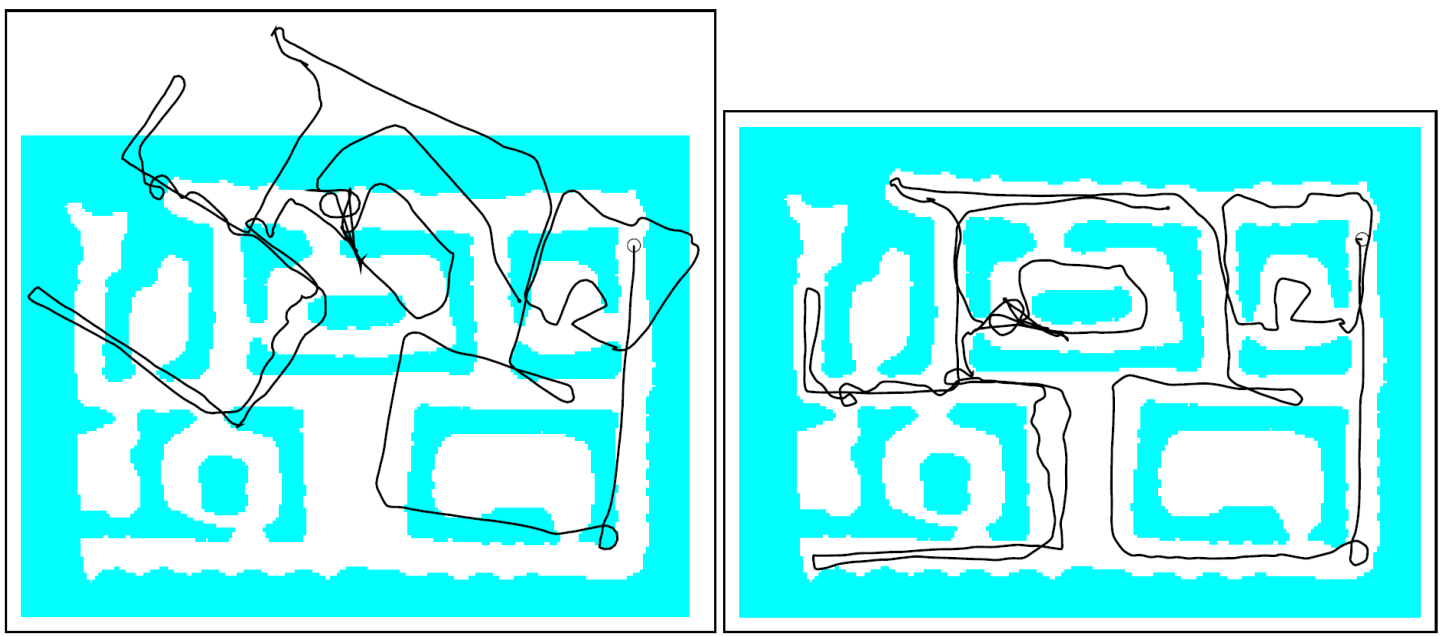
# Localization

- Localization is the problem of robot position estimation
  - relatively with respect to the start robot position
  - in the given coordinate system (in a known map)
- Pose is supposed also with orientation: i.e. $(x, y, \phi)$, resp. $(x, y, z, roll, pitch, yaw)$
- Input: odometer information, data measurements



# Why is localization difficult?

- Position can be measured with no single sensor => position is determined from sensory data
- But sensors are inaccurate (subject of noise)!
- Control commands are realized inaccurately.
- One single measurement is not enough.



# Taxonomy

- Position tracking (continuous, local)
  - The initial position is known.
  - Correction of odometer inaccuracies.
  - The error is bounded.
  - Unimodal distribution (Gaussian).
- Lost robot problem(global, absolute)
  - The initial position is not known.
  - The error is unbounded.
  - Multi-modal distribution.
- kidnapped robot problem
  - Detection of position errors $=>$ correction.
  - Suitable for testing.

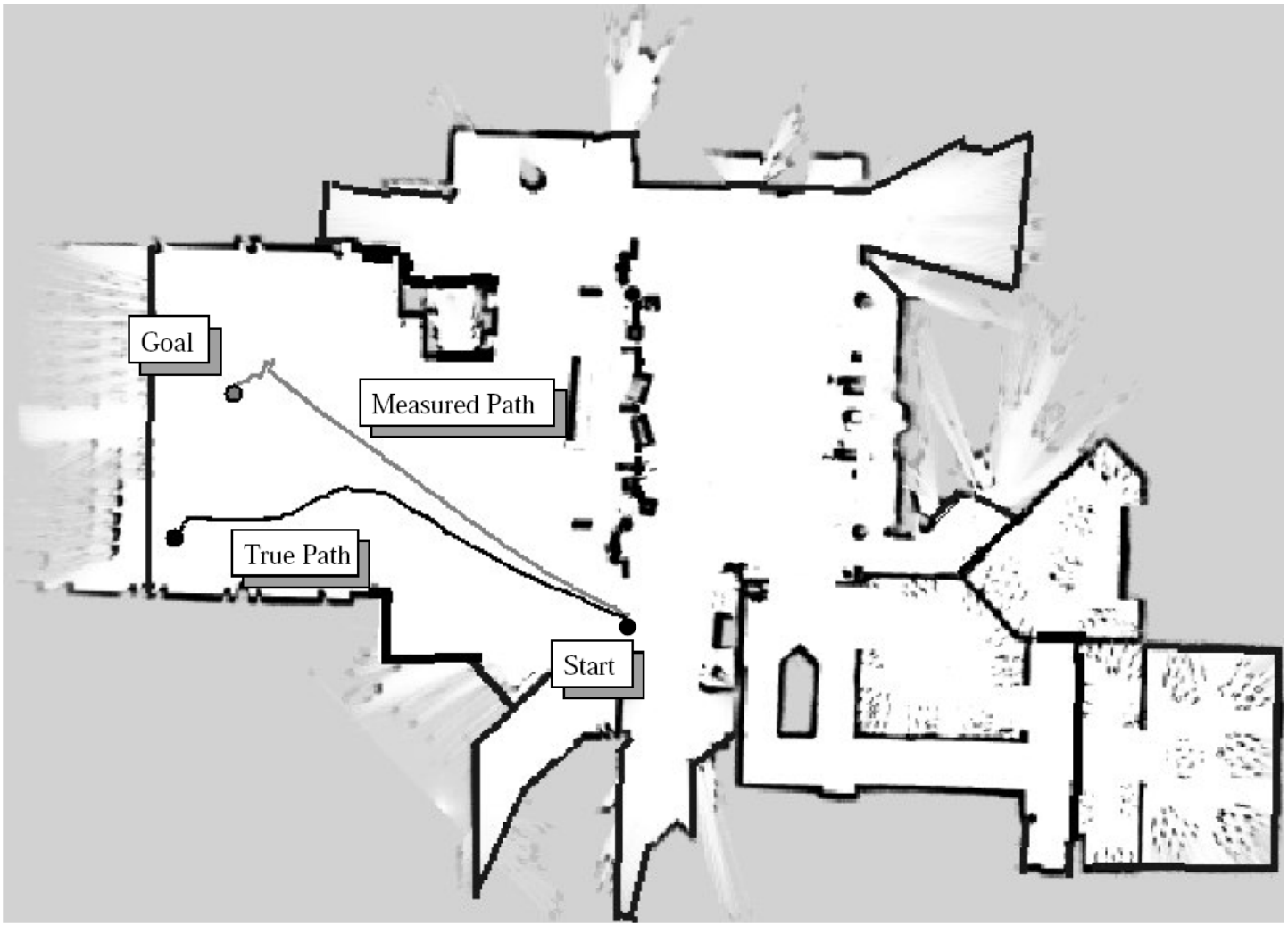
# Taxonomy

- Static environment
  - The robot is the only one what moves.
  - The robot position is the only variable.
  - "Nice" mathematical features.
- Dynamic environment
  - Objects or/and other robots move (change their positions/state).
  - Objects: humans, light (for cameras), doors, ....
  - Two approaches
    - Object's movement can be modeled => it is a part of state description of the localization task.
    - Data about moving objects are filtered.

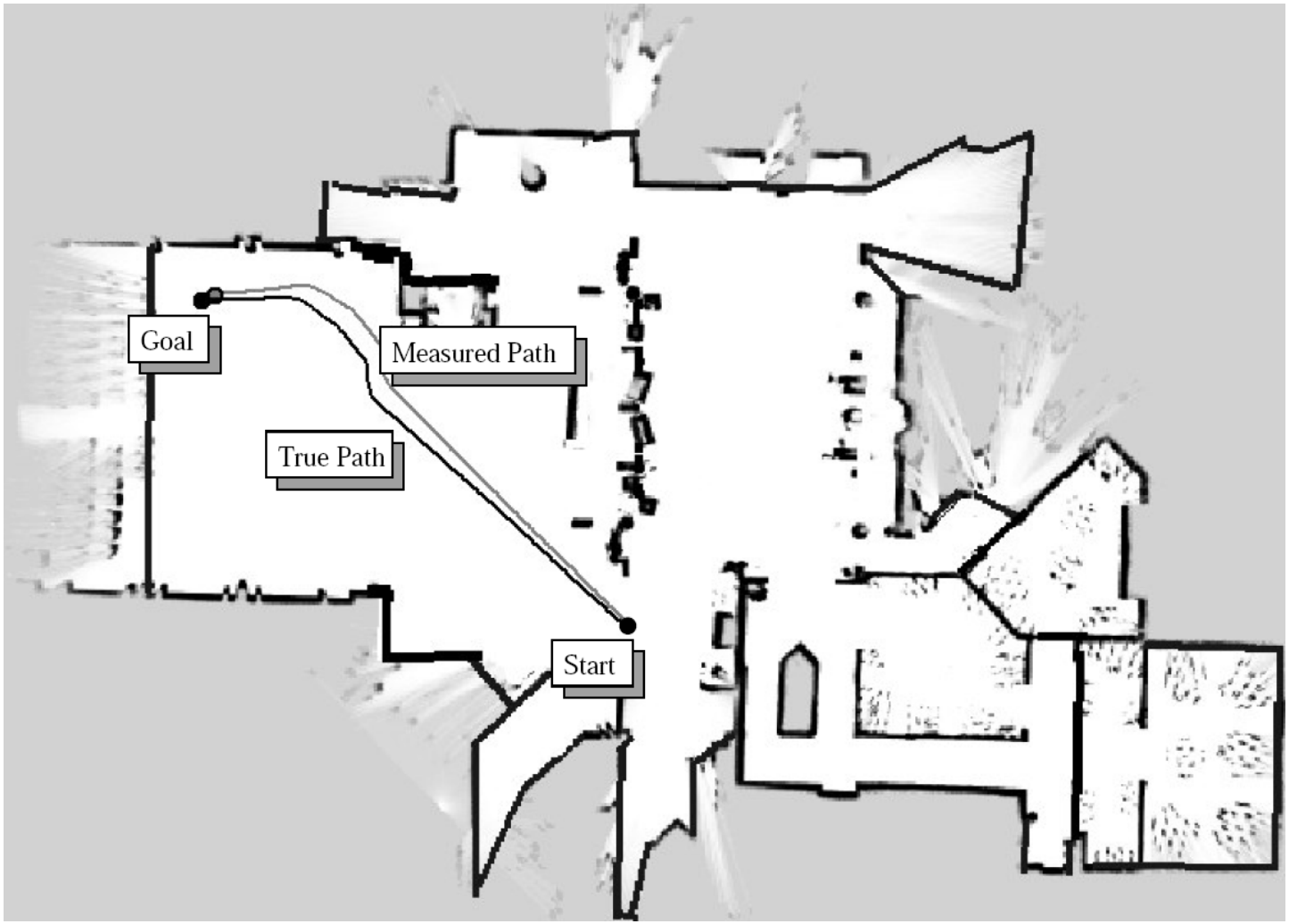
# Taxonomy

- Passive localization
  - Localization module only passively observes, what is happening - it has no influence on the robot control.
  - Localization is a "complementary product of other task.
- Active localization
  - The robot is controlled in order to correct/determine its position.
  - Gives better results ....
  - ... but it can't be applied every time.
  - Goals alternation: the robot is localized sometimes x the primary task is solved sometimes.
  - The goal is generated as a compromise.

# Passive localization



# Active localization

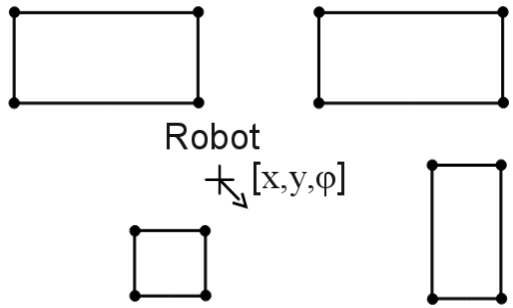


# Taxonomy

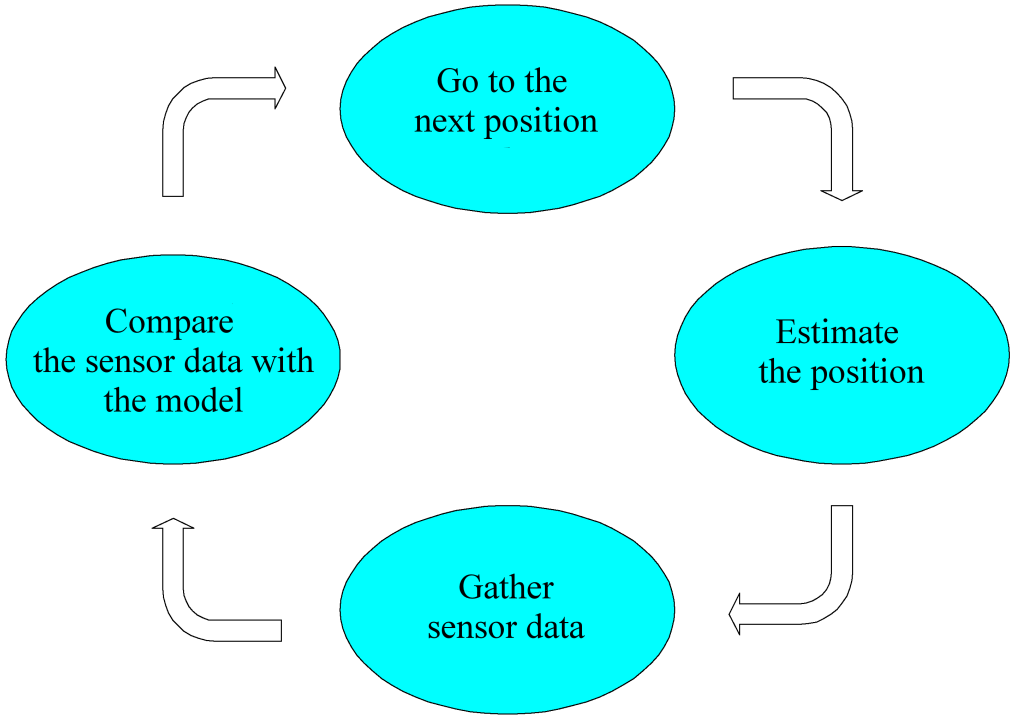
- Single robot
  - The classical problem.
  - No communication is need, everything at one place.
- Multiple robots
  - Every robot is localized separately/independently (it pretends that other robots do not exist).
  - But if the robots are able communicate or detect each other, it is better.
  - Self-localization in the other robot map.
  - My position estimate is known, other's robot position estimates are known as well as relative positions => optimization („rubber bands“).

# We will suppose

- A terrestrial robot operating on a horizontal plane (2D).
- Only one plane of the environment is sensed.
- Majority of objects is static and detectable by sensors
- Distance measuring sensors are used.
- Passive localization.
- Single robot.
- Continuous, absolute localization, and kidnapped robot

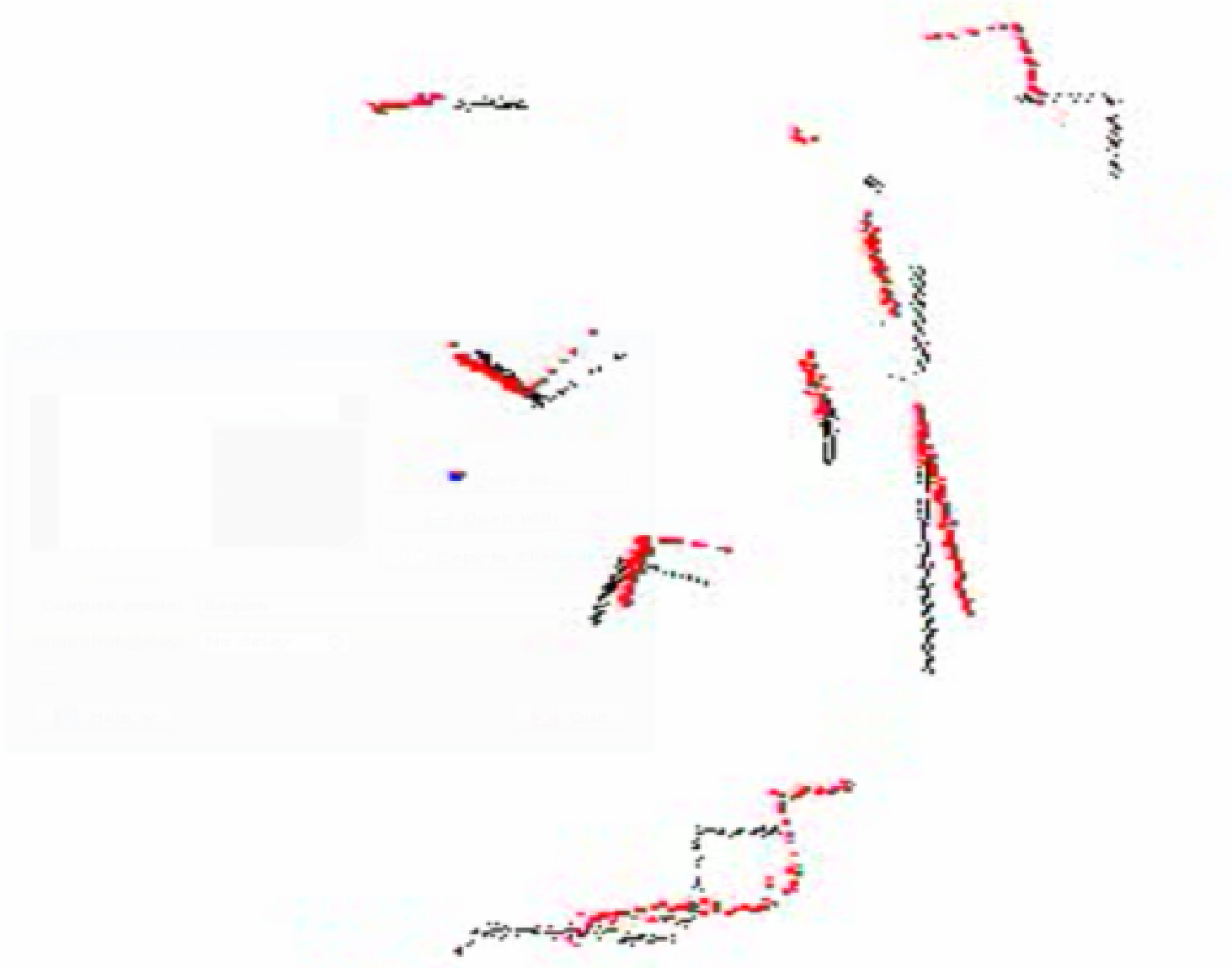


# Continuous localization



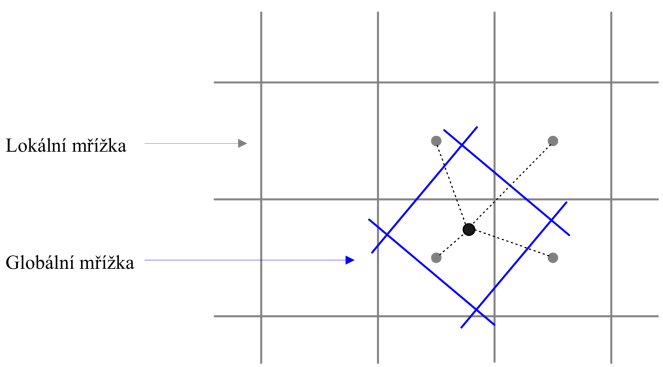

- The position is supposed to be in the close neighborhood of the estimate.
- High precision needed (continuous process).
- Input: local a and global maps (scans).
- Output: transformation minimizing measure of maps discrepancy.

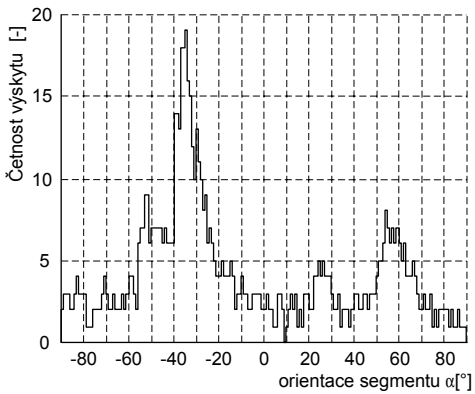
# Example

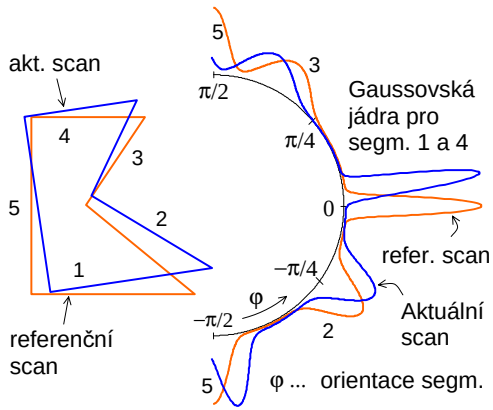


# Continuous localization

- Many methods differing in the representation of the maps (scans)
- It is always optimization problem.
- Approaches:
  - Localization on occupancy grids.
  - Histograms
  - Localization based on processing of geometric primitives (point-to-line, line-to-line, Hough transform)



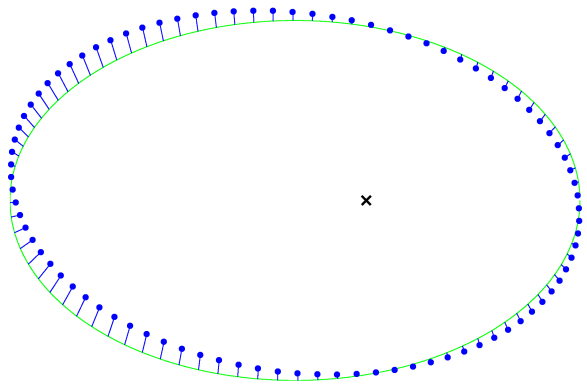



# Iterative Closest Point (ICP)

(Lu, Milios)

- Iterative
  - The actual scan is interpolated with a polyline.
  - Corresponding points are determined (the smallest distance).
  - Transformation is determined (using least squares).

$$E_{dist}(T_x, T_y, \omega) = \sum_i^n |R_\omega P + T - P'|^2$$



- It has the analytical solution:

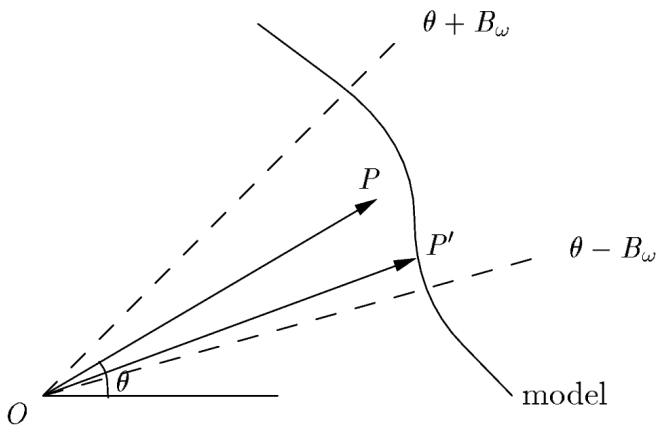
$$\begin{aligned} T_x &= \bar{x}' - (\bar{x}\cos\bar{\omega} - \bar{y}\sin\bar{\omega}) \\ T_y &= \bar{y}' - (\bar{x}\sin\bar{\omega} + \bar{y}\cos\bar{\omega}) \\ \bar{\omega} &= \arctan \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i' - \bar{y}') - \sum_{i=1}^n (y_i - \bar{y})(x_i' - \bar{x}')}{\sum_{i=1}^n (x_i - \bar{x})(x_i' - \bar{x}') + \sum_{i=1}^n (y_i - \bar{y})(y_i' - \bar{y}')} \end{aligned}$$

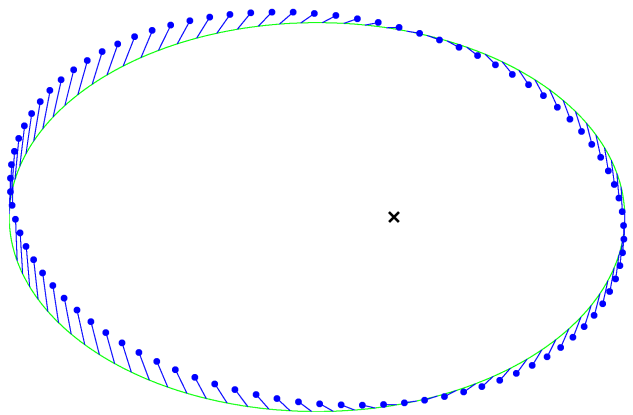
- It converges slowly, especially in the rotational part.

# Iterative Matching Range Point (IMRP)

(Lu, Milios)

- The procedure is the same as in ICP, except the correspondence rule.
- Consider that translation is irrelevant $=>$ $|P| \approx |P'|$ and $\tilde{\phi} \approx \phi + \omega$.
- Neighborhood size for correspondence search is decreasing.
- Initially slower convergence than ICP, but it is faster later on.
- IMRP tries to correct translation error with rotation $=>$ instability.

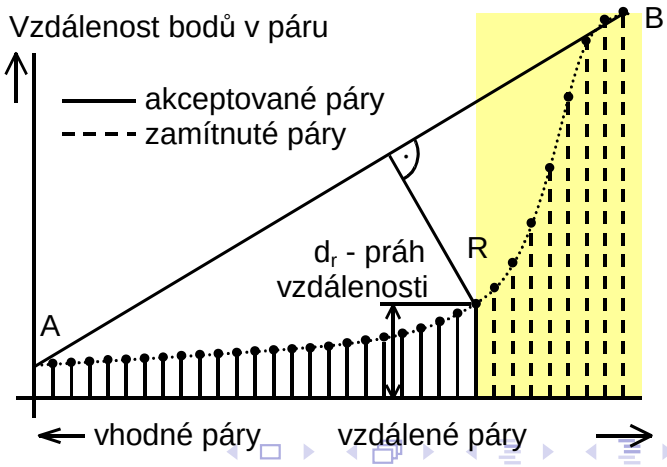


# Iterative Dual Correspondence (IDC)

- Combination of the two rules: translation from ICP, rotation from IMRP
- Algorithm
  - For each $P_i$ from the actual scan:
    - Apply the *closest-point* rule to determine the corr. point $P'$
    - Apply the *matching-range-point* rule $=> P''$
    - Compute the least-squares solution $(\omega_1, T_1)$ from the set of correspondence pairs $(P, P')$
    - Compute the least-squares solution $(\omega_2, T_2)$ from the set of correspondence pairs $(P, P'')$
    - Form $(\omega_2, T_1)$ as the solution
  - Repeat the previous steps until the process converges.

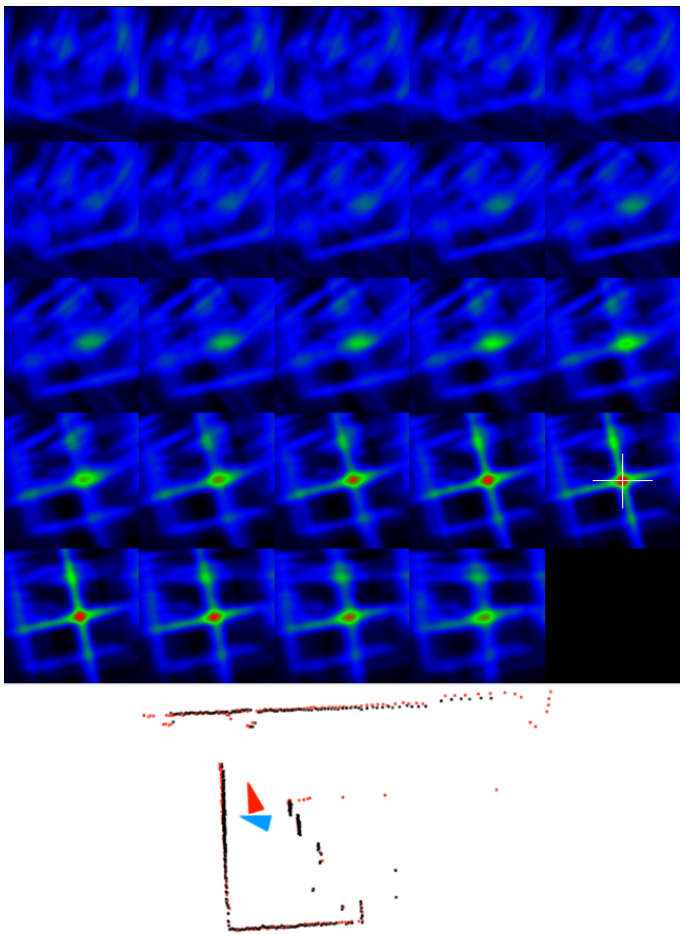
Improvement: "Bad" correspondence pairs are discarded.



# Correlative Scan Matching

(Edwin B. Olson)

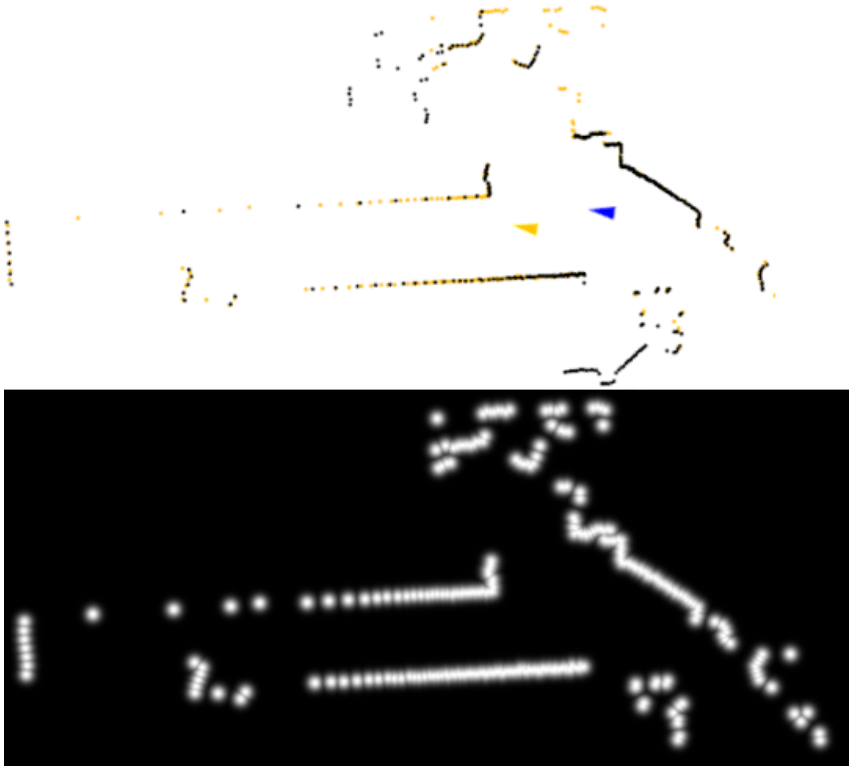
- Shape of a cost function is really complex with number of local extrema.
- Initial position estimation (odometry) can be inaccurate.
- ICP finds a local optimum only.
- Input: two scans
- Goal: find a global optimum



# Correlative Scan Matching

## Look-table building

- For each point in a map compute the probability that the point corresponds to a reference measurement.



# Correlative Scan Matching

## Algorithm

- Brute force
  - Compute correlation for each pose (including orientation) of the scan.
  - The pose with the highest correlation is chosen.
- Computing 2D slices
  - A huge amount of time is spent with scan transformation.
  - Idea: rotation is done in the outer-most loop. The inner loops simply translate the query points.

# Correlative Scan Matching

## Algorithm

- Multi-level resolution
  - Two lookup tables: low resolution (LRT) and high resolution table (HRT)
  - Cell in LRT is the maximum value of corresponding cells in HRT
  - This guarantees that we will not miss the maximum.
  - Algorithm
    - Compute correspondences in LRT.
    - Find the best voxel in $LRT_B$ v LRB not yet considered.
    - If the value of $LRT_B$ ¡ $H_{best}$ terminate ($H_{best}$ is the maximum).
    - Evaluate the voxel in HRT and set $H_{best}$ if new maximum is found.

# Correlative Scan Matching

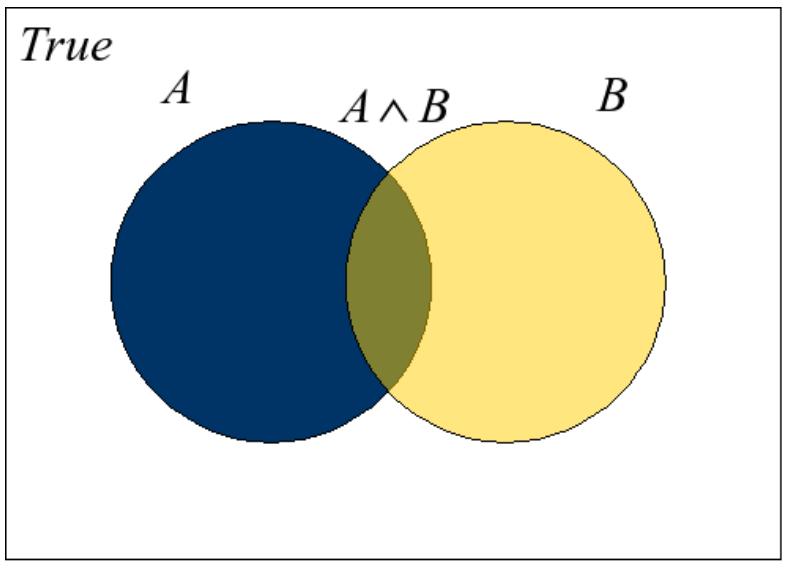
## Results

PC: Intel Core2 6600 @ 2.4 Ghz

| | ICP | ICL | Hill-Climb | Correlative (Naive) | Correlative (2D Slices) | Correlative (Multi-Res) | Correlative (GPU 7600GS) | Correlative (GPU GTX260) |
|---|---|---|---|---|---|---|---|---|
| 0.5 m, 20 deg | 56 ms | 64 ms | 1.1 ms | 246 ms | 27 ms | 8.4 ms | 98 ms | 26 ms |
| 2.0 m, 40 deg | 99 ms | 105 ms | 1.3 ms | 7512 ms | 692 ms | 20.8 ms | 1563 ms | 289 ms |
| 4.0 m, 90 deg | 145 ms | 174 ms | 1.0 ms | 65282 ms | 5029 ms | 86.1 ms | 13166 ms | 2012 ms |

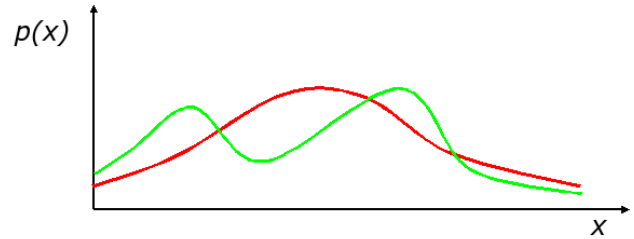
# Gentle introduction to probability theory

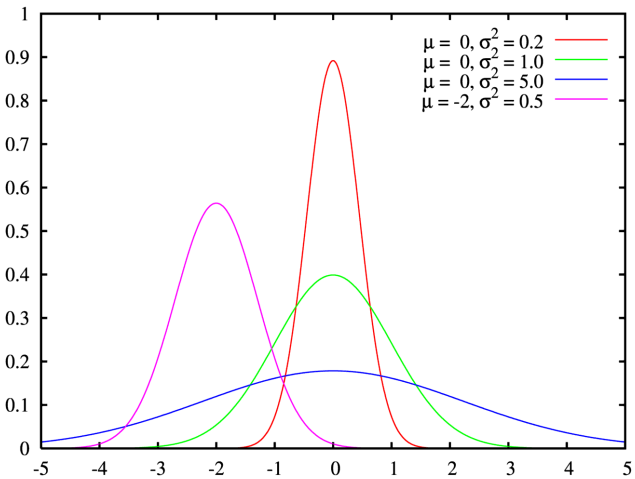
- Idea: explicit representation of uncertainty using calculus of the probability theory
- p(X=x) probability that the random variable $X$ is $x$
- $0 \leq p(x) \leq 1$
- $p(true) = 1$, $p(false) = 0$
- $p(A \vee B) = p(A) + p(B) - p(A \wedge B)$



# Discrete and continuous random variable

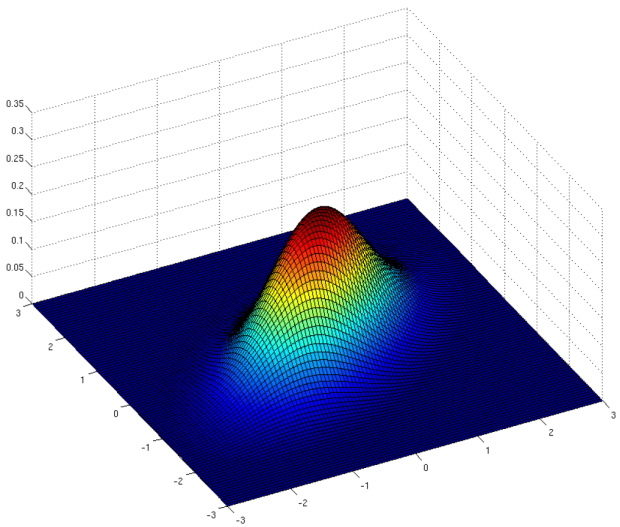
- **Discrete**: $X$ is countable, i.e. $X = x_1, x_2, \ldots, x_n$
- **Spojitá**: $X$ can have an uncountable number of values (from some interval)
- $p$ is probability density
- Various distributions
- Most known: Normal (Gaussian)
- $p(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$

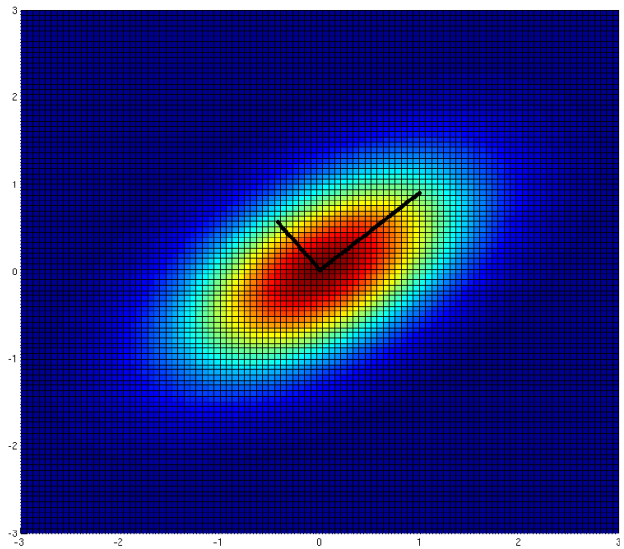


# Multi-dimensional normal distribution

$$p(\mathbf{x} = x_1, \ldots, x_k) = \frac{1}{\sqrt{(2\pi)^k |\Sigma|}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \Sigma^{-1} (\mathbf{x}-\boldsymbol{\mu})},$$





- Eigenvalues and eigenvectors of the covariance matrix define an ellipse.

# Joint and conditional probability distribution

- $p(X = x \text{ a } Y = y) = p(x, y)$
- If $X$ and $Y$ are independent then

$$p(x, y) = p(x)p(y)$$

- $p(x|y)$ is probability $x$ given $y$

$$p(x|y) = \frac{p(x, y)}{p(y)}$$

$$p(x, y) = p(x|y)p(y)$$

- If $X$ a $Y$ are independent then

$$p(x|y) = p(x)$$

# Total probability theorem

## Discrete case

$$\sum_x p(x) = 1$$

$$p(x) = \sum_y p(x, y)$$

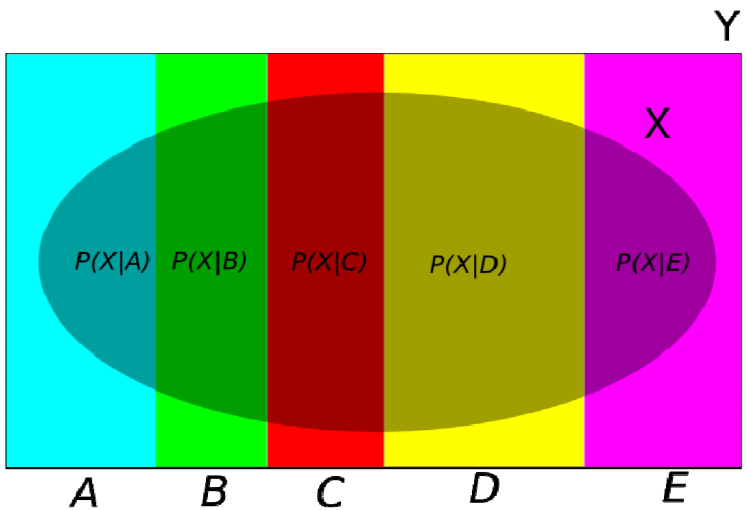
$$p(x) = \sum_y p(x|y)p(y)$$

## Continuous space

$$\int_x p(x)dx = 1$$

$$p(x) = \int_y p(x, y)dy$$

$$p(x) = \int_y p(x|y)p(y)dy$$



# Bayes' theorem

$$p(x, y) = p(x|y)p(y) = p(y|x)p(x)$$

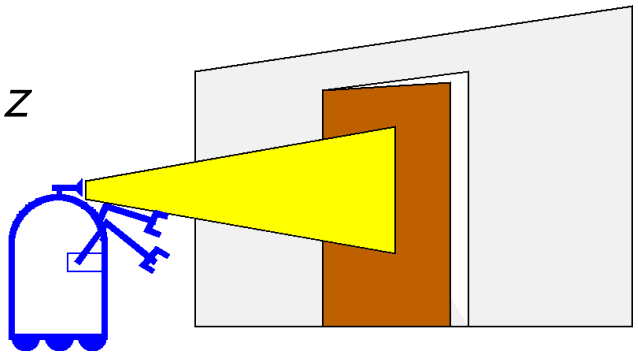
$$\Rightarrow$$

$$\boxed{p(x|y) = \frac{p(y|x)p(x)}{p(y)} = \frac{\mathit{likelihood} \cdot \mathit{prior}}{\mathit{evidence}}}$$

$$p(x|y) = \frac{p(y|x)p(x)}{p(y)} = \eta p(y|x)p(x)$$

$$\eta = p(y)^{-1} = \frac{1}{\sum_x p(y|x)p(x)}$$

# Simple example of state estimation



- Assume a robot obtains measurement $z$
- What is $p(open|z)$?
- $p(open|z)$ is diagnostic
- $p(z|open)$ is causal
- Often causal knowledge is easier to obtain (counting frequencies)
- Bayes rule allows us to use causal:

$$p(open|z) = \frac{p(z|open)p(open)}{p(z)}$$

# Example - open doors

- $p(z|open) = 0.6$ $p(z|\neg open) = 0.3$
- $p(open) = p(\neg) = 0.5$

$$p(open|z) = \frac{p(z|open)p(open)}{p(z|open)p(open) + p(z|\neg open)p(\neg open)}$$

$$p(open|z) = \frac{0.6 \cdot 0.5}{0.6 \cdot 0.5 + 0.3 \cdot 0.5} = \frac{2}{3} = 0.67$$

- $z$ raises probability that the door is open.

# Example - second measurement

- $p(z_2|open) = 0.5$ $p(z_2|\neg open) = 0.6$
- $p(open|z_1) = \frac{2}{3}$

$$p(open|z_2 z_1) = \frac{p(z_2|open)p(open|z_1)}{p(z_2|open)p(open|z_1) + p(z_1|\neg open)p(\neg open|z_1)}$$

$$= \frac{\frac{1}{2} \cdot \frac{2}{3}}{\frac{1}{2} \cdot \frac{2}{3} + \frac{3}{5} \cdot \frac{1}{3}} = \frac{5}{8} = 0.625$$

- $z_2$ lowers the probability that the door is open.

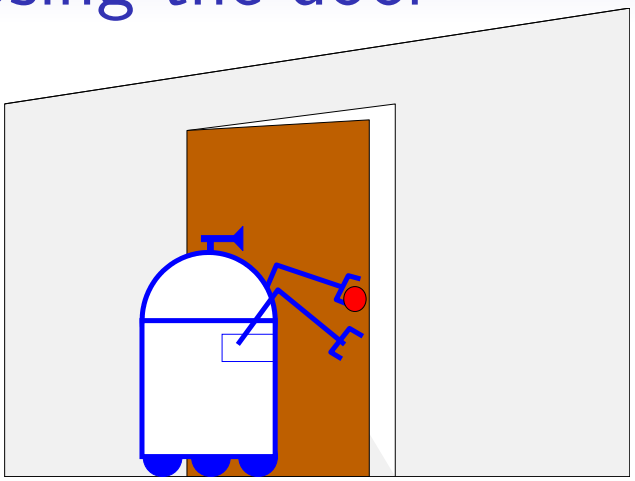
# Actions

- Often the world is dynamic since
  - actions carried out by the robot,
  - actions carried out by other agents,
  - or just the time passing by change the world (plants grow).
- Actions are never carried out with absolute certainty.
- In contrast to measurements, actions generally decrease the uncertainty.
- To incorporate the outcome of an action $u$ into the current "belief", we use the conditional pdf

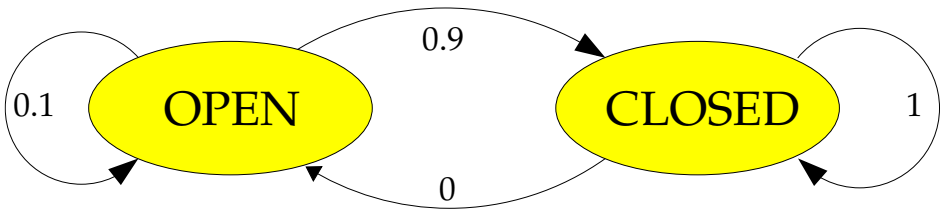
$$p(x|u, x')$$

- This term specifies the pdf that executing $u$ changes the state from $x'$ to $x$.

# Continuing the example - closing the door



$p(x|u, x')$ for $u =$ "close door"




$$p(x, u) = \sum_{x'} p(x|u, x')p(x')$$

If the door is open, the action "close door" succeeds in 90% of all cases.

# Continuing the example - closing the door

$$
\begin{aligned}
p(closed|u) &= \sum_{x'} p(closed|u,x')p(x') \\
&= p(closed|u,open)p(open) \\
&+ p(closed|u,closed)p(closed) \\
&= \frac{9}{10}\cdot\frac{5}{8} + \frac{1}{1}\cdot\frac{3}{8} = \frac{15}{16}
\end{aligned}
$$

$$
\begin{aligned}
p(open|u) &= \sum_{x'} p(open|u,x')p(x') \\
&= p(open|u,open)p(open) \\
&+ p(open|u,closed)p(closed) \\
&= \frac{1}{10}\cdot\frac{5}{8} + \frac{0}{1}\cdot\frac{3}{8} = \frac{1}{16} \\
&= 1 - p(closed|u)
\end{aligned}
$$