IRO Homework 6: Shortest path with complete coverage of targets

Karel Zimmermann

Discretized world consist of (i) a rectangular grid created by N nodes, connected by K edges and (ii) V target elements (e.g. voxels). Traversing of an k-th edge yields coverage of a subset of target elements. This subset is determined by V-dimensional binary vector $\mathbf{a}_k$, which contains 1 for covered elements and 0 otherwise. Find the shortest path from node 1 to node N, which covers all target elements.

1. Download $V \times K$ matrix `A.mat` consisting of column concatenation

$$\mathtt{A} = [\mathbf{a}_1, \ldots \mathbf{a}_K]$$

.

2. Download $K \times 2$ matrix `E.mat` of edges.

3. Formulate the problem as the integer linear program:

   - Define scalar binary variable x_{ij} which corresponds to the edge e_{ij} connecting nodes i and j. x_{ij} is equal to 1 if the edge is selected to lye on the optimal path. K-dimensional vector $\mathbf{x}$ containing all these variables is the unknown variable.
   - Criterion is sum of edges lying on the optimal path, i.e. $\mathbf{1}^\top \mathbf{x}$.
   - Equalities assure that the optimal path:
     (a) starts in node 1, i.e. $\sum_j x_{j1} - \sum_j x_{1j} = -1$,
     (b) ends in node N, i.e. $\sum_j x_{jN} - \sum_j x_{Nj} = 1$,
     (c) if path enters any other node, it must also leave it, i.e. $\sum_j x_{ji} - \sum_j x_{ij} = 0$, for all $i = 2 \ldots (N-1)$.
   - Inequalities assures that all targets are visible, i.e.

$$\mathtt{A}\mathbf{x} \geq \mathbf{1}$$

.