IRO Homework 1: Maximum likelihood identification of the torque-to-velocity model

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Forward velocity v of a tracked robot often depends non-linearly on input motor torque u. Since the velocity depends also on other unknown quantities such as terrain-robot friction coefficient or robots pitch angle, resulting model is probabilistic. Identify unknown parameter a of the torque-to-velocity model defined as the following normal distribution of velocities v for a given torque u and parameter a

$$p(v|u,a) = \mathcal{N}(1 - \frac{a}{1+e^u}, \sigma^2),$$

where σ^2 is constant unknown variance. Input to your parameter identification algorithm is a set of i.i.d. measurements $(u_1, v_1, \dots u_n, v_n)$. The output is estimated parameter a^*, which maximize joint likelihood of measured data defined as $L(a) = \prod_{i=1}^n p(v_i|u_i, a)$.

1. Find matrix $\mathtt{A}$ and vector $\mathbf{b}$ such that the $L(a)$ maximization is transformed to the searching of L_2 solution of the overdetermined set of linear equations, i.e. such that $a^* = \arg\max_a L(a) = \arg\min_a \|\mathtt{A}a - \mathbf{b}\|_2$

2. Download `ML_u_v.mat` data file, with measurements $(u_1, v_1, \dots u_n, v_n)$ and find a^* by solving the overdetermined set of linear equations.

3. Draw (i) the mean velocity $v(u) = 1 - \frac{a^*}{1+e^u}$ as a function of torque u and (ii) measurements $(u_1, v_1, \dots u_n, v_n)$ into a common figure.

4. **Bonus:** Derive and implement and visualize the ML estimate of the unknown variance σ^2.