

Introduction to Visual Odometry

Karel Zimmermann

Czech Technical University in Prague

Faculty of Electrical Engineering, Department of Cybernetics

Center for Machine Perception

`http://cmp.felk.cvut.cz/~zimmerk, zimmerk@fel.cvut.cz`

Some images and codes taken from D.Scaramuzza

Localization sensors I

There are many localization-suitable sensors. Which pros, cons?

Localization sensors I

There are many localization-suitable sensors. Which pros, cons?

- ◆ **GPS:** Global Positioning System
 - Inaccurate.
 - Does not work indoor

Localization sensors I

There are many localization-suitable sensors. Which pros, cons?

- ◆ **GPS:** Global Positioning System
 - Inaccurate.
 - Does not work indoor
- ◆ **IMU:** Accelerometers+Gyroscope+Magnetometers.
 - Fuse all sensors data to get 6DOF (dead-reckoning).
 - Suffers from drift.

Localization sensors I

There are many localization-suitable sensors. Which pros, cons?

◆ **GPS:** Global Positioning System

- Inaccurate.
- Does not work indoor

◆ **IMU:** Accelerometers+Gyroscope+Magnetometers.

- Fuse all sensors data to get 6DOF (dead-reckoning).
- Suffers from drift.

◆ **Wi-fi:**

- Map Wi-Fi routers in advance, than guess the location based on IDs and signal strength.
- Does not work when routers are unavailable.
- Inaccurate.

Localization sensors II

- ◆ **Odometry:** Integrate wheel motion and steering via (kinematic) model.
 - Does not work on slippage terrain.
 - Drift+Inaccurate (show video !!!).

Localization sensors II

- ◆ **Odometry:** Integrate wheel motion and steering via (kinematic) model.
 - Does not work on slippage terrain.
 - Drift+Inaccurate (show video !!!).

- ◆ **What about camera?**
 - cheap+light
 - every mobile device has a camera.
 - high frame-rate.

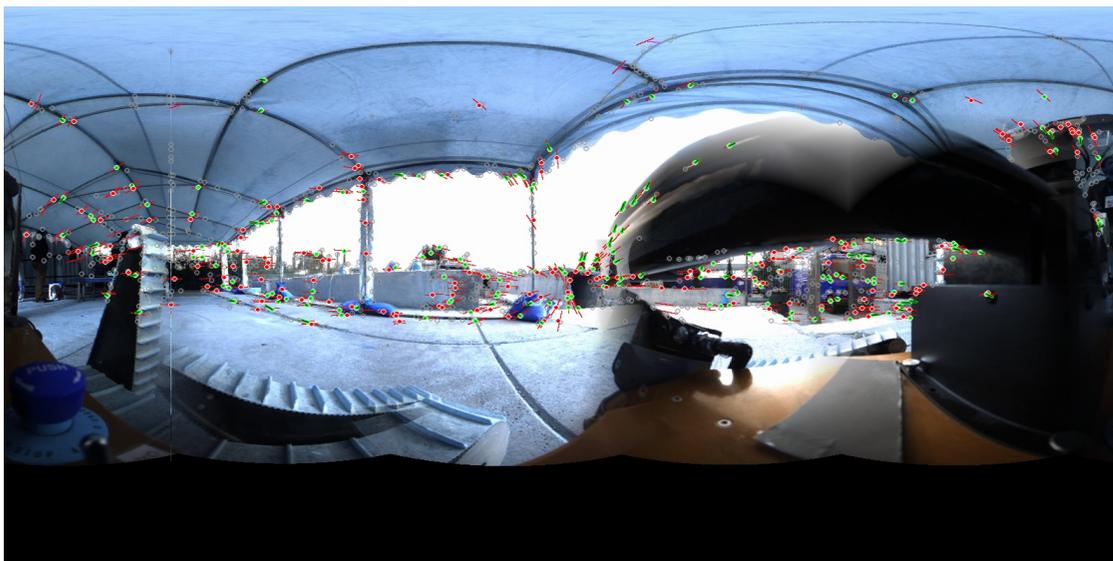
Localization sensors II

- ◆ **Odometry:** Integrate wheel motion and steering via (kinematic) model.
 - Does not work on slippage terrain.
 - Drift+Inaccurate (show video !!!).
- ◆ **What about camera?**
 - cheap+light
 - every mobile device has a camera.
 - high frame-rate.
- ◆ **What about depth sensors?** - show video !!!
 - stereo camera
 - structured light approach (e.g. Kinect)
 - time-of-flight approach (e.g. Velodyne)

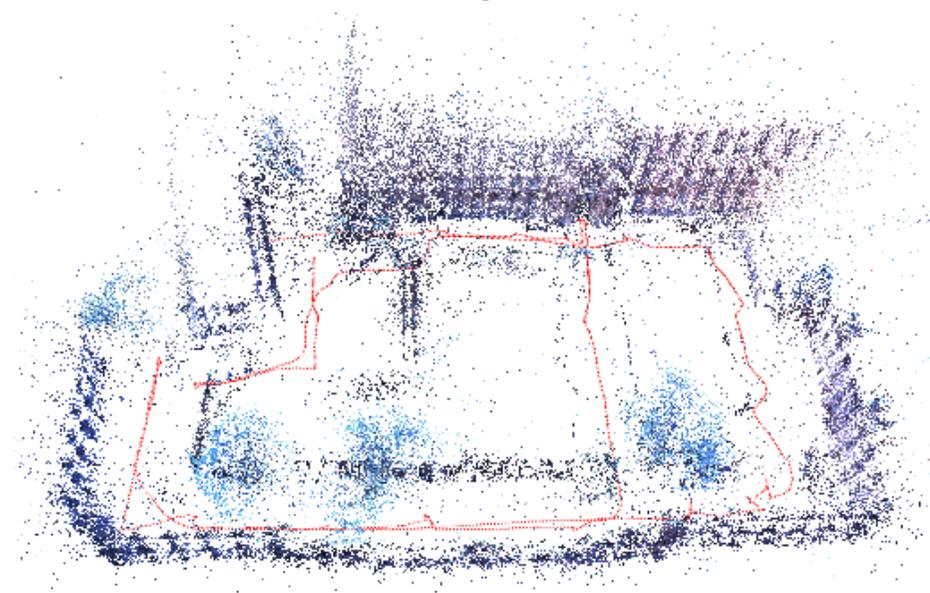
Task definition

- ◆ **Input:** Camera images.
- ◆ **Output:** Real-time robot location.

input

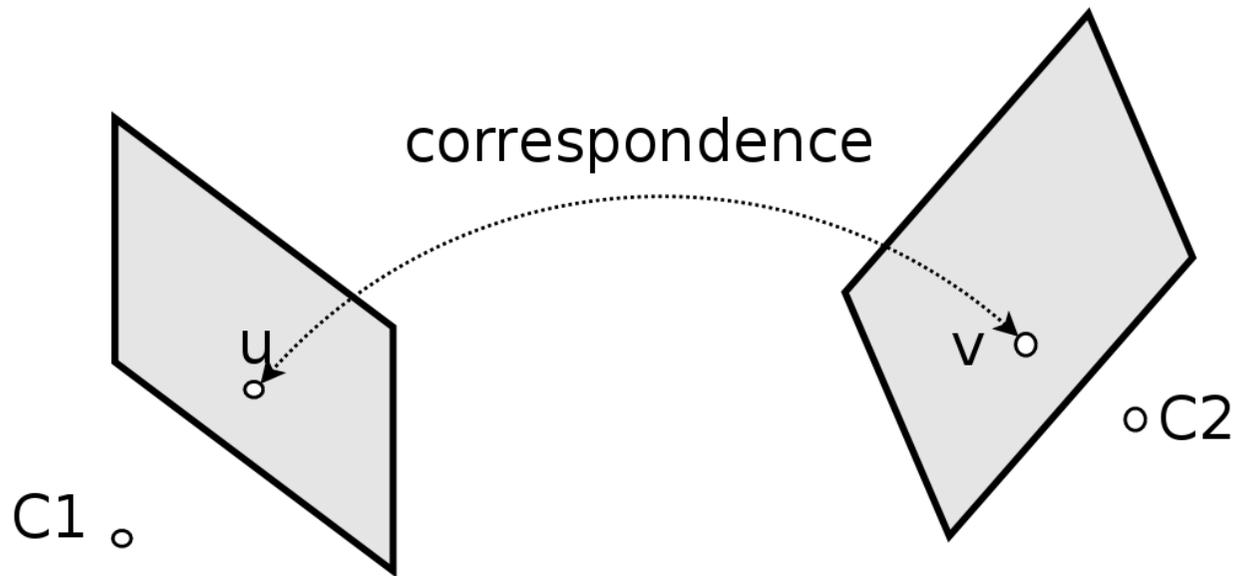


output



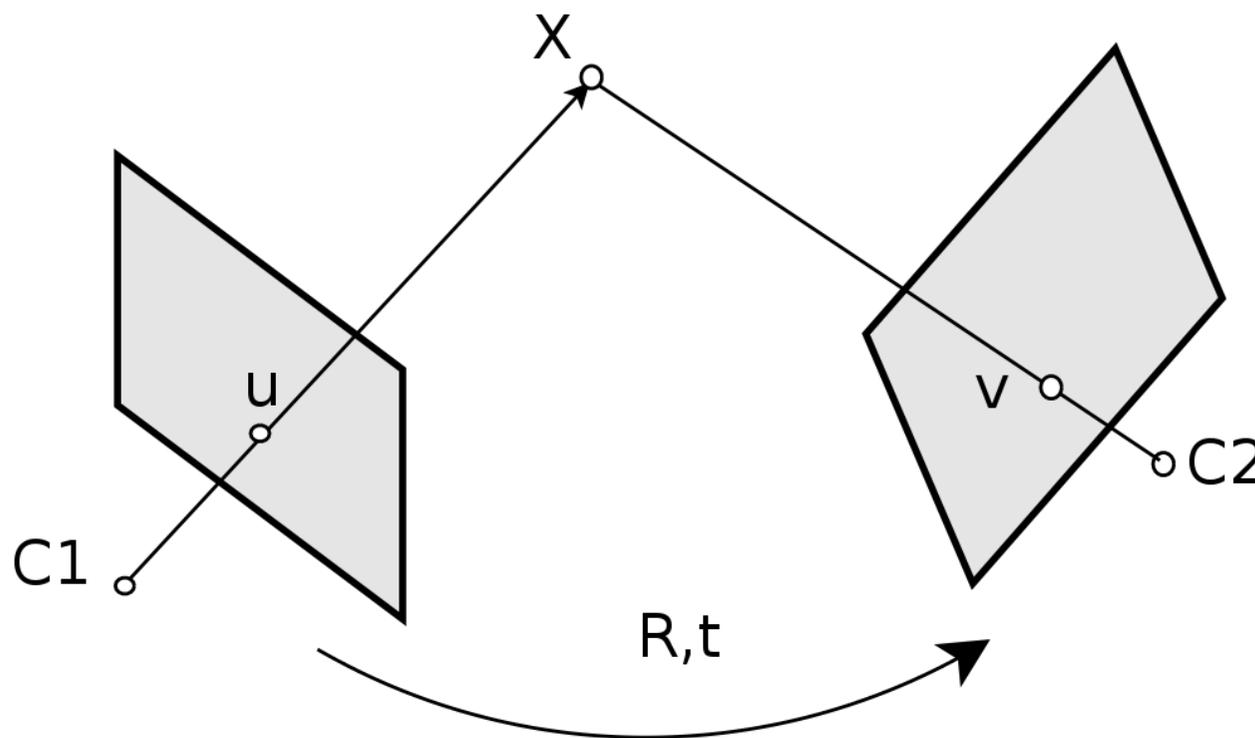
Main principle

- ◆ Find correspondences in two consecutive frames.



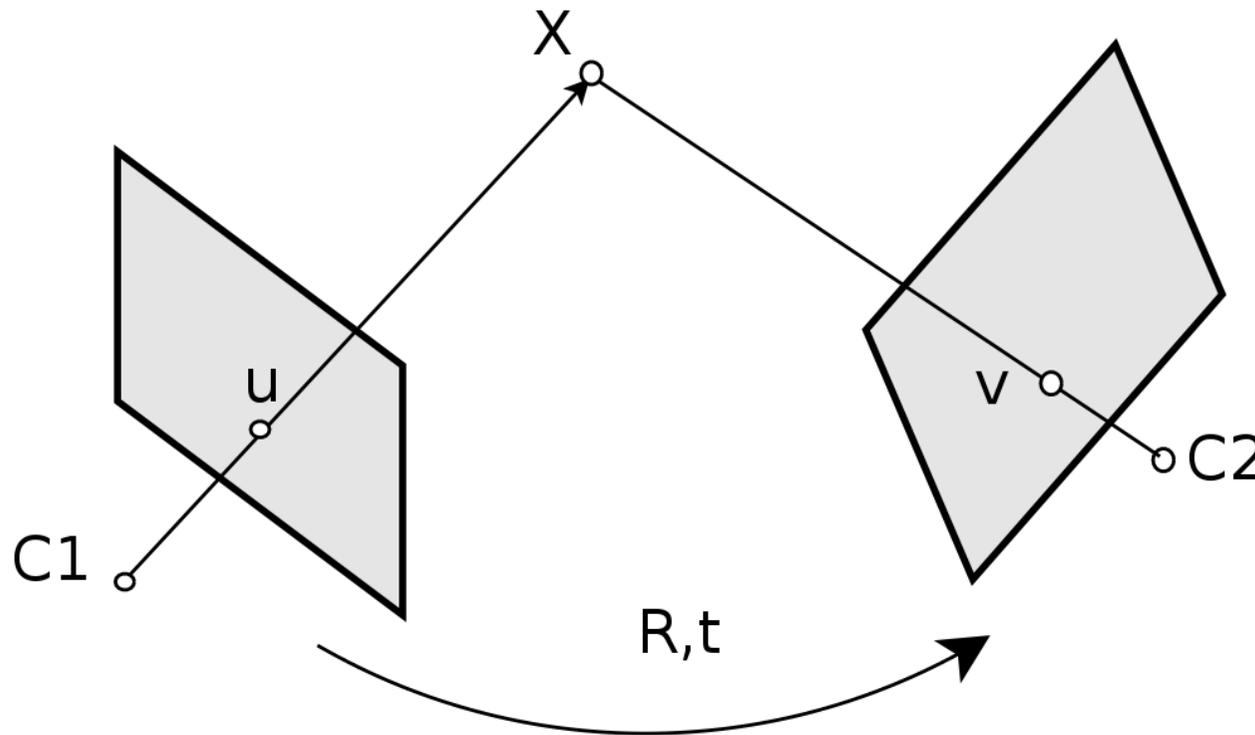
Main principle

- ◆ Find correspondences in two consecutive frames.
- ◆ If the world is **static** and correspondences are **correct** then estimate camera motion R and t (and 3D reconstruction X) minimizing reprojection error.



Main principle

- ◆ Find correspondences in two consecutive frames.
- ◆ If the world is **static** and correspondences are **correct** then estimate camera motion R and t (and 3D reconstruction X) minimizing reprojection error.



- ◆ Since the world is **dynamic** and most of the correspondences are **incorrect** we need robust method - show RANSAC presentation!!

Algorithm at glance

1. Get image I_k .
2. Compute correspondences between I_{k-1} and I_k (either feature matching or tracking).
3. Find correct correspondences and compute essential matrix \mathbf{E} .
4. Decompose \mathbf{E} into \mathbf{R}_k and \mathbf{t}_k .
5. Compute 3D model (points X).
6. Rescale \mathbf{t}_k according to relative scale r .
7. $k = k + 1$

Feature point detection

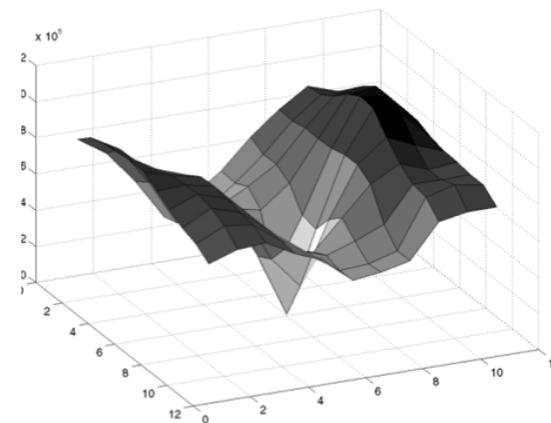
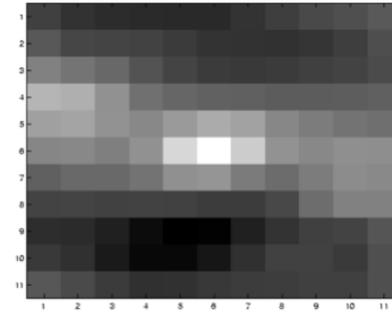
- ◆ Which points are suitable?



Feature point detection

- ◆ Feature points must be well distinguishable from its neighbourhood.

$$E(u, v) = \sum_{x,y} \left(I(x + u, y + v) - I(x, y) \right)^2 \approx [u \ v] \mathbf{M} \begin{bmatrix} u \\ v \end{bmatrix}$$

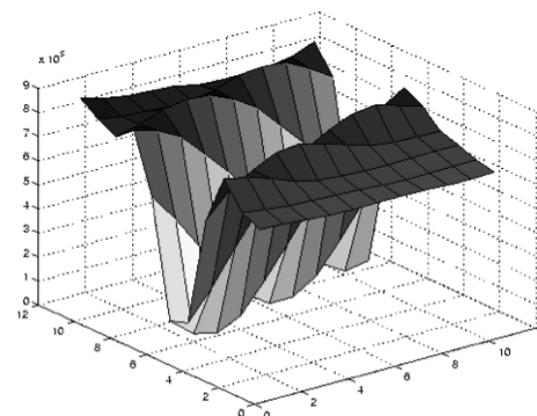
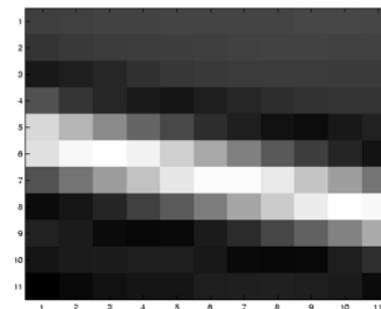


λ_1 and λ_2 are large

Feature point detection

- ◆ Feature points must be well distinguishable from its neighbourhood.

$$E(u, v) = \sum_{x,y} \left(I(x + u, y + v) - I(x, y) \right)^2 \approx [u \ v] M \begin{bmatrix} u \\ v \end{bmatrix}$$

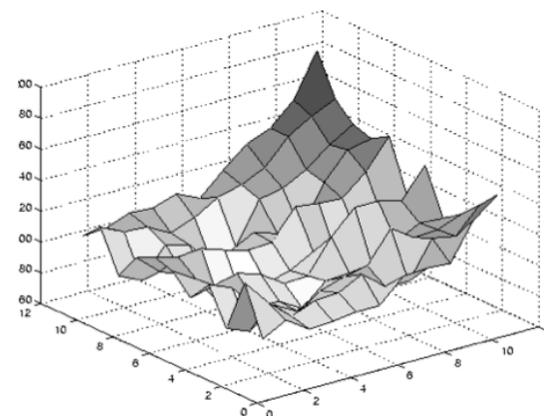
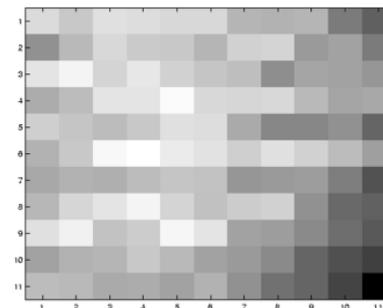


large λ_1 , small λ_2

Feature point detection

- ◆ Feature points must be well distinguishable from its neighbourhood.

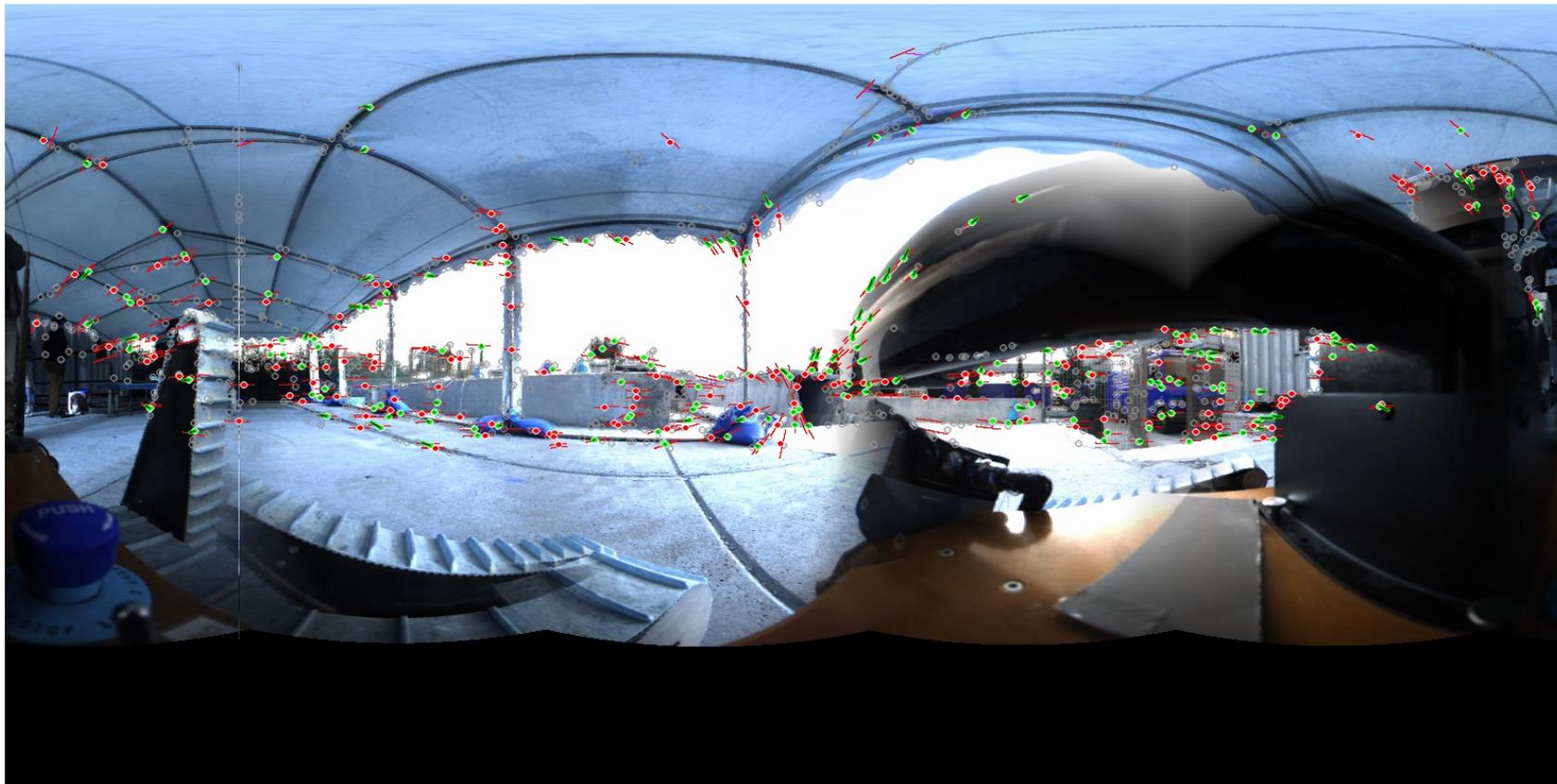
$$E(u, v) = \sum_{x,y} \left(I(x + u, y + v) - I(x, y) \right)^2 \approx [u \ v] \mathbf{M} \begin{bmatrix} u \\ v \end{bmatrix}$$



small λ_1 , small λ_2

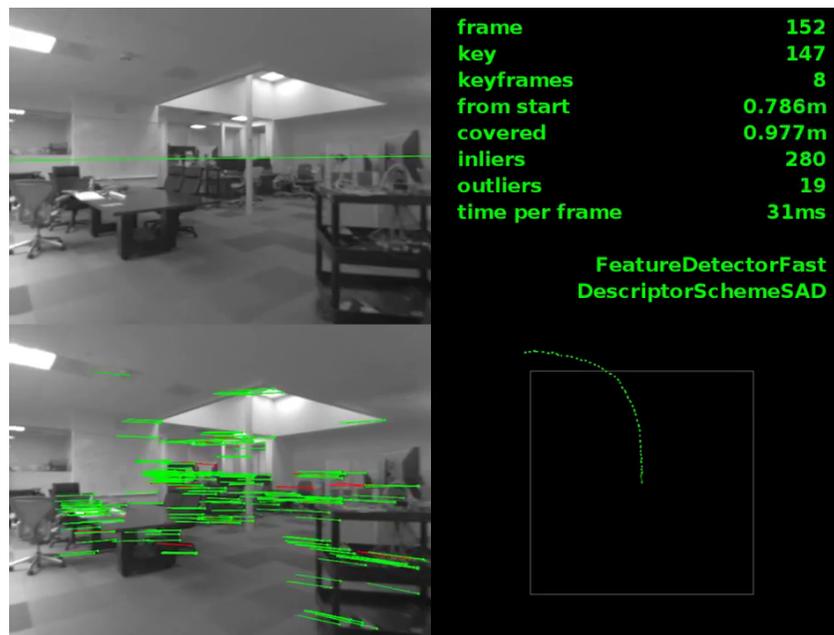
Feature point detection

- ◆ Detected feature points
- ◆ Show video `Vodom_video_Prato_roof.avi`



Estimating correspondences

- ◆ There are two ways:
 - Tracking - for high **temporal** resolution
OpenCV Lucas-Kanade tracker
 - Descriptor and matching - for high **spatial** resolution
OpenCV: SIFT, SURF etc ...
- ◆ show video vo_ros_PR2.avi



Algorithm at glance

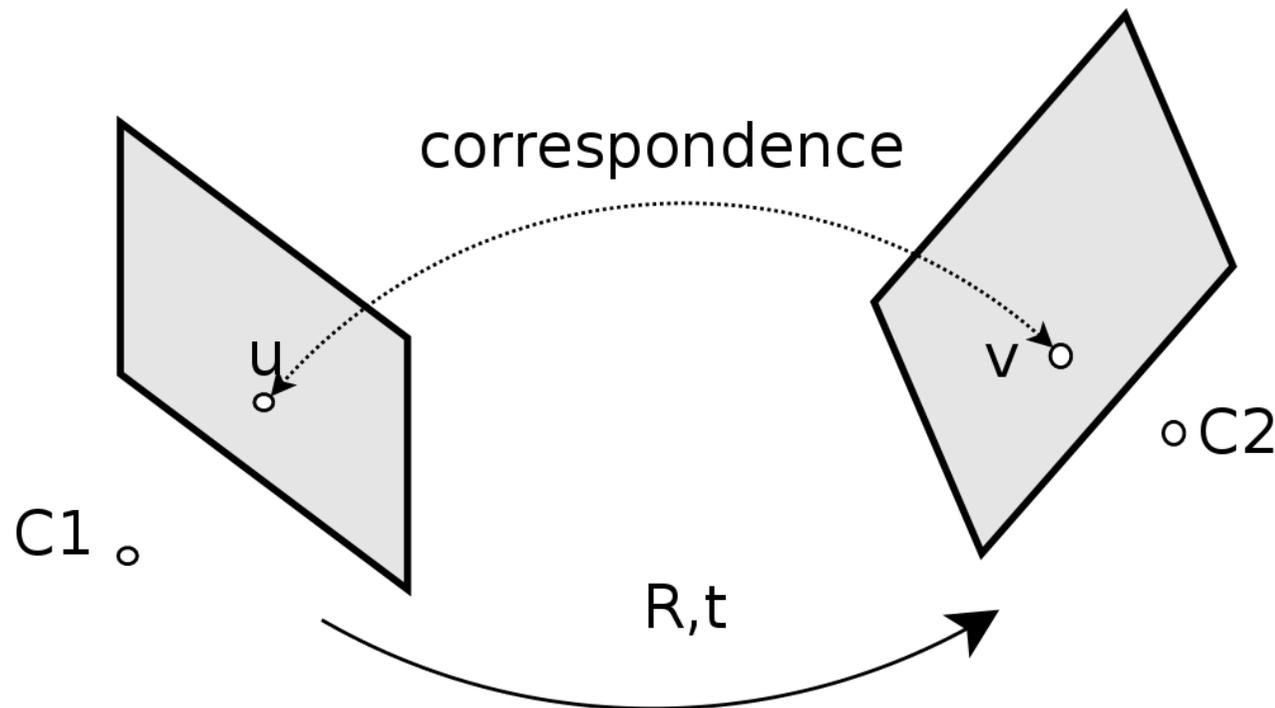
1. Get image I_k .
2. Compute correspondences between I_{k-1} and I_k (either feature matching or tracking).
3. Find correct correspondences and compute essential matrix \mathbf{E} .
4. Decompose \mathbf{E} into \mathbf{R}_k and \mathbf{t}_k .
5. Compute 3D model (points X).
6. Rescale \mathbf{t}_k according to relative scale r .
7. $k = k + 1$

Compute essential matrix

- ◆ Forget for a moment that we have already estimated correspondences.

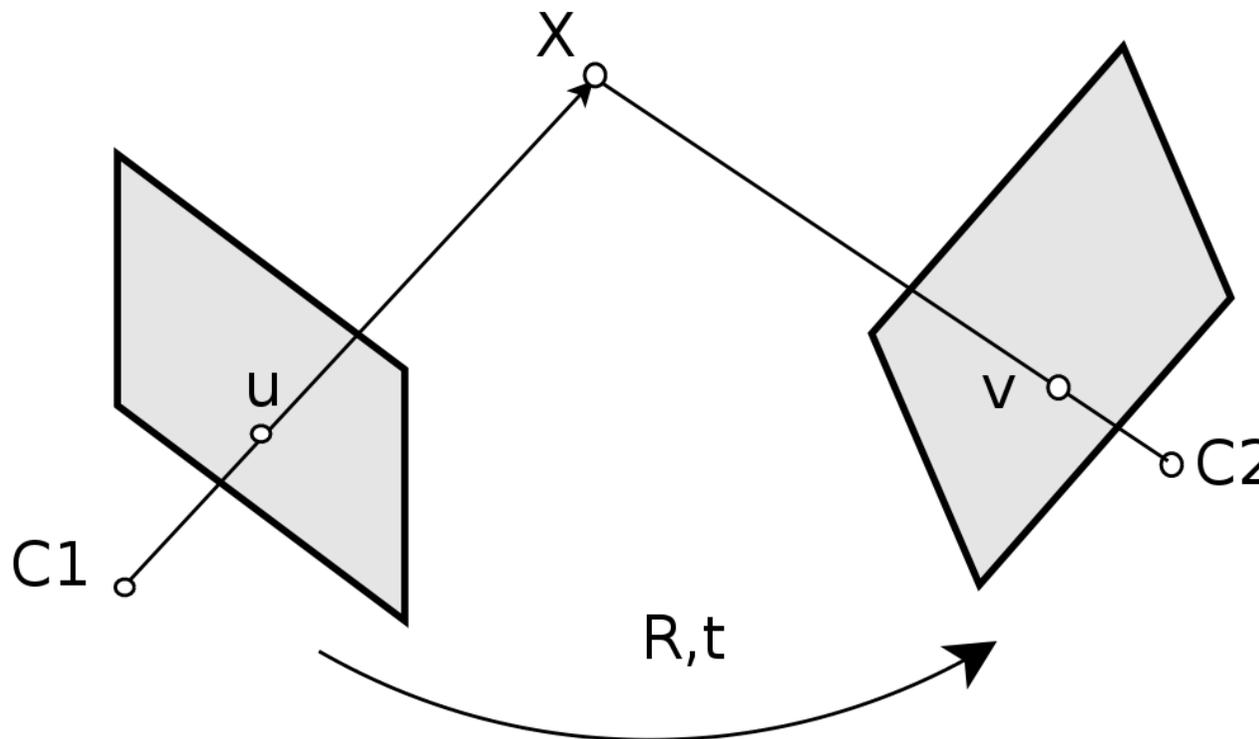
Compute essential matrix

- ◆ Forget for a moment that we have already estimated correspondences.
- ◆ Given cameras C_1, C_2 (including their rotations) and point u , where is corresponding point v ?



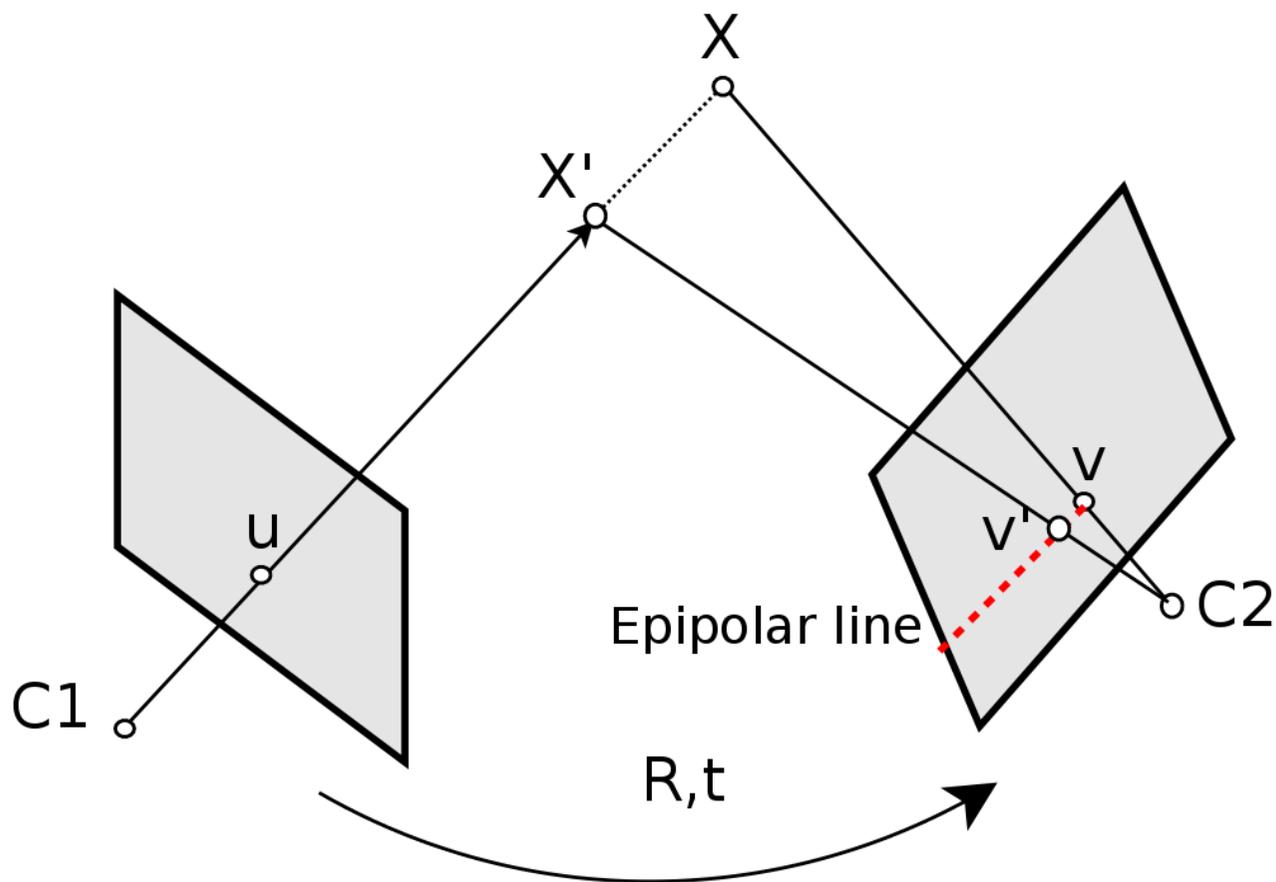
Compute essential matrix

- ◆ Forget for a moment that we have already estimated correspondences.
- ◆ Given cameras C_1, C_2 (including their rotations) and point u , where is corresponding point v ?
- ◆ That depends on 3D point X .



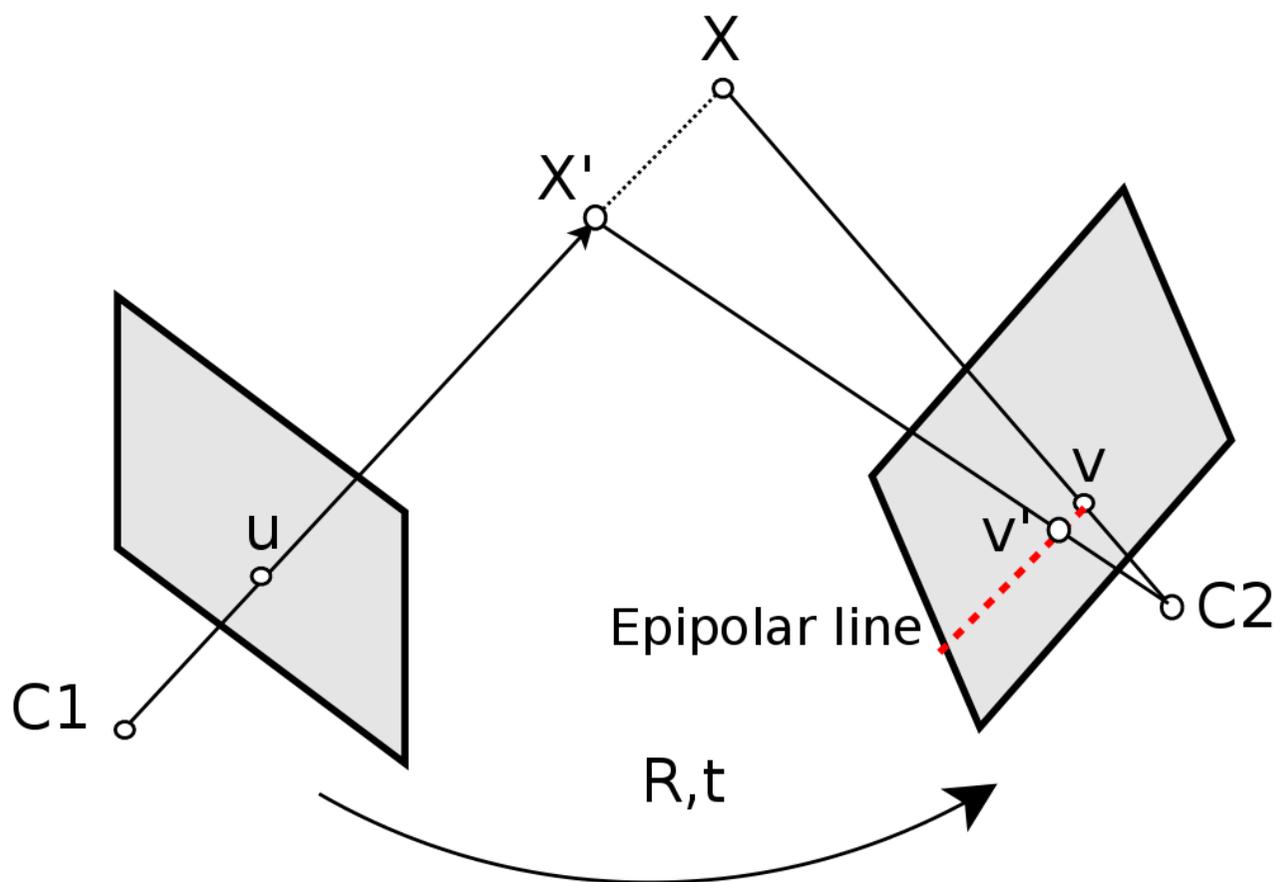
Compute essential matrix

- ◆ Forget for a moment that we have already estimated correspondences.
- ◆ Given cameras C_1, C_2 (including their rotations) and point u , where is corresponding point v ?
- ◆ That depends on 3D point X .



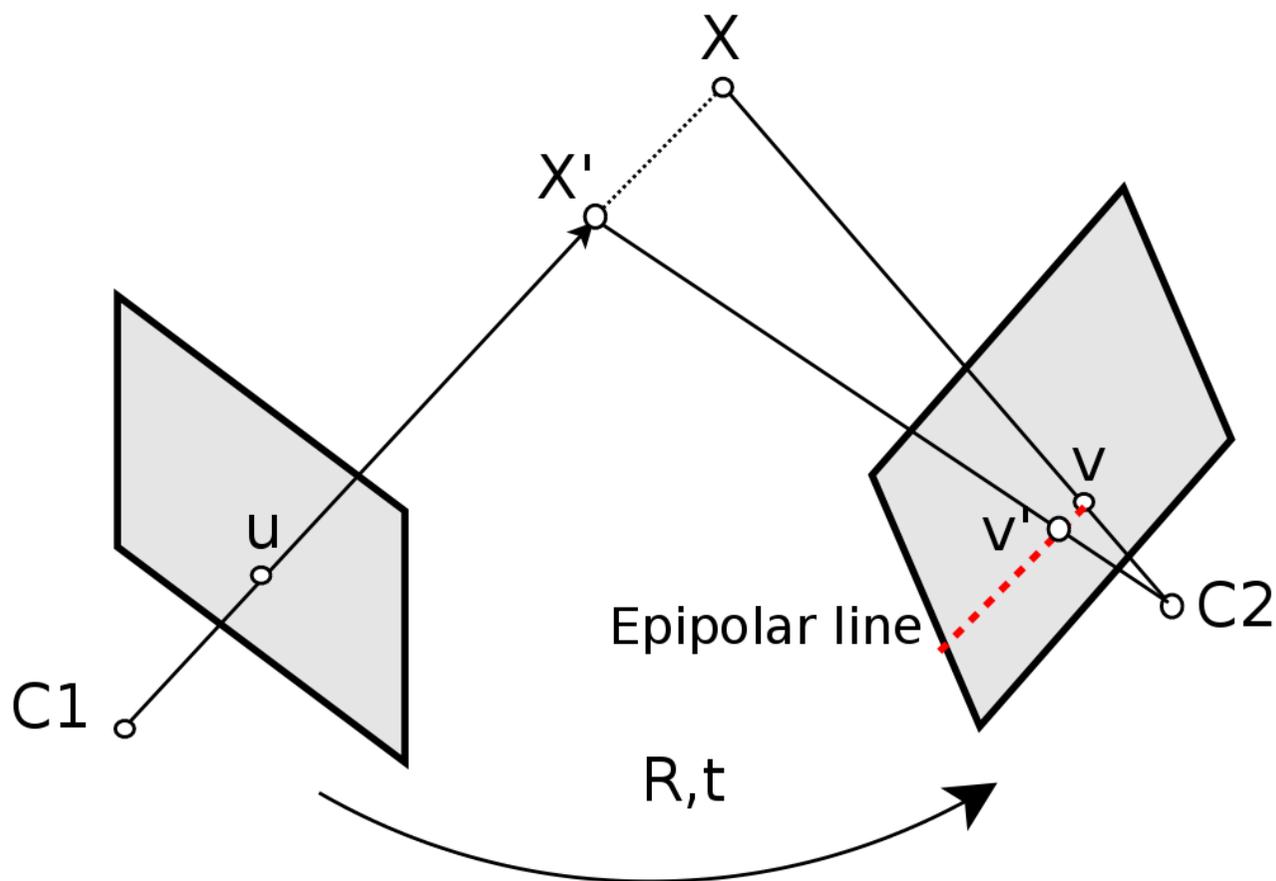
Compute essential matrix

- ◆ Forget for a moment that we have already estimated correspondences.
- ◆ Given cameras C_1, C_2 (including their rotations) and point u , where is corresponding point v ?
- ◆ That depends on 3D point X .
- ◆ All possible correspondences lie on epipolar line $\{v \mid u^T E v = 0\}$.



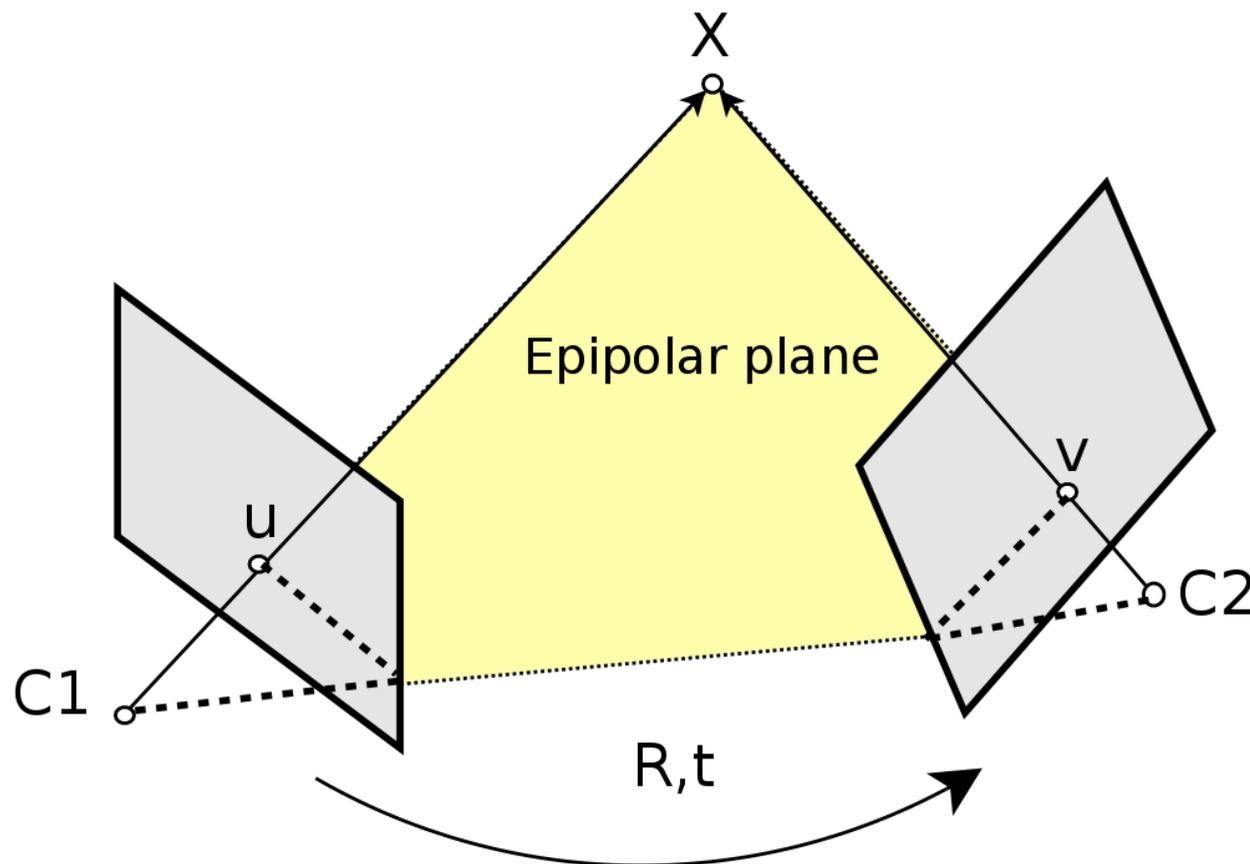
Compute essential matrix

- ◆ Forget for a moment that we have already estimated correspondences.
- ◆ Given cameras C_1, C_2 (including their rotations) and point u , where is corresponding point v ?
- ◆ That depends on 3D point X .
- ◆ All possible correspondences lie on epipolar line $\{v \mid u^T E v = 0\}$.



Compute essential matrix

- ◆ All possible correspondences lie on epipolar line $\{\mathbf{v} \mid \mathbf{u}^\top \mathbf{E} \mathbf{v} = 0\}$.
- ◆ Note:
 - This holds only if \mathbf{K} is identity matrix (or \mathbf{u}, \mathbf{v} are decalibrated).
 - Putting projection rays of all possible 3D points \mathbf{X} together yields epipolar plane (yellow).
 - There exist unique decomposition $\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R} \Rightarrow$ camera motion!



Compute essential matrix

- ◆ Let us assume that we have several correspondences.

Compute essential matrix

- ◆ Let us assume that we have several correspondences.
- ◆ Essential matrix \mathbf{E} is just a solution of (overdetermined) homogeneous system of linear equations.

Compute essential matrix

- ◆ Let us assume that we have several correspondences.
- ◆ Essential matrix \mathbf{E} is just a solution of (overdetermined) homogeneous system of linear equations.
- ◆ For each correspondence pair \mathbf{u}, \mathbf{v} , the following holds:

$$\mathbf{u}^\top \mathbf{E} \mathbf{v} = \mathbf{u}^\top \begin{bmatrix} \mathbf{e}_1^\top \\ \mathbf{e}_2^\top \\ \mathbf{e}_3^\top \end{bmatrix} \mathbf{v} = \mathbf{u}^\top \begin{bmatrix} \mathbf{e}_1^\top \mathbf{v} \\ \mathbf{e}_2^\top \mathbf{v} \\ \mathbf{e}_3^\top \mathbf{v} \end{bmatrix} = [u_1 \mathbf{e}_1^\top \mathbf{v} + u_2 \mathbf{e}_2^\top \mathbf{v} + u_3 \mathbf{e}_3^\top \mathbf{v}] =$$

Compute essential matrix

- ◆ Let us assume that we have several correspondences.
- ◆ Essential matrix \mathbf{E} is just a solution of (overdetermined) homogeneous system of linear equations.

- ◆ For each correspondence pair \mathbf{u}, \mathbf{v} , the following holds:

$$\begin{aligned} \mathbf{u}^\top \mathbf{E} \mathbf{v} &= \mathbf{u}^\top \begin{bmatrix} \mathbf{e}_1^\top \\ \mathbf{e}_2^\top \\ \mathbf{e}_3^\top \end{bmatrix} \mathbf{v} = \mathbf{u}^\top \begin{bmatrix} \mathbf{e}_1^\top \mathbf{v} \\ \mathbf{e}_2^\top \mathbf{v} \\ \mathbf{e}_3^\top \mathbf{v} \end{bmatrix} = [u_1 \mathbf{e}_1^\top \mathbf{v} + u_2 \mathbf{e}_2^\top \mathbf{v} + u_3 \mathbf{e}_3^\top \mathbf{v}] = \\ &= [u_1 \mathbf{v}^\top \ u_2 \mathbf{v}^\top \ u_3 \mathbf{v}^\top] \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{bmatrix} = 0 \end{aligned}$$

- ◆ It must hold for all correspondence pairs $\mathbf{u}_i, \mathbf{v}_i$, therefore:

$$\begin{bmatrix} u_{11} \mathbf{v}_1^\top & u_{12} \mathbf{v}_1^\top & u_{13} \mathbf{v}_1^\top \\ u_{21} \mathbf{v}_2^\top & u_{22} \mathbf{v}_2^\top & u_{23} \mathbf{v}_2^\top \\ \vdots & & \end{bmatrix} \begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{bmatrix} = \mathbf{0}$$

Compute essential matrix

- ◆ It is just homogeneous set of linear equations:

$$\underbrace{\begin{bmatrix} u_{11}\mathbf{v}_1^\top & u_{12}\mathbf{v}_1^\top & u_{13}\mathbf{v}_1^\top \\ u_{21}\mathbf{v}_2^\top & u_{22}\mathbf{v}_2^\top & u_{23}\mathbf{v}_2^\top \\ \vdots & & \end{bmatrix}}_A \underbrace{\begin{bmatrix} \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{bmatrix}}_e = \mathbf{0}$$

- ◆ Again we want to avoid trivial solution $\mathbf{e}_1 = \mathbf{e}_2 = \mathbf{e}_3 = \mathbf{0}$.
- ◆ We solve the following optimization task (constrained LSQ)

$$\arg \min_e \|\mathbf{A}\mathbf{e}\| \quad \text{subject to} \quad \|\mathbf{e}\| = 1$$

- ◆ the solution is singular vector of matrix \mathbf{A} corresponding to the smallest singular value (can be found via SVD or eigenvectors/eigenvalues of $\mathbf{A}\mathbf{A}^\top$)

Compute essential matrix

- ◆ Since L2 is sensitive to outliers, RANSAC is used to find inliers (i.e. correct correspondences).
- ◆ Since **algebraic error** is minimized (instead of geometric error), coordinates have to be normalized.

Algorithm at glance

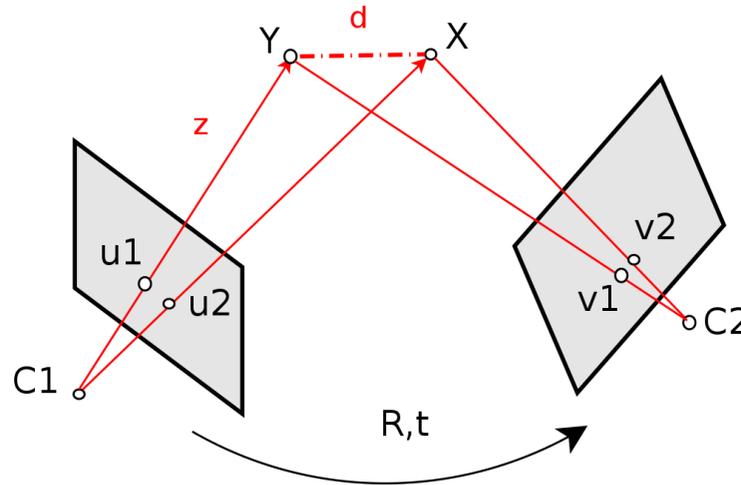
1. Get image I_k .
2. Compute correspondences between I_{k-1} and I_k (either feature matching or tracking).
3. Find correct correspondences and compute essential matrix \mathbf{E} .
4. Decompose \mathbf{E} into \mathbf{R}_k and \mathbf{t}_k .
5. Compute 3D model (points X).
6. Rescale \mathbf{t}_k according to relative scale r .
7. $k = k + 1$

Decompose E into R and t

- ◆ Once you find E , you can estimate camera motion by decomposing $E = [t]_{\times} R$ via
- ◆ SVD ($E = U\Sigma V^T$) as follows: $[t]_{\times} = VW\Sigma V^T$, $R = UW^{-1}V^T$, **but !!!:**

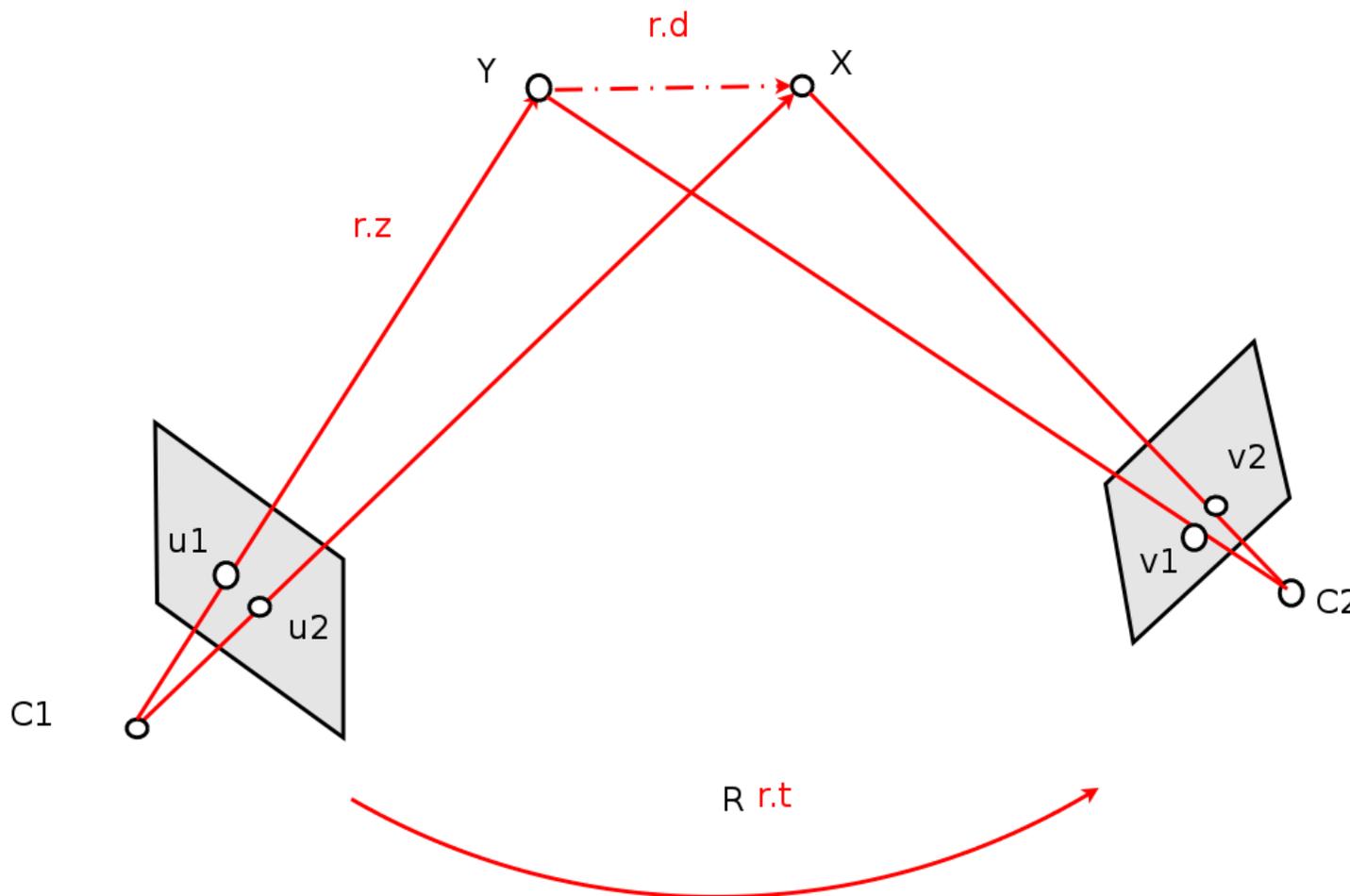
Decompose E into R and t

- ◆ Once you find E , you can estimate camera motion by decomposing $E = [t]_{\times} R$ via
- ◆ SVD ($E = U\Sigma V^T$) as follows: $[t]_{\times} = VW\Sigma V^T$, $R = UW^{-1}V^T$, **but !!!**:
 - Scale is unknown (if something is equal to zero, than any scalar multiplication of it is equal to zero as well).



Estimating camera motion

- ◆ Once you find \mathbf{E} , you can estimate camera motion by decomposing $\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$ via
- ◆ SVD ($\mathbf{E} = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T$) as follows: $[\mathbf{t}]_{\times} = \mathbf{V} \mathbf{W} \mathbf{\Sigma} \mathbf{V}^T$, $\mathbf{R} = \mathbf{U} \mathbf{W}^{-1} \mathbf{V}^T$, **but !!!**:
 - Scale is unknown (if something is equal to zero, then any scalar multiplication of it is equal to zero as well).



Estimating camera motion

- ◆ Once you find \mathbf{E} , you can estimate camera motion by decomposing $\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$ via
- ◆ SVD ($\mathbf{E} = \mathbf{U}\Sigma\mathbf{V}^{\top}$) as follows: $[\mathbf{t}]_{\times} = \mathbf{V}\mathbf{W}\Sigma\mathbf{V}^{\top}$, $\mathbf{R} = \mathbf{U}\mathbf{W}^{-1}\mathbf{V}^{\top}$, **but !!!**:
 - Scale is unknown (if something is equal to zero, then any scalar multiplication of it is equal to zero as well).
 - We search for 8 unknowns ($\dim(\mathbf{e}) = 9$ minus scale) \Rightarrow at least 8 correspondences needed \Rightarrow 8-point algorithm.

Estimating camera motion

- ◆ Once you find \mathbf{E} , you can estimate camera motion by decomposing $\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$ via
- ◆ SVD ($\mathbf{E} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\top}$) as follows: $[\mathbf{t}]_{\times} = \mathbf{V}\mathbf{W}\mathbf{\Sigma}\mathbf{V}^{\top}$, $\mathbf{R} = \mathbf{U}\mathbf{W}^{-1}\mathbf{V}^{\top}$, **but !!!**:
 - Scale is unknown (if something is equal to zero, then any scalar multiplication of it is equal to zero as well).
 - We search for 8 unknowns ($\dim(\mathbf{e}) = 9$ minus scale) \Rightarrow at least 8 correspondences needed \Rightarrow 8-point algorithm.
 - However you want to find only camera translation (3 DoFs) and rotation (3 DoFs) minus scale \Rightarrow 5-point algorithm [Nister 2003].

Estimating camera motion

- ◆ Once you find \mathbf{E} , you can estimate camera motion by decomposing $\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$ via
- ◆ SVD ($\mathbf{E} = \mathbf{U}\Sigma\mathbf{V}^{\top}$) as follows: $[\mathbf{t}]_{\times} = \mathbf{V}\mathbf{W}\Sigma\mathbf{V}^{\top}$, $\mathbf{R} = \mathbf{U}\mathbf{W}^{-1}\mathbf{V}^{\top}$, **but !!!**:
 - Scale is unknown (if something is equal to zero, then any scalar multiplication of it is equal to zero as well).
 - We search for 8 unknowns ($\dim(\mathbf{e}) = 9$ minus scale) \Rightarrow at least 8 correspondences needed \Rightarrow 8-point algorithm.
 - However you want to find only camera translation (3 DoFs) and rotation (3 DoFs) minus scale \Rightarrow 5-point algorithm [Nister 2003].
 - Why is it such a big deal???

Estimating camera motion

- ◆ Once you find \mathbf{E} , you can estimate camera motion by decomposing $\mathbf{E} = [\mathbf{t}]_{\times} \mathbf{R}$ via
- ◆ SVD ($\mathbf{E} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^{\top}$) as follows: $[\mathbf{t}]_{\times} = \mathbf{V}\mathbf{W}\mathbf{\Sigma}\mathbf{V}^{\top}$, $\mathbf{R} = \mathbf{U}\mathbf{W}^{-1}\mathbf{V}^{\top}$, **but !!!**:
 - Scale is unknown (if something is equal to zero, then any scalar multiplication of it is equal to zero as well).
 - We search for 8 unknowns ($\dim(\mathbf{e}) = 9$ minus scale) \Rightarrow at least 8 correspondences needed \Rightarrow 8-point algorithm.
 - However you want to find only camera translation (3 DoFs) and rotation (3 DoFs) minus scale \Rightarrow 5-point algorithm [Nister 2003].
 - Why is it such a big deal???
 - Because, usually you have 80% outliers and you need to use RANSAC (explain in the end of the lecture).

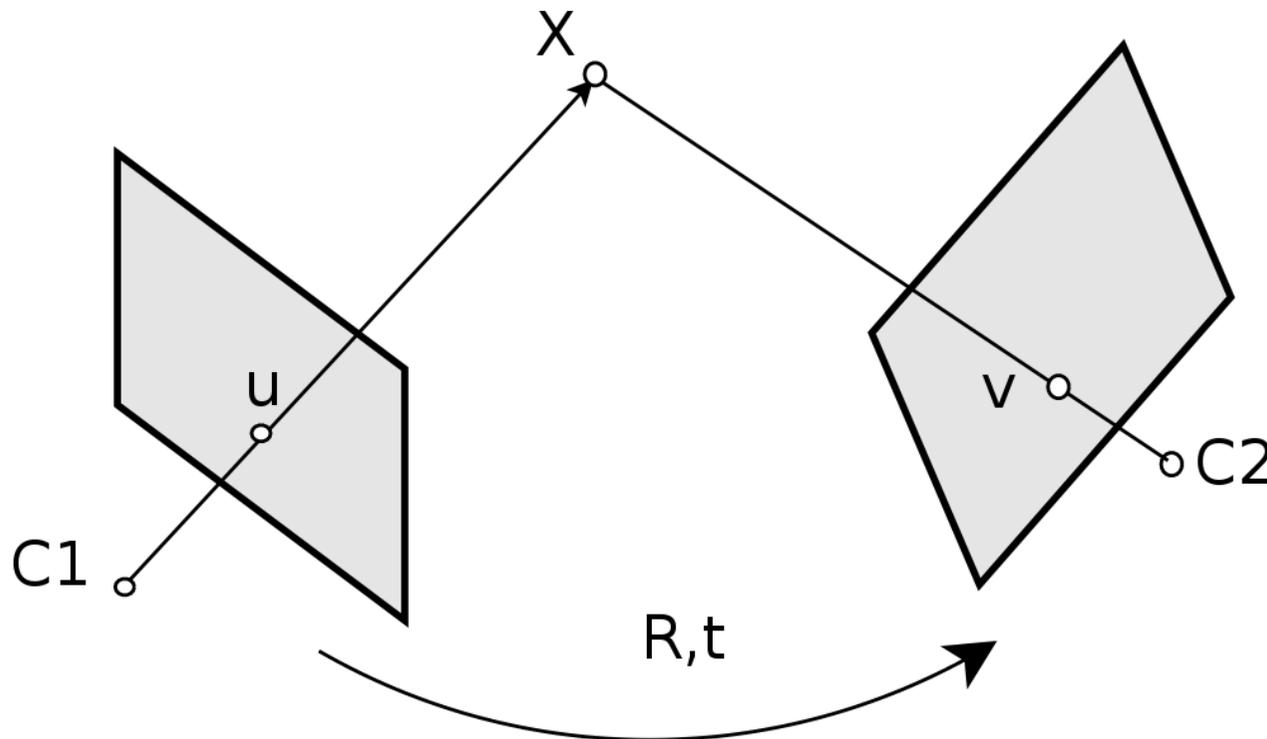
Algorithm at glance

1. Get image I_k .
2. Compute correspondences between I_{k-1} and I_k (either feature matching or tracking).
3. Find correct correspondences and compute essential matrix \mathbf{E} .
4. Decompose \mathbf{E} into \mathbf{R}_k and \mathbf{t}_k .
5. Compute 3D model (points X).
6. Rescale \mathbf{t}_k according to relative scale r .
7. $k = k + 1$

Compute 3D model

- ◆ Scene point X is observed by two cameras P and Q .
- ◆ Let $\mathbf{u} = [u_1 \ u_2]^\top$ and $\mathbf{v} = [v_1 \ v_2]^\top$ are projections of X in P and Q .
- ◆ Then

$$u_1 = \frac{\mathbf{p}_1^\top \mathbf{X}}{\mathbf{p}_3^\top \mathbf{X}} \Rightarrow u_1 \mathbf{p}_3^\top \mathbf{X} - \mathbf{p}_1^\top \mathbf{X} = 0$$



Compute 3D model

- ◆ Scene point X is observed by two cameras P and Q .
- ◆ Let $\mathbf{u} = [u_1 \ u_2]^\top$ and $\mathbf{v} = [v_1 \ v_2]^\top$ are projections of X in P and Q .
- ◆ Then

$$u_1 = \frac{\mathbf{p}_1^\top \mathbf{X}}{\mathbf{p}_3^\top \mathbf{X}} \Rightarrow u_1 \mathbf{p}_3^\top \mathbf{X} - \mathbf{p}_1^\top \mathbf{X} = 0$$

- ◆ and similarly ...

$$u_2 = \frac{\mathbf{p}_2^\top \mathbf{X}}{\mathbf{p}_3^\top \mathbf{X}} \Rightarrow u_2 \mathbf{p}_3^\top \mathbf{X} - \mathbf{p}_2^\top \mathbf{X} = 0$$

$$v_1 = \frac{\mathbf{q}_1^\top \mathbf{X}}{\mathbf{q}_3^\top \mathbf{X}} \Rightarrow v_1 \mathbf{q}_3^\top \mathbf{X} - \mathbf{q}_1^\top \mathbf{X} = 0$$

$$v_2 = \frac{\mathbf{q}_2^\top \mathbf{X}}{\mathbf{q}_3^\top \mathbf{X}} \Rightarrow v_2 \mathbf{q}_3^\top \mathbf{X} - \mathbf{q}_2^\top \mathbf{X} = 0$$

Compute 3D model

- ◆ Which is 4×4 homogeneous system of linear equations:

$$\begin{bmatrix} u_1 \mathbf{p}_3^\top - \mathbf{p}_1^\top \\ u_2 \mathbf{p}_3^\top - \mathbf{p}_2^\top \\ v_1 \mathbf{q}_3^\top - \mathbf{q}_1^\top \\ v_2 \mathbf{q}_3^\top - \mathbf{q}_2^\top \end{bmatrix} \mathbf{X} = \mathbf{0}$$

Compute 3D model

- ◆ Which is 4×4 homogeneous system of linear equations:

$$\begin{bmatrix} u_1 \mathbf{p}_3^\top & - \mathbf{p}_1^\top \\ u_2 \mathbf{p}_3^\top & - \mathbf{p}_2^\top \\ v_1 \mathbf{q}_3^\top & - \mathbf{q}_1^\top \\ v_2 \mathbf{q}_3^\top & - \mathbf{q}_2^\top \end{bmatrix} \mathbf{X} = \mathbf{0}$$

- ◆ Algebraic error is minimized (it often yields small geometric error).
- ◆ To obtain a better 3D model, the reprojection (geometric) error is locally minimized by Levenberg-Marquardt method:

$$\arg \min_{\mathbf{P}^i, \mathbf{X}_j} \sum_{i,j} d(\mathbf{P}^i \mathbf{X}_j, \mathbf{u}_j^i)^2$$

- ◆ It is often called the bundle adjustment.

Algorithm at glance

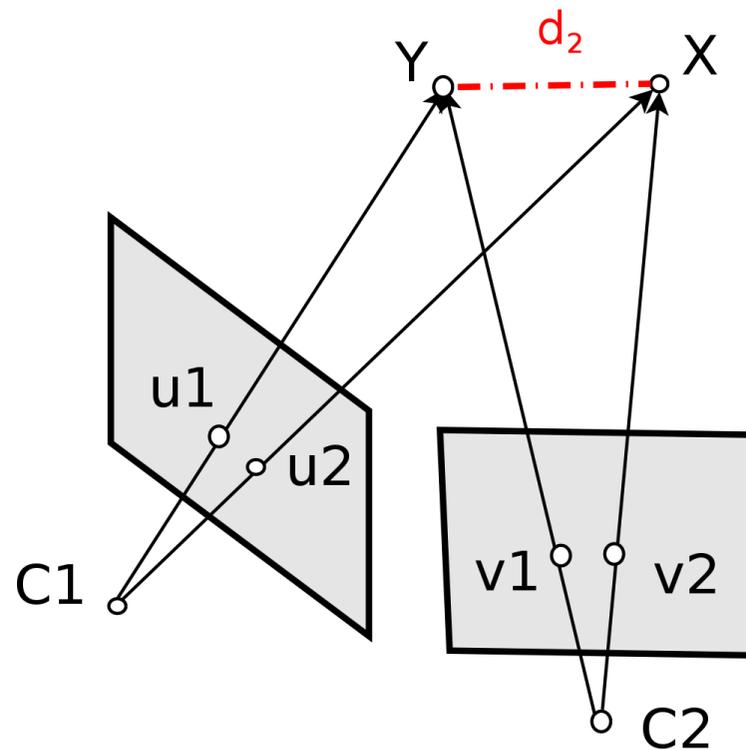
1. Get image I_k .
2. Compute correspondences between I_{k-1} and I_k (either feature matching or tracking).
3. Find correct correspondences and compute essential matrix \mathbf{E} .
4. Decompose \mathbf{E} into \mathbf{R}_k and \mathbf{t}_k .
5. Compute 3D model (points X).
6. Rescale t_k according to relative scale r .
7. $k = k + 1$

Estimating camera motion - relative scale

1. You cannot get absolute scale (without calibration object).

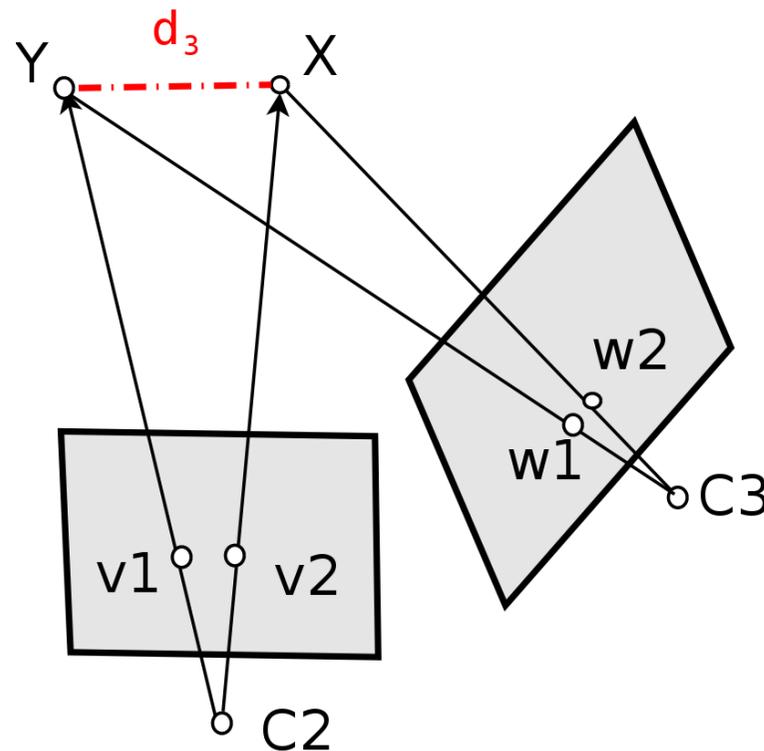
Estimating camera motion - relative scale

1. You cannot get absolute scale (without calibration object).
2. If you estimate motion (and 3D model) from C_1, C_2



Estimating camera motion - relative scale

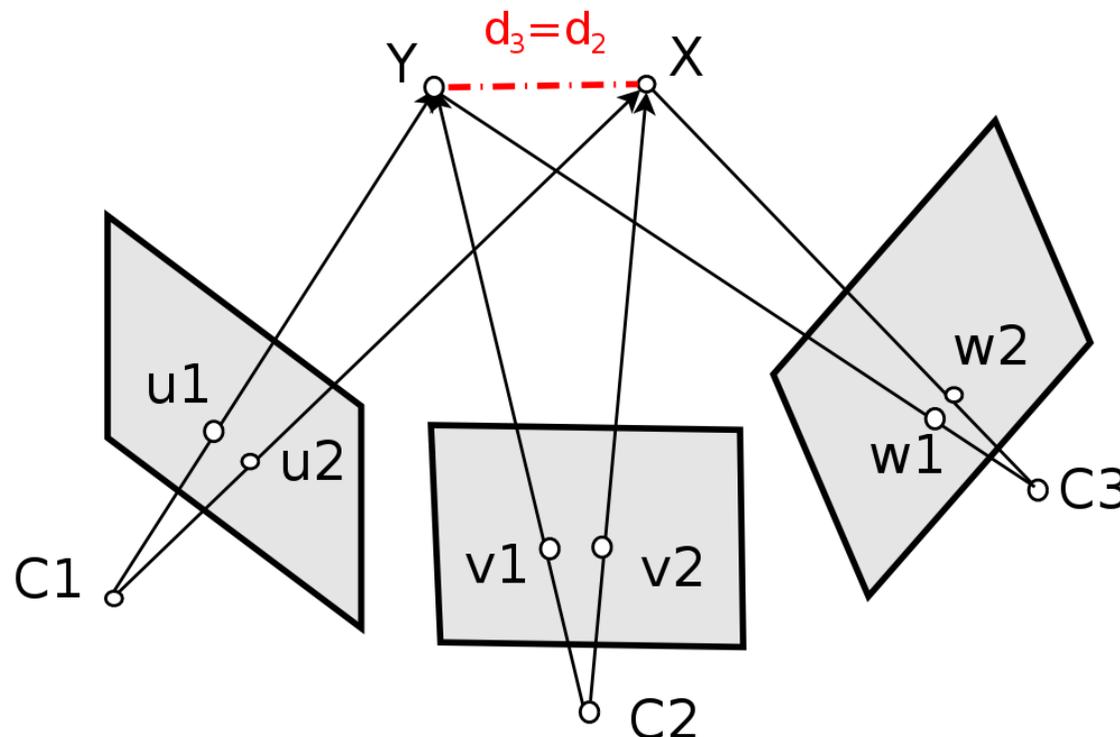
1. You cannot get absolute scale (without calibration object).
2. If you estimate motion (and 3D model) from C_1, C_2 and than from C_2, C_3 you can have completely different scale.



Estimating camera motion - relative scale

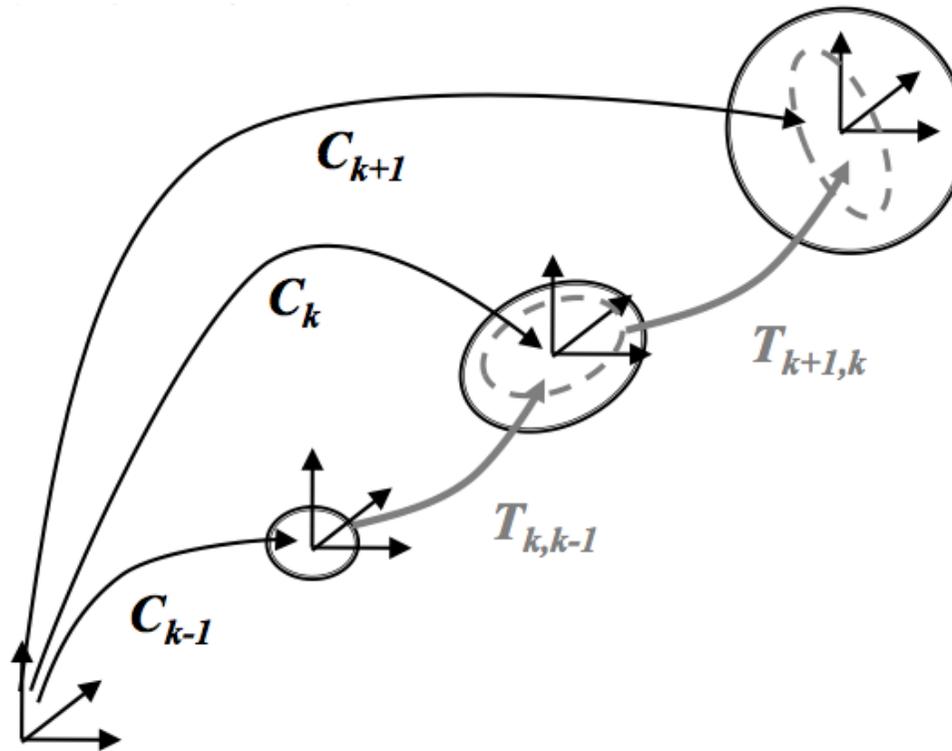
1. You cannot get absolute scale (without calibration object).
2. If you estimate motion (and 3D model) from C_1, C_2 and than from C_2, C_3 you can have completely different scale.
3. You want to keep the same relative scale r by rescaling t (and 3D)

$$r = \frac{d_k}{d_{k-1}} = \frac{\|X_k - Y_k\|}{\|X_{k-1} - Y_{k-1}\|}$$

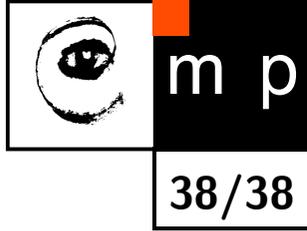


Drift

- ◆ Error accumulates over time \Rightarrow drift \Rightarrow loop-closure needed.
- ◆ Keyframe detection (avoid motion estimation for small motion or pure rotation) - show video `vo_ros_PR2.avi`

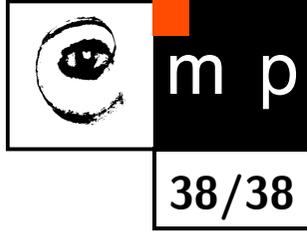


Visual Odometry (VO), Structure from Motion (SFM), Visual Simultaneous Localisation and Mapping (VSLAM)



- ◆ SFM (3D from **unordered** set of images) show video `sfm_colloseum.avi`

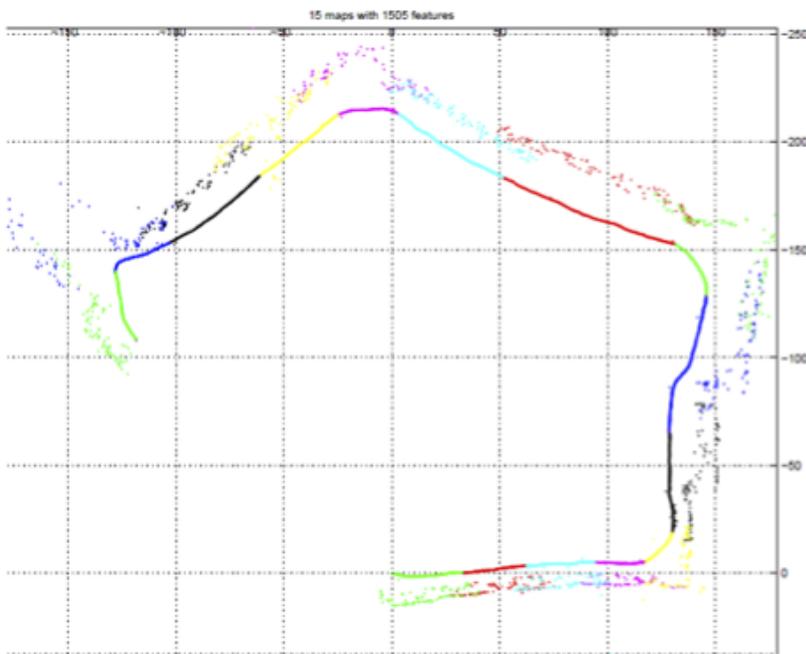
Visual Odometry (VO), Structure from Motion (SFM), Visual Simultaneous Localisation and Mapping (VSLAM)



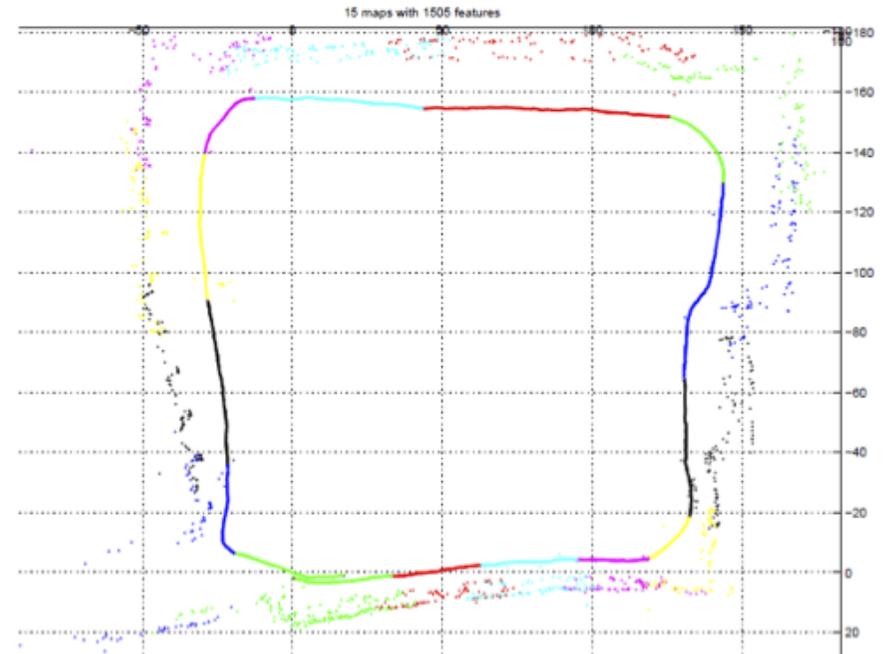
- ◆ SFM (3D from **unordered** set of images) show video `sfm_colloseum.avi`
- ◆ VO **sequential** and **real-time** camera motion estimation.

Visual Odometry (VO), Structure from Motion (SFM), Visual Simultaneous Localisation and Mapping (VSLAM)

- ◆ SFM (3D from **unordered** set of images) show video `sfm_colloseum.avi`
- ◆ VO **sequential** and **real-time** camera motion estimation.
- ◆ VSLAM is VO with loop closer (bundle adjustment) - show video `vo_kinect.avi`



Before loop closing



After loop closing