Practical geometric optics

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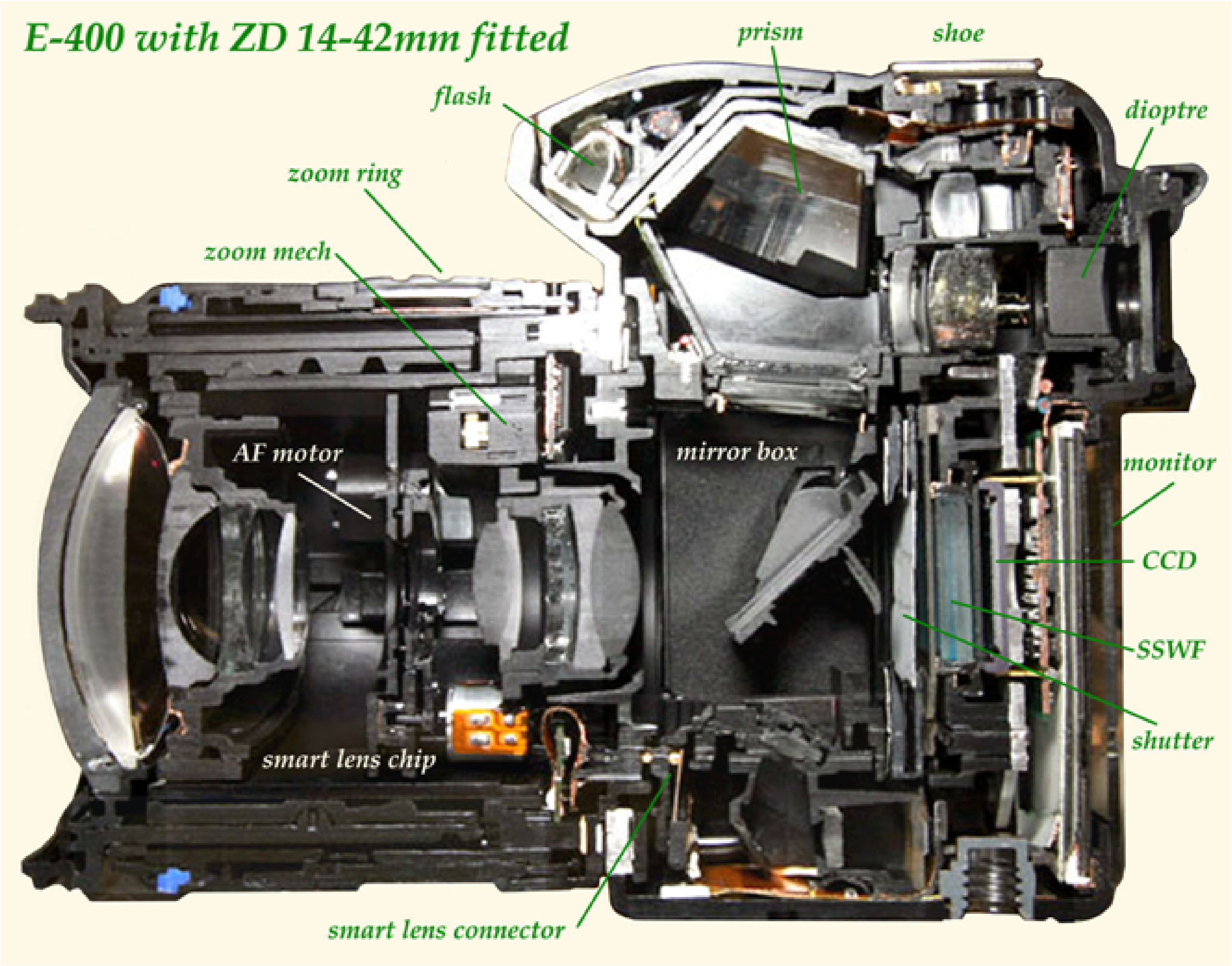
Outline of the talk:

- Motivation, lens of a camera.
- Why are lenses needed?
- Geometric optics as a simplified model.
- Lens from a physical point of view
- Depth of focus. Depth of field.
- Lenses.
- Lenses abberations.

Single-lens reflex camera, a cross-section



Basic elements of a camera

- **The lens**. Fixed or varying focal lenght.
- The shutter. Mechanic or electronic.
- Optical viewer (absent with cheaper cameras).
- Matrix of light sensitive sensors. CCD or CMOS.
- The amplifier modifying the signal from sensors.
- Analog/digital converter.
- A computer converting raw image data to a viewable representation.
- LCD display for viewing images.
- Memory medium, often removable.
- Energy source, battery or rechargeable battery.

The job of a lens

- The optical system (lens) focuses the incoming energy (photons) and creates the image on the image sensor.
- The measured physical entity is the irradiance $[\mathrm{W\,m^{-2}}]$ (informally brightness or gray value from a human perception point of view).
- The lens should mimic the ideal perspective transformation as much as possible (also projective transformation, pin-hole model).
- We will constrain to geometric optics in this simplified optics explanation. We leave wave and quantum optics models aside.

Approximation by geometric optics

It is one of several possible approximations.

Assumptions:

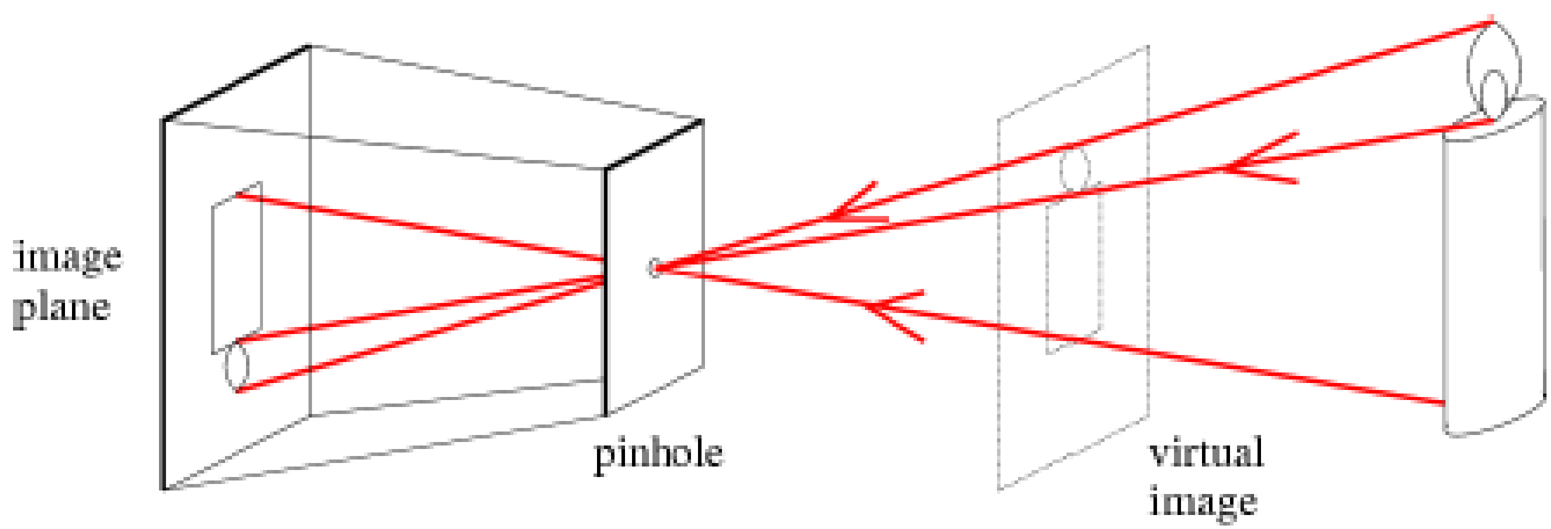
- The involved wavelengths of the electromagnetic irradiation (here a frequency sub-band of it = light) are very small with respect to sizes of used optical and mechanical elements.
- The energy of photons (from the quantum theory point of view) are small with respect to energetic sensitivity of involved sensors.

Geometric optics is a rough approximation. Geometric optics is important for daily life technology. It is also interesting from the point of view of the historic development of opinions in physics.

Recommended reading: *Feynman R.P, Leighton R.B., Sands M.: Feynman Lectures on Physics, 3 volumes, (1963-1965)*.

A pin-hole camera

- 15th century, the architect Filippo Brunelleschi from Italian Florence (1377-1446), a tool for drawing perspective images.
- 16th century, pin-hole camera, in Latin camera obscura.
- 1822 The Frenchman J.-N. Niepce added a photographic plate to the pinhole camera ⇒ the first photograph was born.



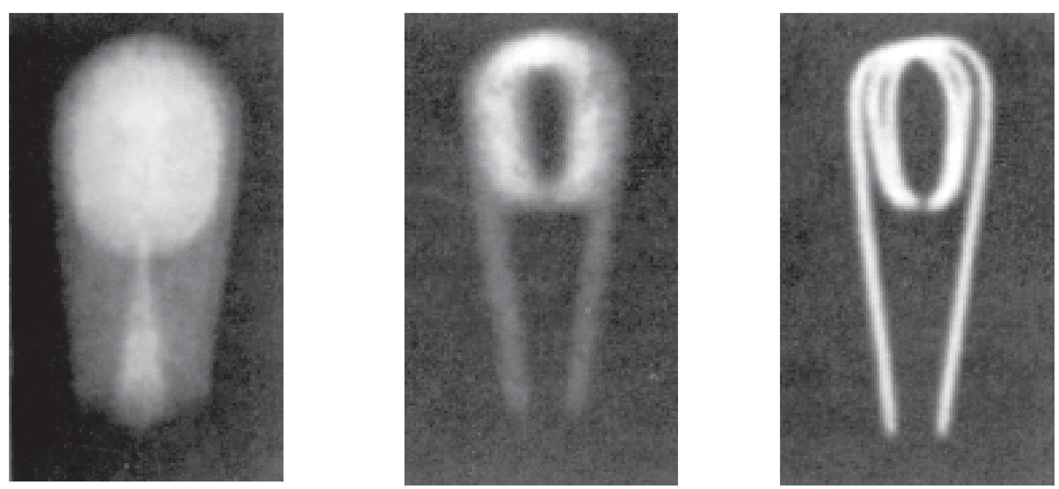
The size of a hole in a pin-hole camera

The interplay of contradictory phenomena.

a. The bigger hole passes more light but blurs the image.

b. The small hole causes diffraction and the image will be blurred too.

c. The optimum exists in which the image is least blurred. Example: For f=100 [mm] and λ=500 [nm], the optimal diameter of the hole is 0.32 [mm].

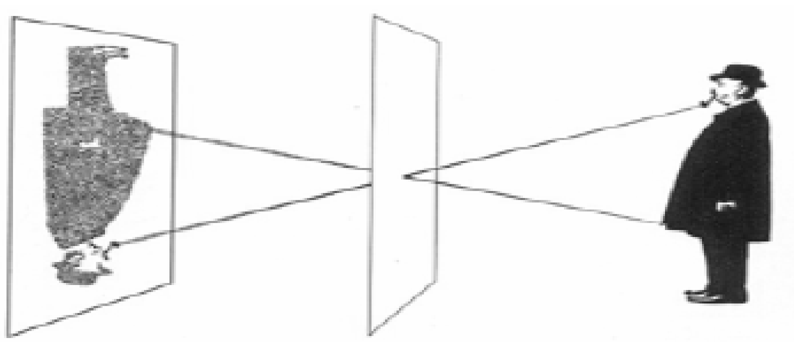


a b c

Why are lenses used?

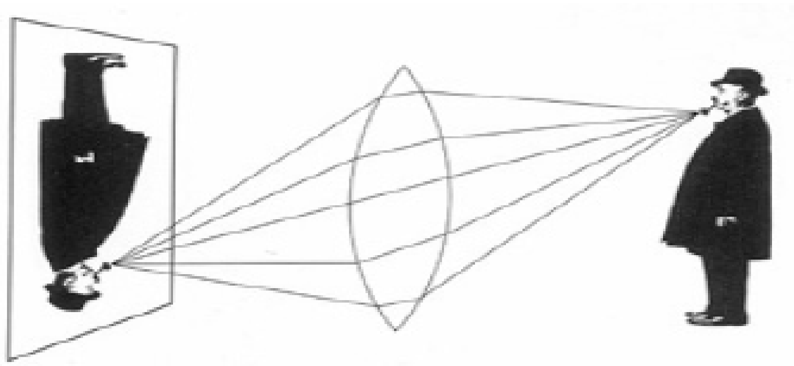
The pin-hole camera:

- Collects only a few photons (light).
- Troubles due to diffraction due to the hole.
- No abberations.



The lens:

- Collect more photons (light).
- Have to be in focus.
- Suffers from abberations.



The lens from a physics point of view

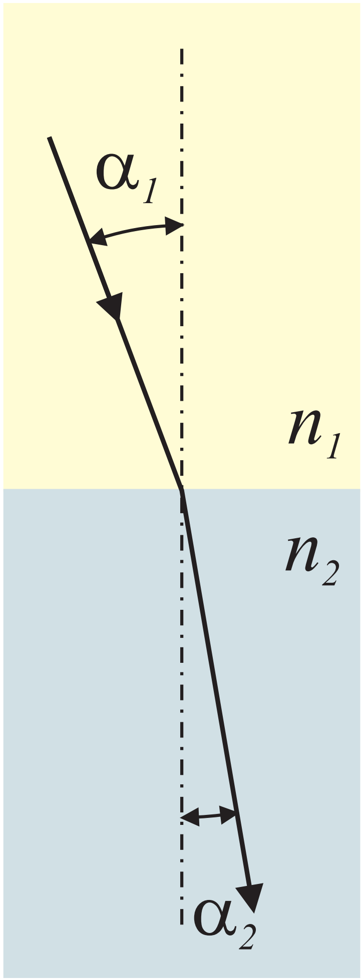
- Behavior of a lens was explained by the Dutch mathematatitian Willebrord van Roijen Snell (1580–1626), who formulated the light refraction law on the boundary of separation of two contacting substances in the year 1621.

$$n = \frac{n_1}{n_2} = \frac{\sin \alpha_2}{\sin \alpha_1} \text{ , kde } n \text{ je index lomu.}$$

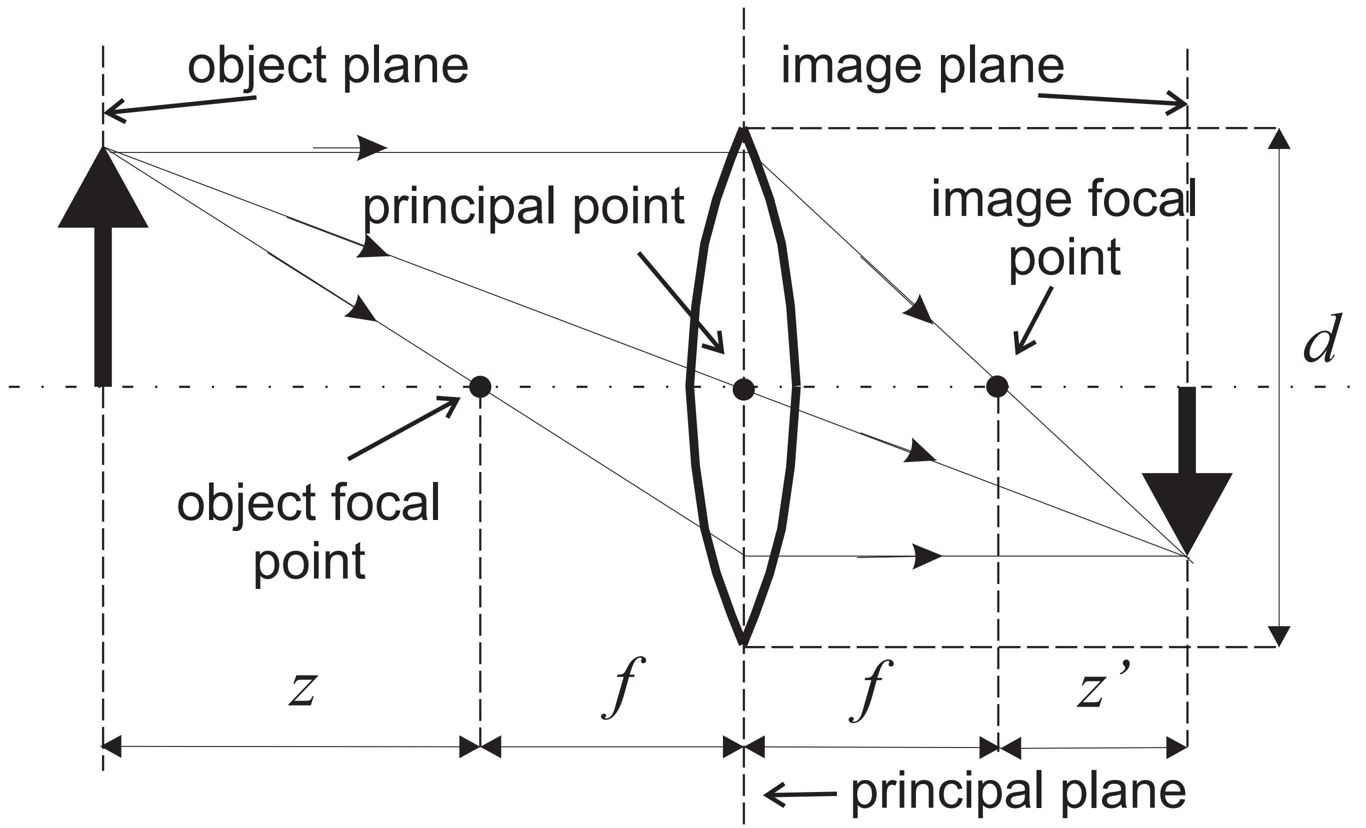
- n for a yellow light λ=589 [nm] on the boundary between the vacuum and X:

  X = air 1,0002; water 1.333; crown glass (a small diffusion of light, a small refraction index) 1.517; lead optical glass 1.655; diamond 2.417.

- There is an elegant derivation of the Snell's refraction law, which uses (approximate) Fermatt's principle of the shortest time from the year 1650, see Feynman's Lectures on Physics.



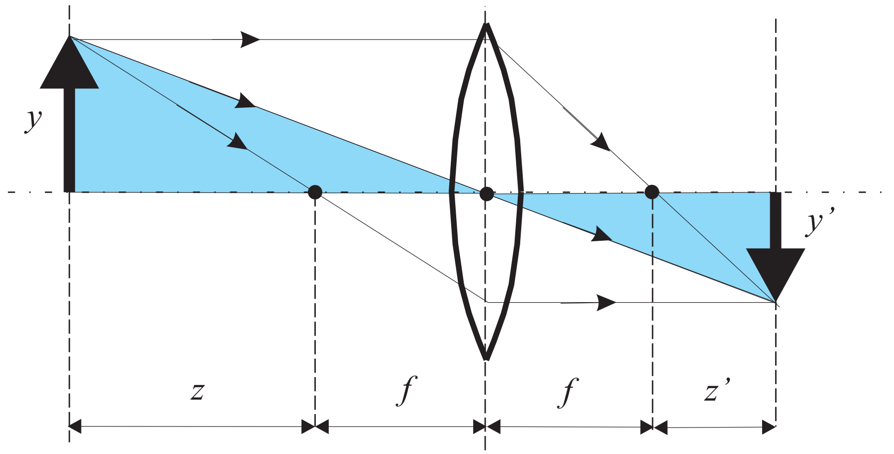
A thin lens

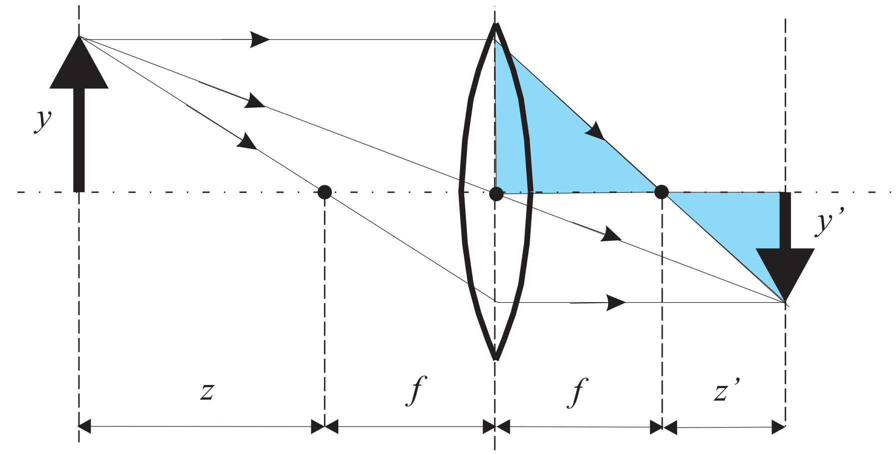


The thin lens equation in a Newtonian form $\frac{1}{f} = \frac{1}{z' + f} + \frac{1}{f + z}$

or rewritten to a simpler expression $f^2 = z\, z'$

A single lens, derivation, similar △





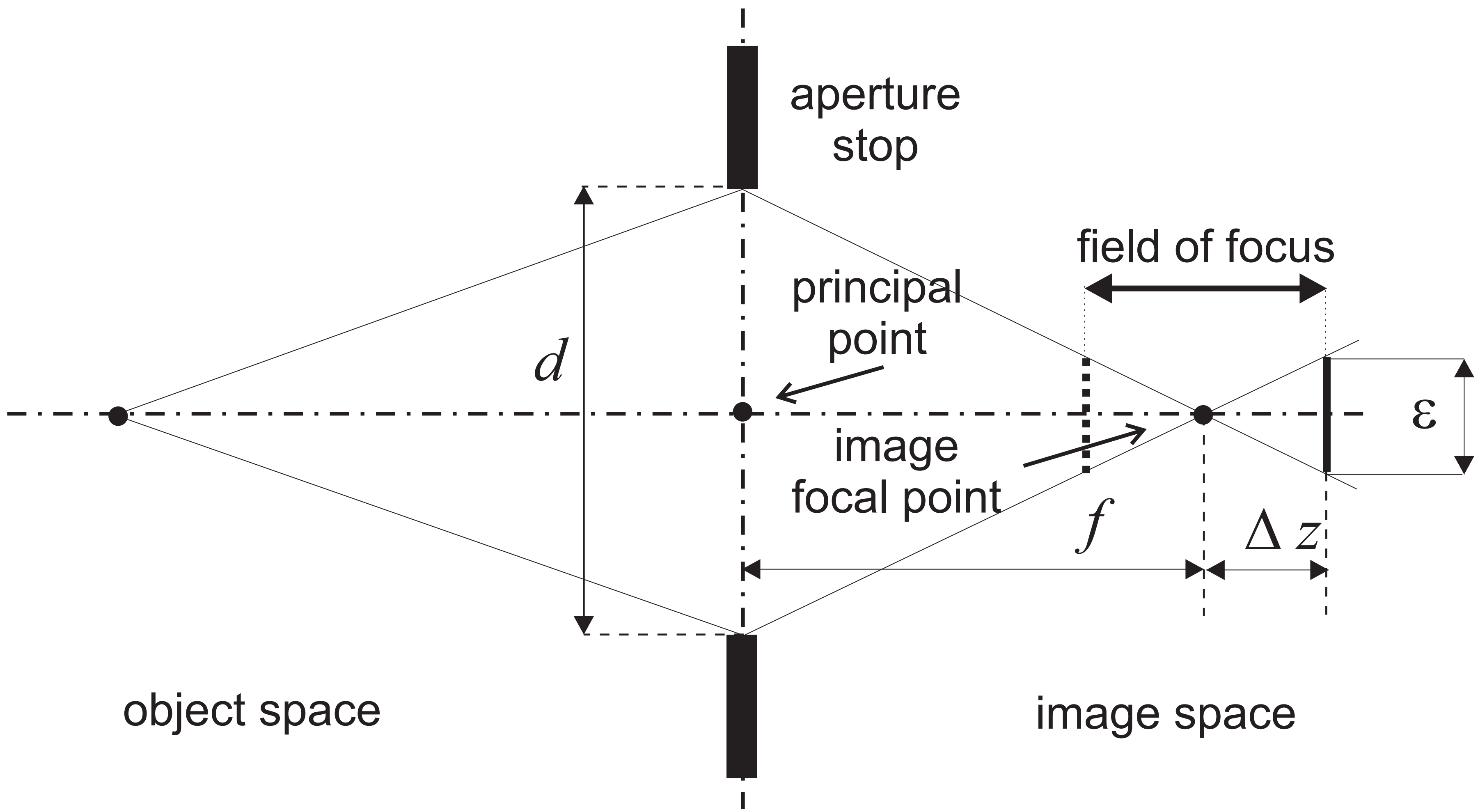
$$\frac{y'}{y} = \frac{z'+f}{z+f} \qquad \frac{y'}{y} = \frac{z'}{f}$$

By connecting the two equations:

$$\begin{aligned} \frac{z'+f}{z+f} &= \frac{z'}{f} \\ f(z'+f) &= z'(z+f) \\ fz' + f^2 &= zz' + fz' \\ f^2 &= z\,z' \end{aligned}$$

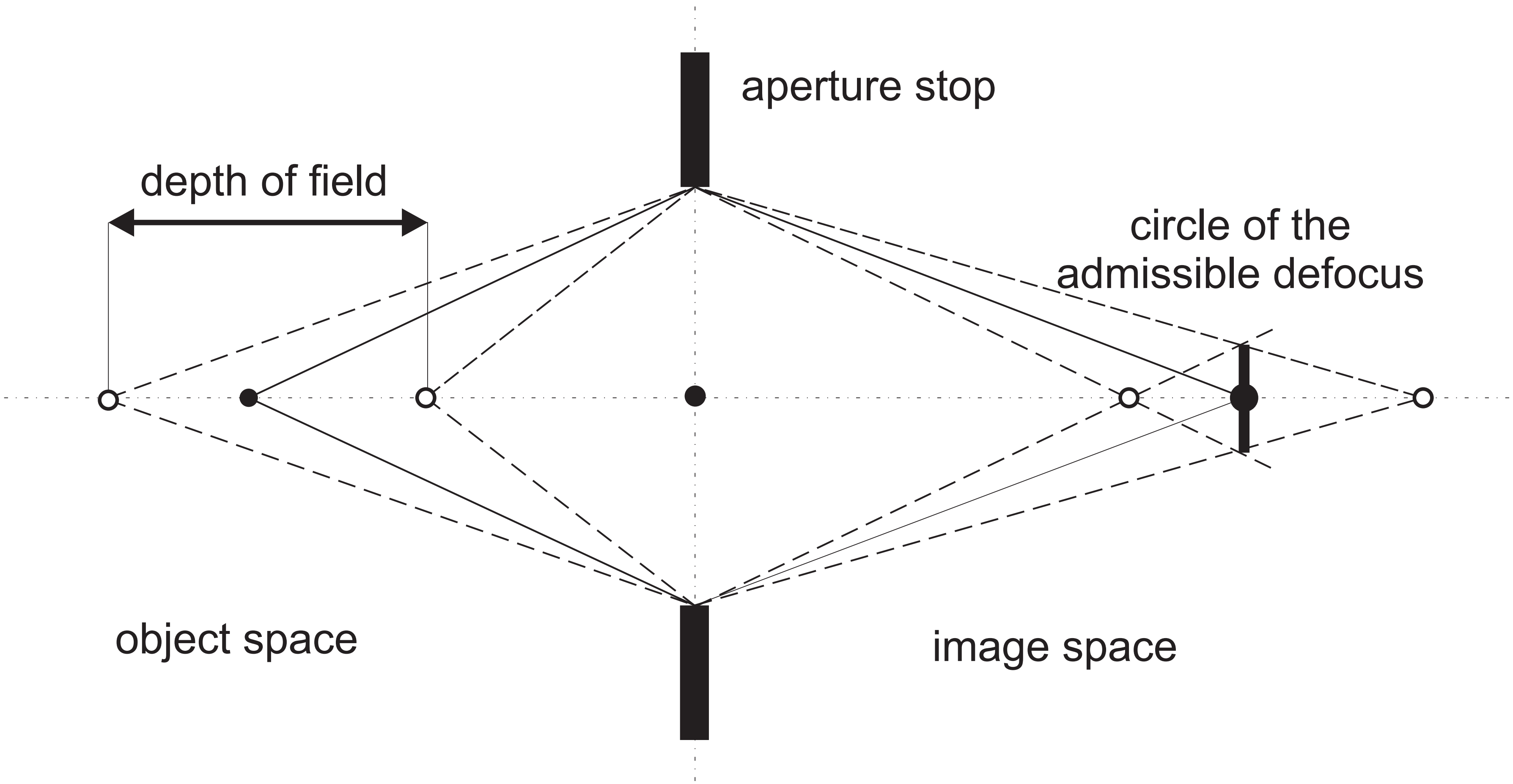
Depth of focus

Depth of focus explains, why is it possible to shift the image plane to the right a little in the direction of optical axis in the image space and still have the image in focus. It is because of a finite size of one pixel on a sensor or a definite size of the grain in the film.



Depth of field

Depth of field determines the range of distances from the center of projection in the object space, in which the objects are shown in focus. This is the parameter, which is of practical interest for the photographer.



Depth of field, pictorial illustration

http://www.cambridgeincolour.com/tutorials/depth-of-field.htm

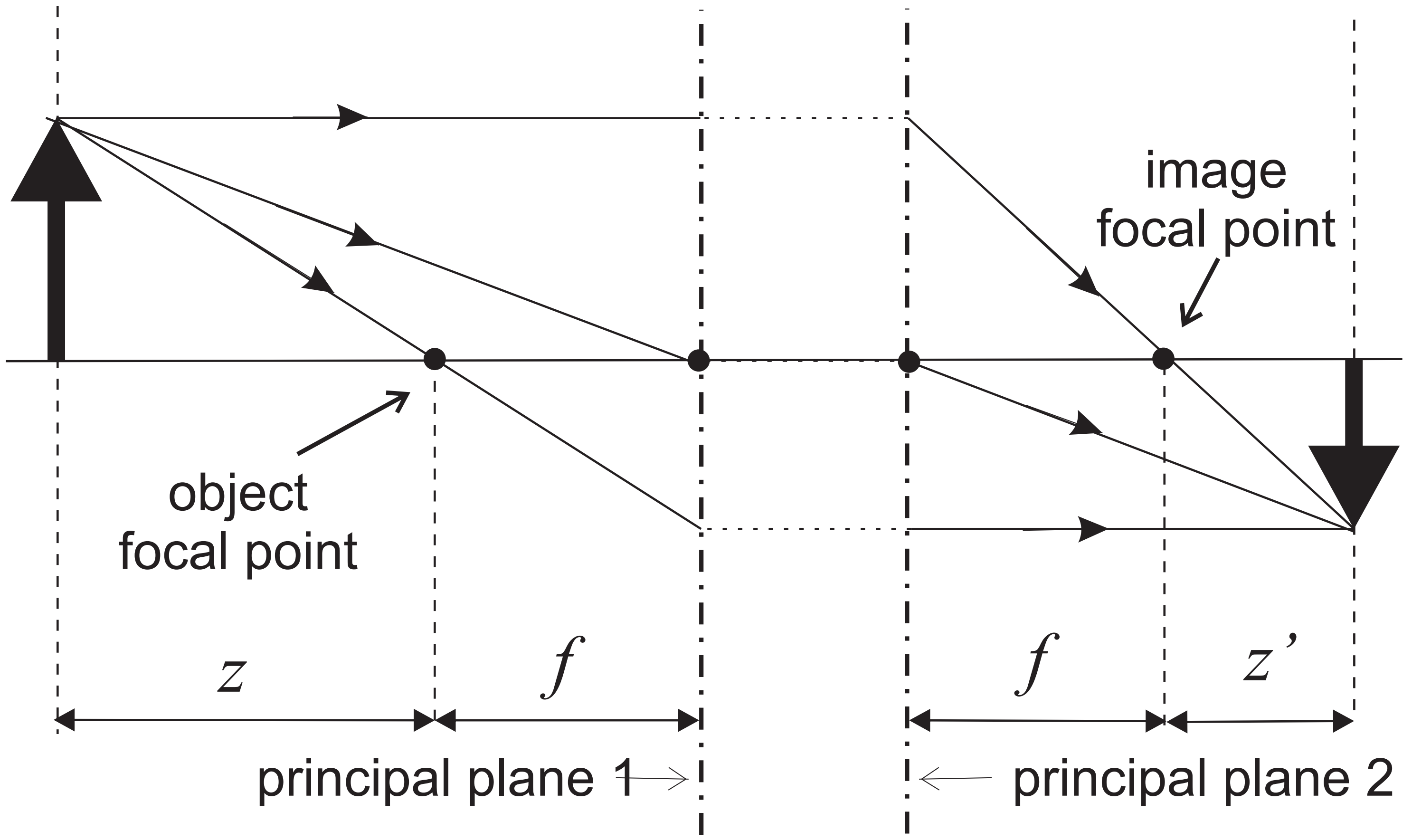
Influence of the aperture stop to the field of focus

a large aperture, small depth of field

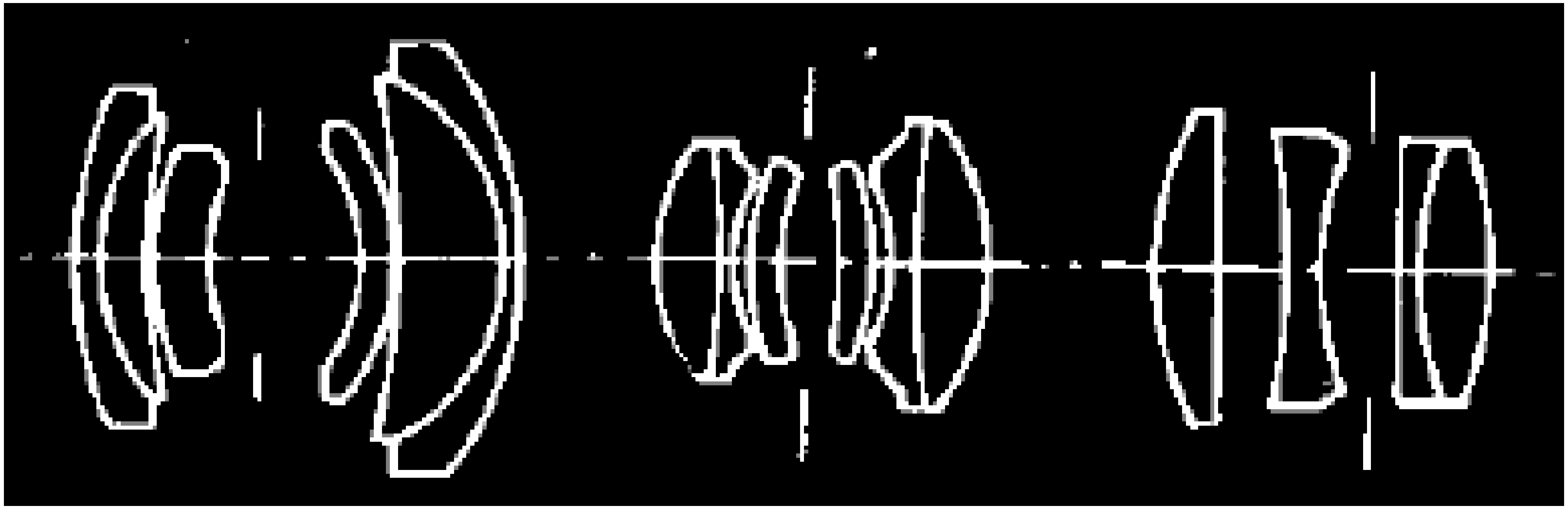
a small aperture, short depth of field

Influence of the focal length to the depth of field

Thick (composed) lens
the approximation of the optical system



A composed lens



- The composed lens is used to suppress optical abberations.
- Principal abberations: vignetting (natural, optical, mechanical), chromatic abberations and radial distortion.

Natural vignetting

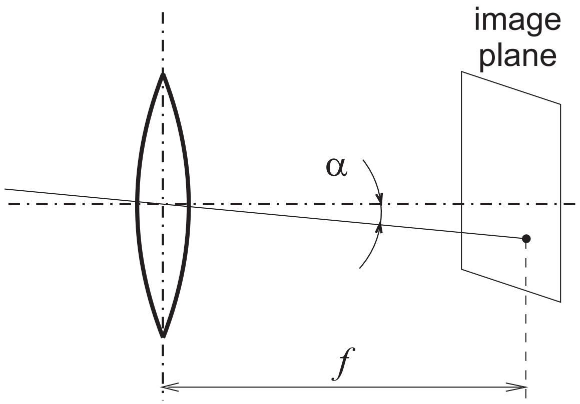
- The term $\cos^4 \alpha$ describes a systematic optical abberation named natural vignetting.

  The derivation of the related Irradiance equation can be found in the lecture about image capturing from the physics point of view.

- The natural vignetting phenomenon describes the situation, in which the rays refracted at higher angle α with respect to the optical axis.

- This error (natural vignetting) is more pronounced with wide angle lenses than with telephoto ones.

- Naural vignetting is a systematic error. It can be undone if the camera and the scene is radiometrically calibrated.



original

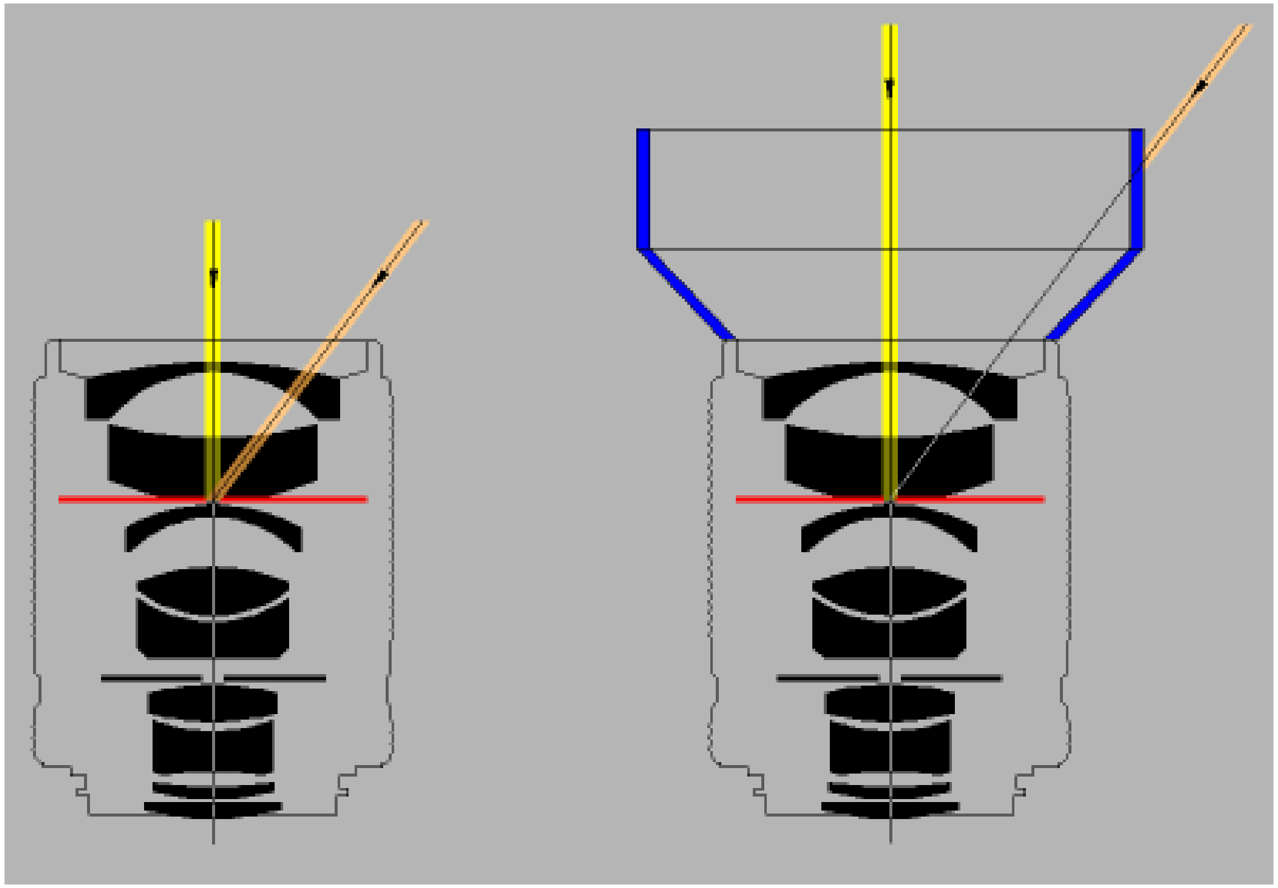
vignetting

Optical vignetting

- The compound lens thickness thickness ranges from several millimeters to several centimeters. This is the reason, that why not all rays can hit the lens aperture opening.
- The phenomenon is more pronounce for for a more open aperture stops.

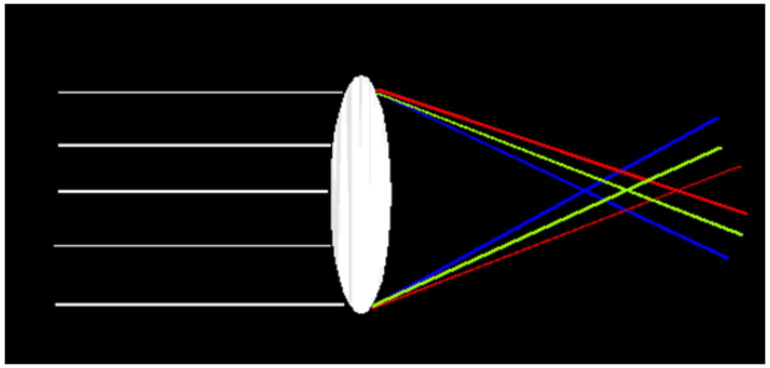
Mechanical vignetting

- Only inattentive users suffer from mechanical vignetting.
- The lens hood must match the particular lens..



Chromatic abberations

- Caused by the dependence of the lens refraction index on the light wavelength.
- This property is desired in the prism for the light decomposition. However, it is undesirable for lenses.
- It causes color errors more pronounced at the margins of the image, i.e. more distant from the optical axis.
- The abberation is rectified while manufacturing the lens. A pair of doublets is used as a building element, i.e. the lens is composed of two pieces from two different materials, the crown and the lead (flint) glasses, optical glasses with, respectively, relatively low and high refraction indices.



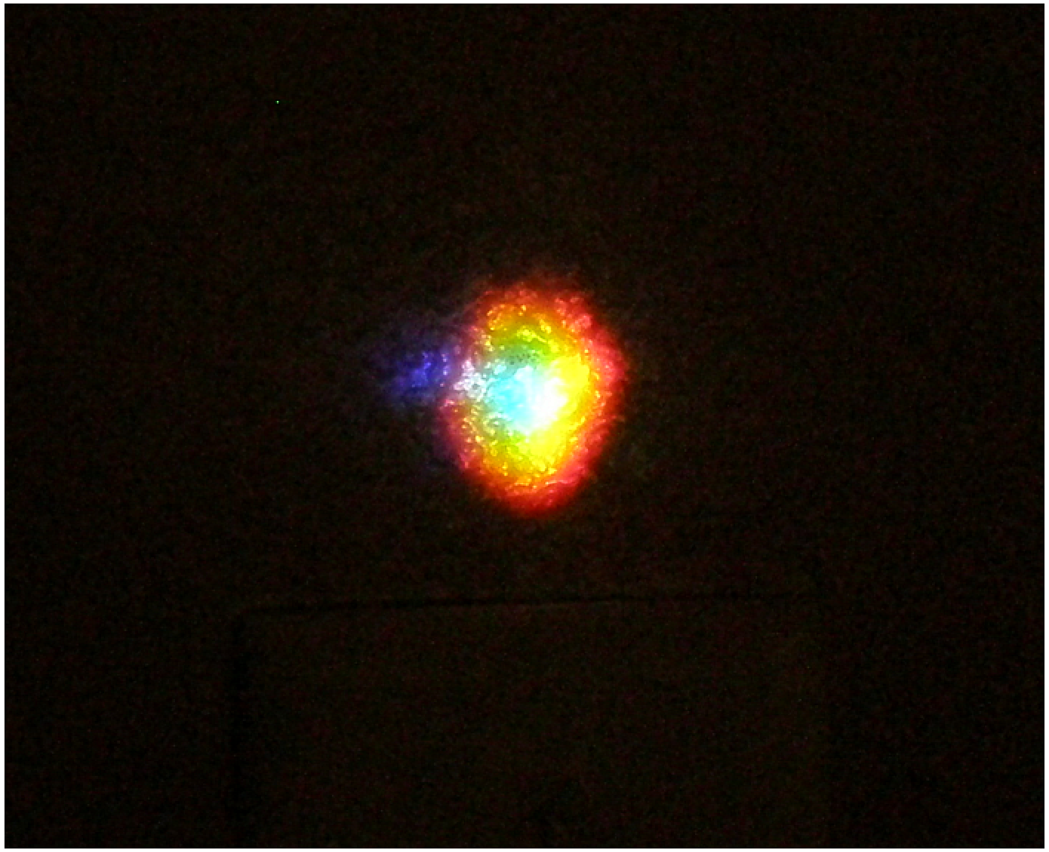
Chromatic abberation, a practical view

blízko optické osy, střed obrazu

daleko od optické osy, okraj obrazu

Chromatic abberation, extreme illustration

Bad quality lens of a peephole
in the US motel.

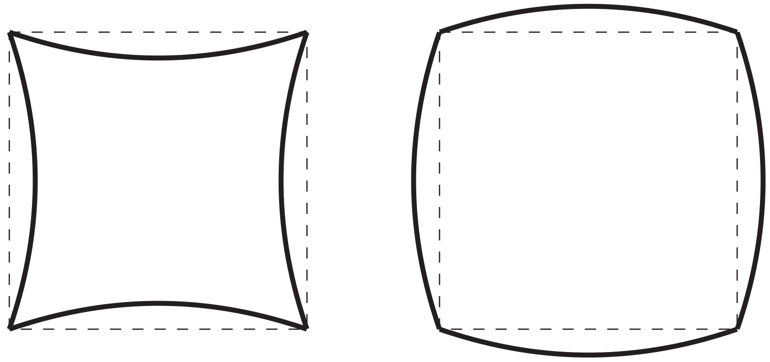


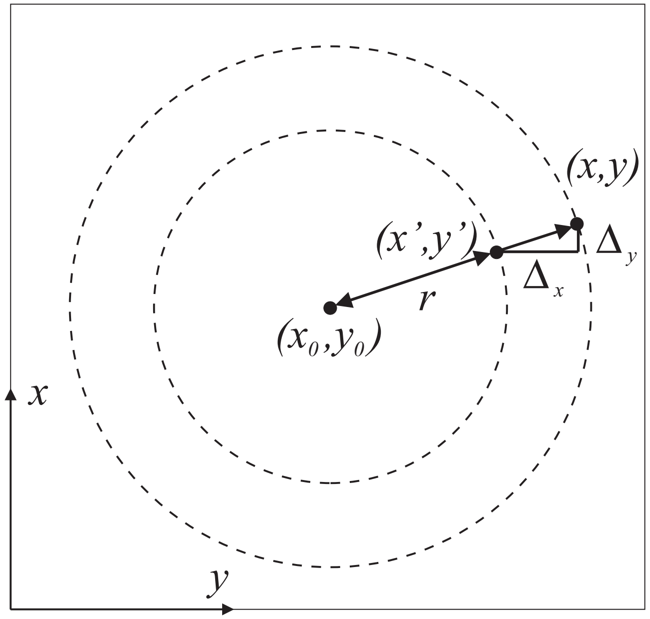
Projected sunset
on the opposite side of a dark roon.

Radial distortion

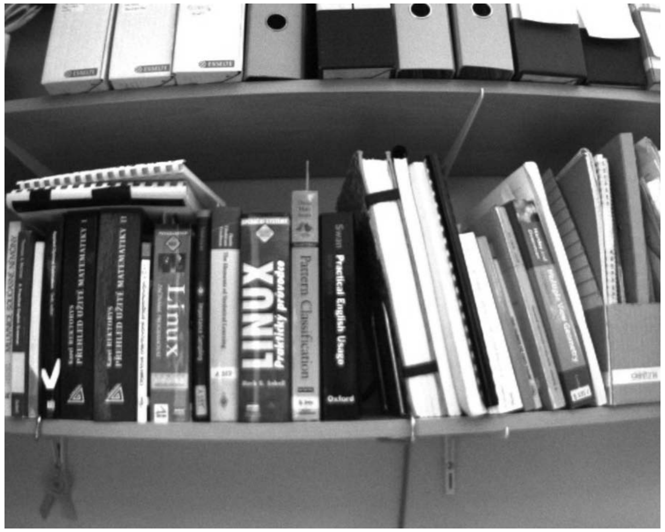
- It is the prevalent distortion. It is pronounced more with wide-angle lenses.
- (x', y') are uncorrected point coordinates measured in the image; (x, y) are corrected coordinates; (x_0, y_0) are coordinates of the principal point; (Δ_x, Δ_y) are elements of the correction, and r is the radius, $r = \sqrt{(x' - x_0)^2 + (y' - y_0)^2}$.
- The distortion is often approximated by a polynomial of the even order (why?), often only 2nd order.

$$\begin{aligned} \Delta_x &= (x' - x_0)\left(\kappa_1 r^2 + \kappa_2 r^4 + \kappa_3 r^6\right) , \\ \Delta_y &= (y' - y_0)\left(\kappa_1 r^2 + \kappa_2 r^4 + \kappa_3 r^6\right) . \end{aligned}$$





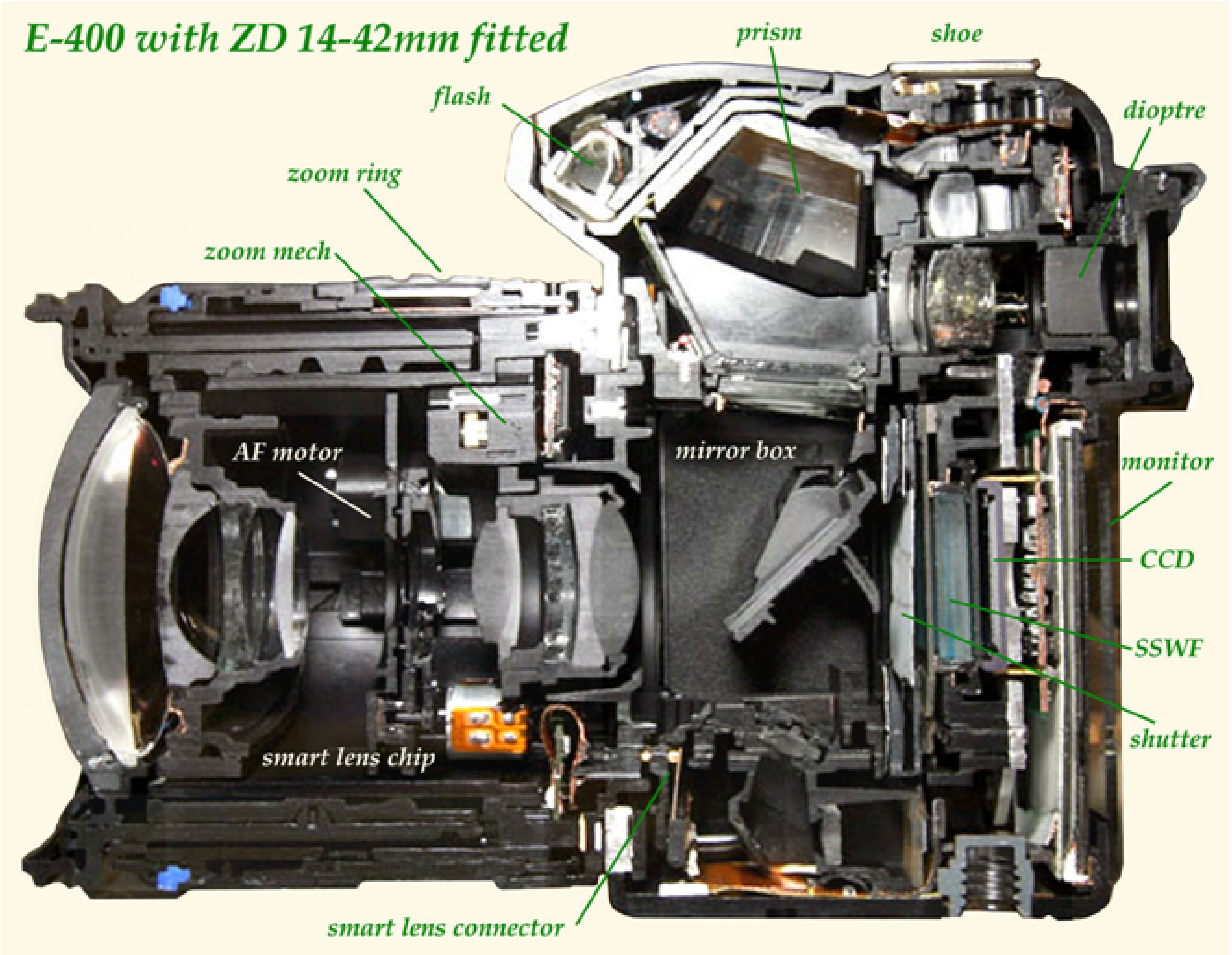
Radial distortion, practical illustration



barrel

without distortion

pincushion


E-400 with ZD 14-42mm fitted
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shoe
flash
dioptre
zoom ring
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AF motor
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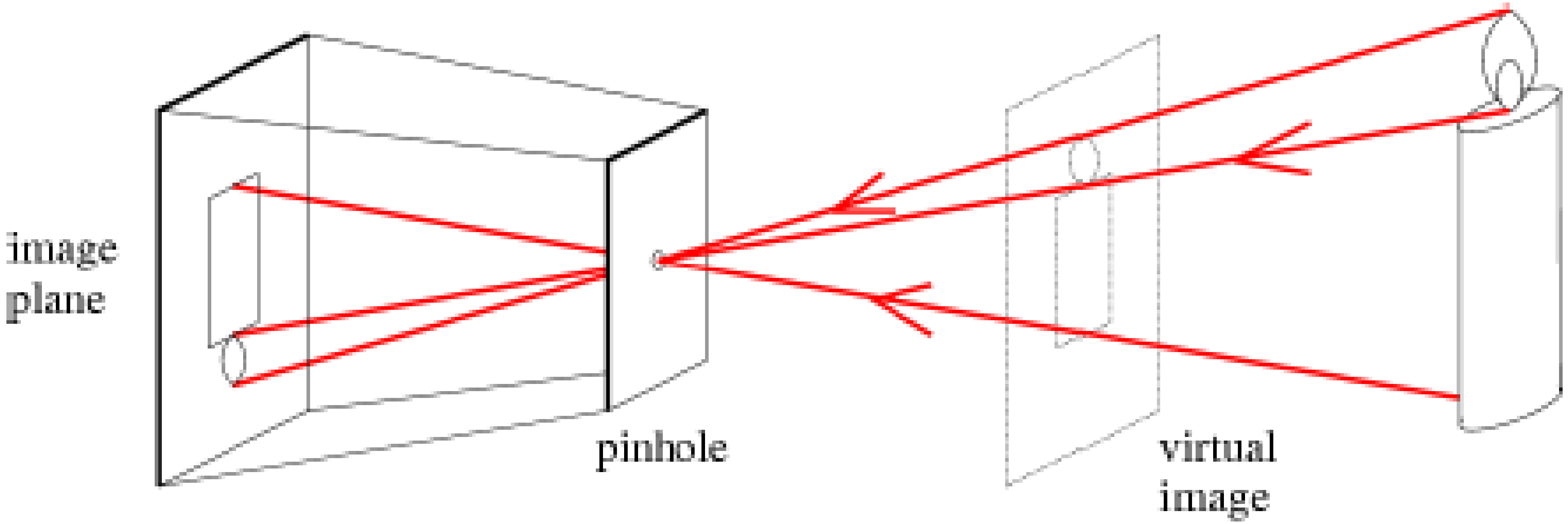
monitor
CCD
SSWF
shutter
smart lens chip
smart lens connector



image
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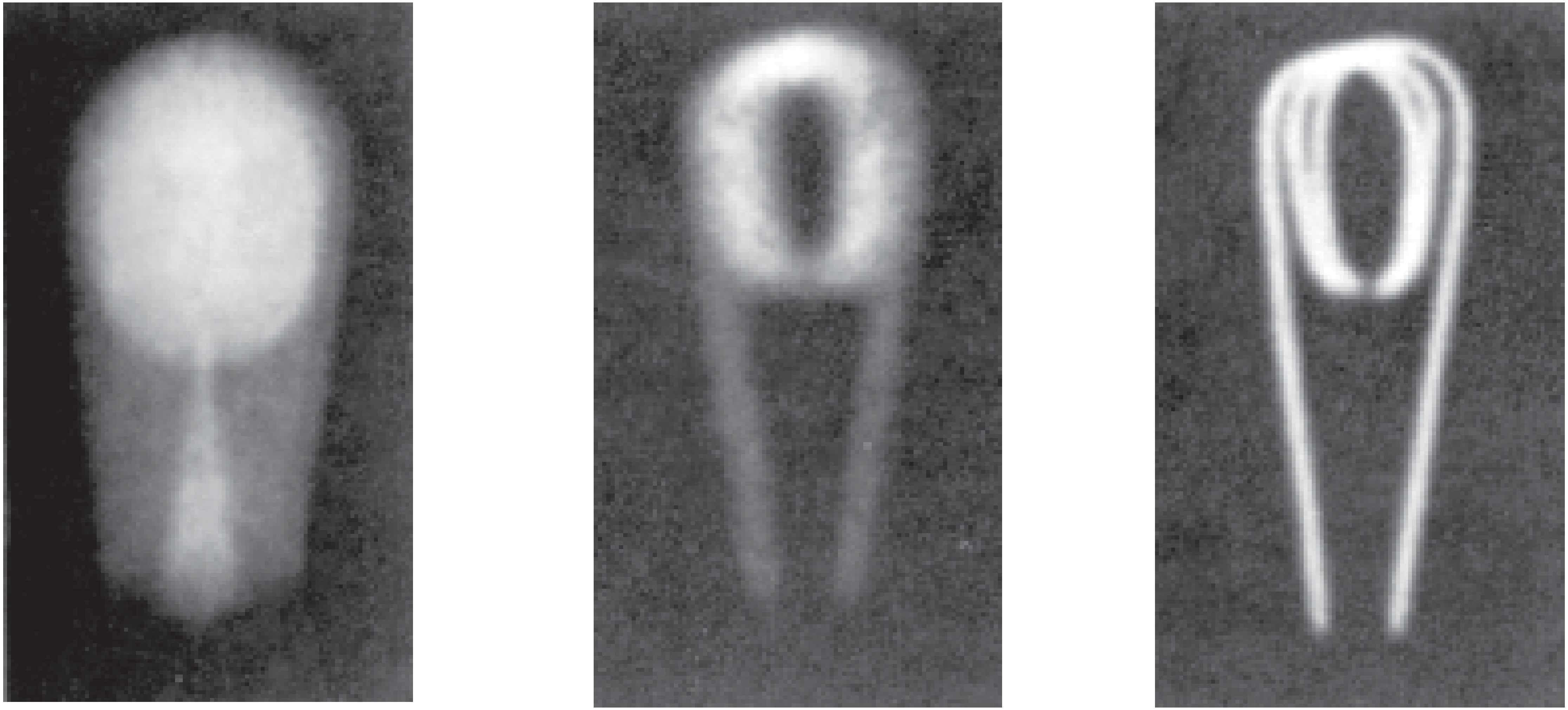
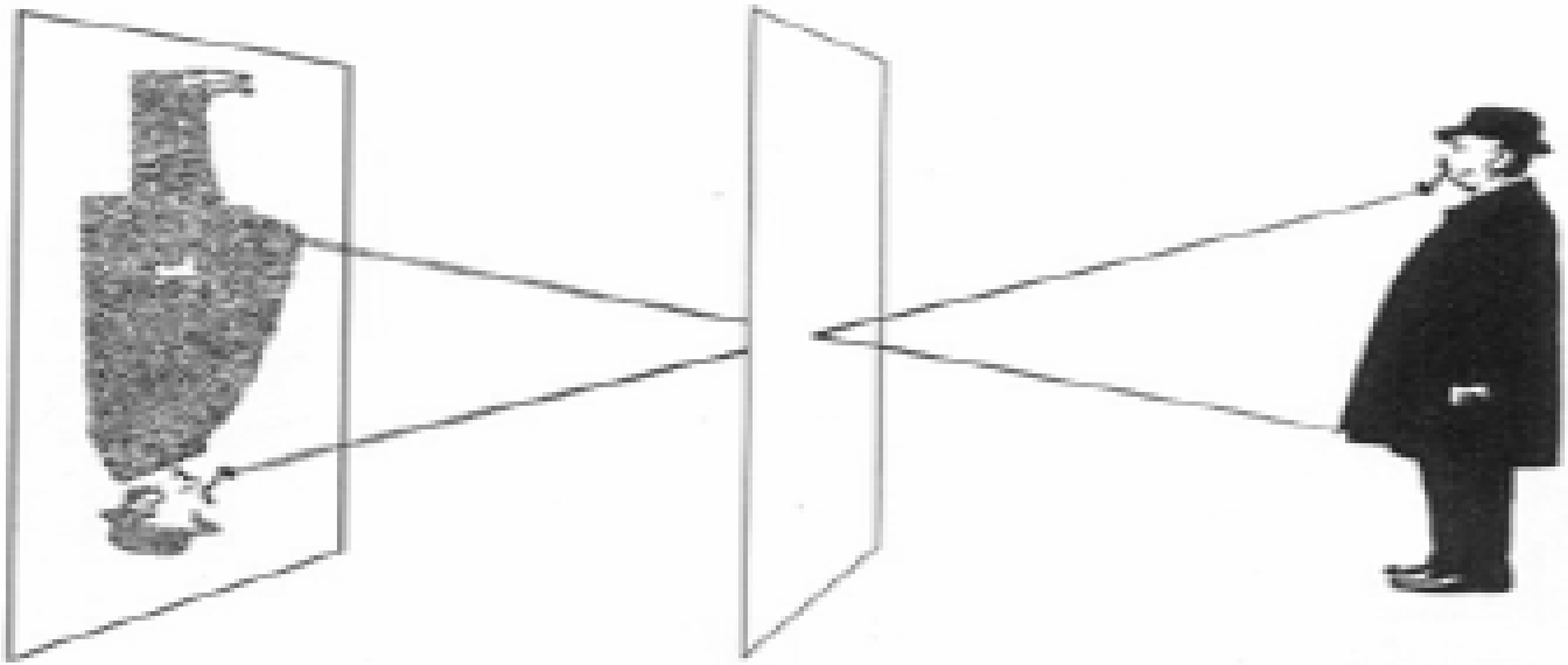
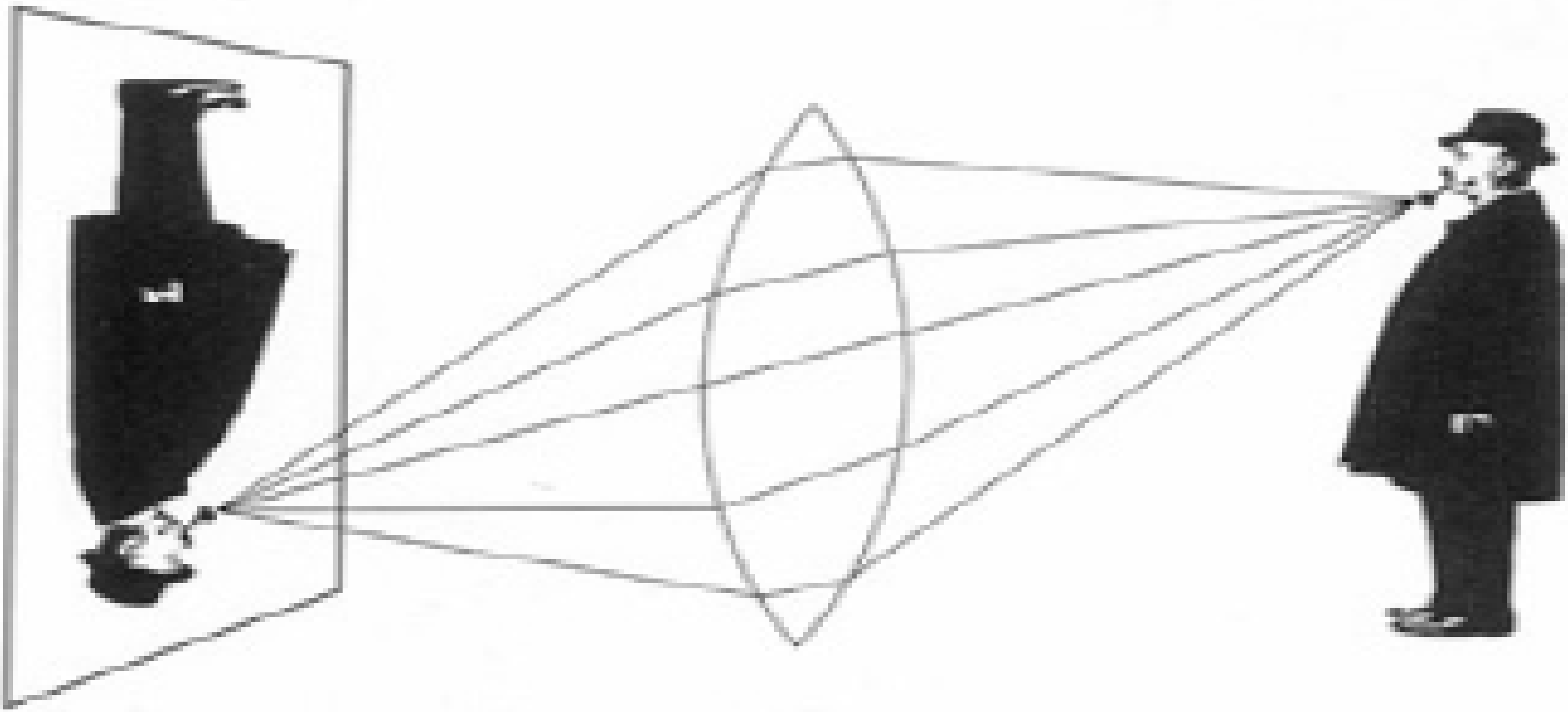
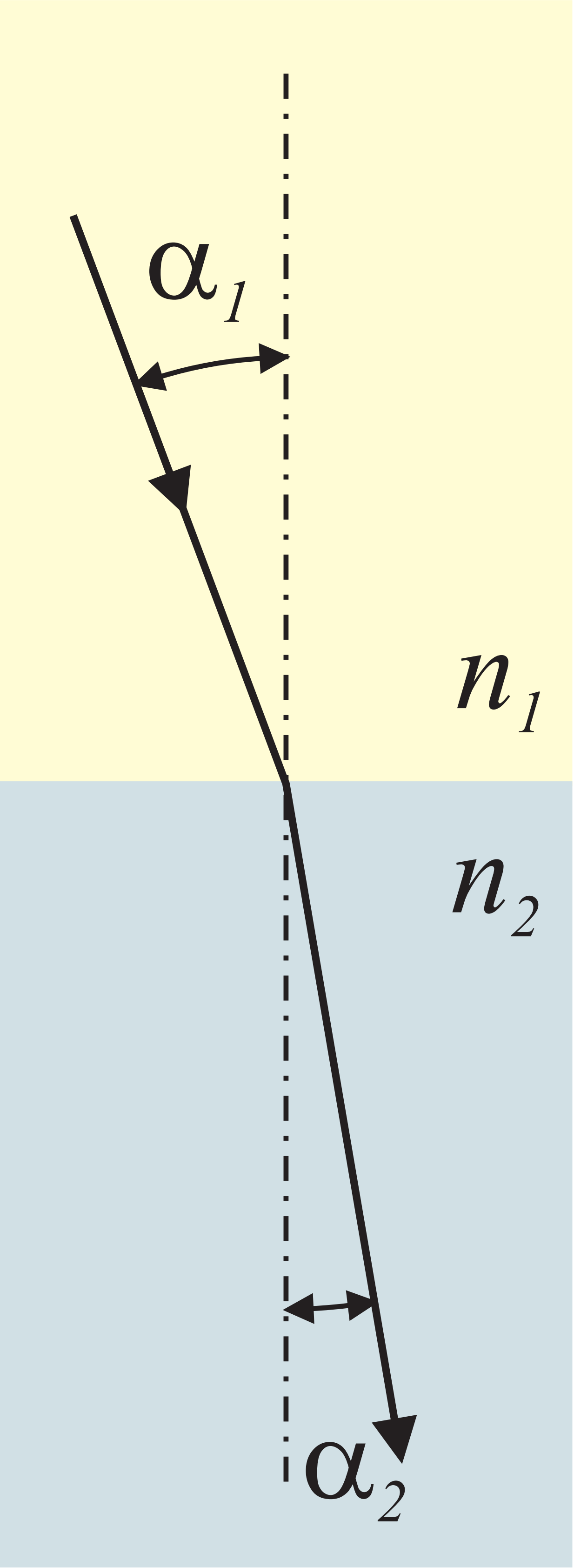
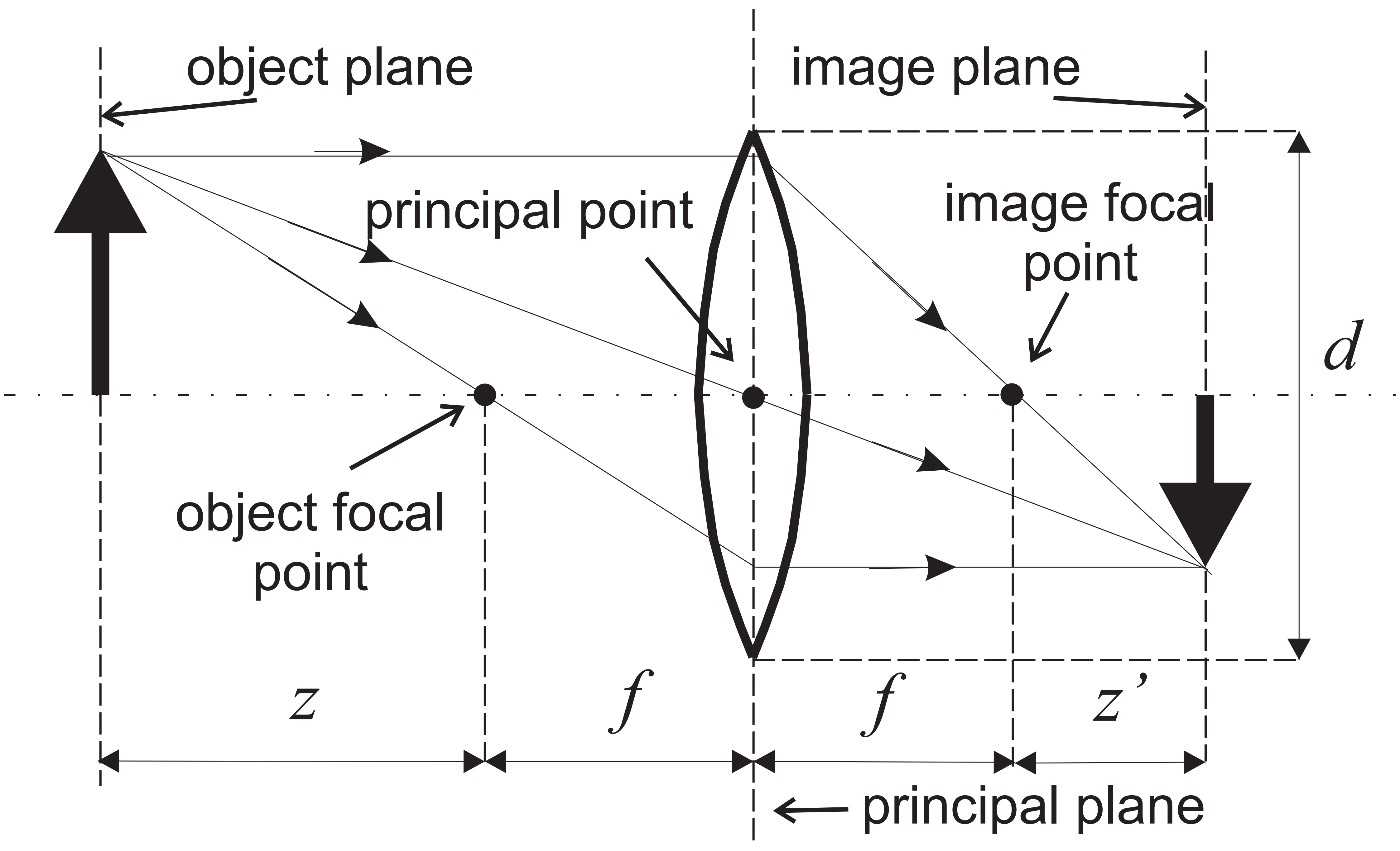


a b c

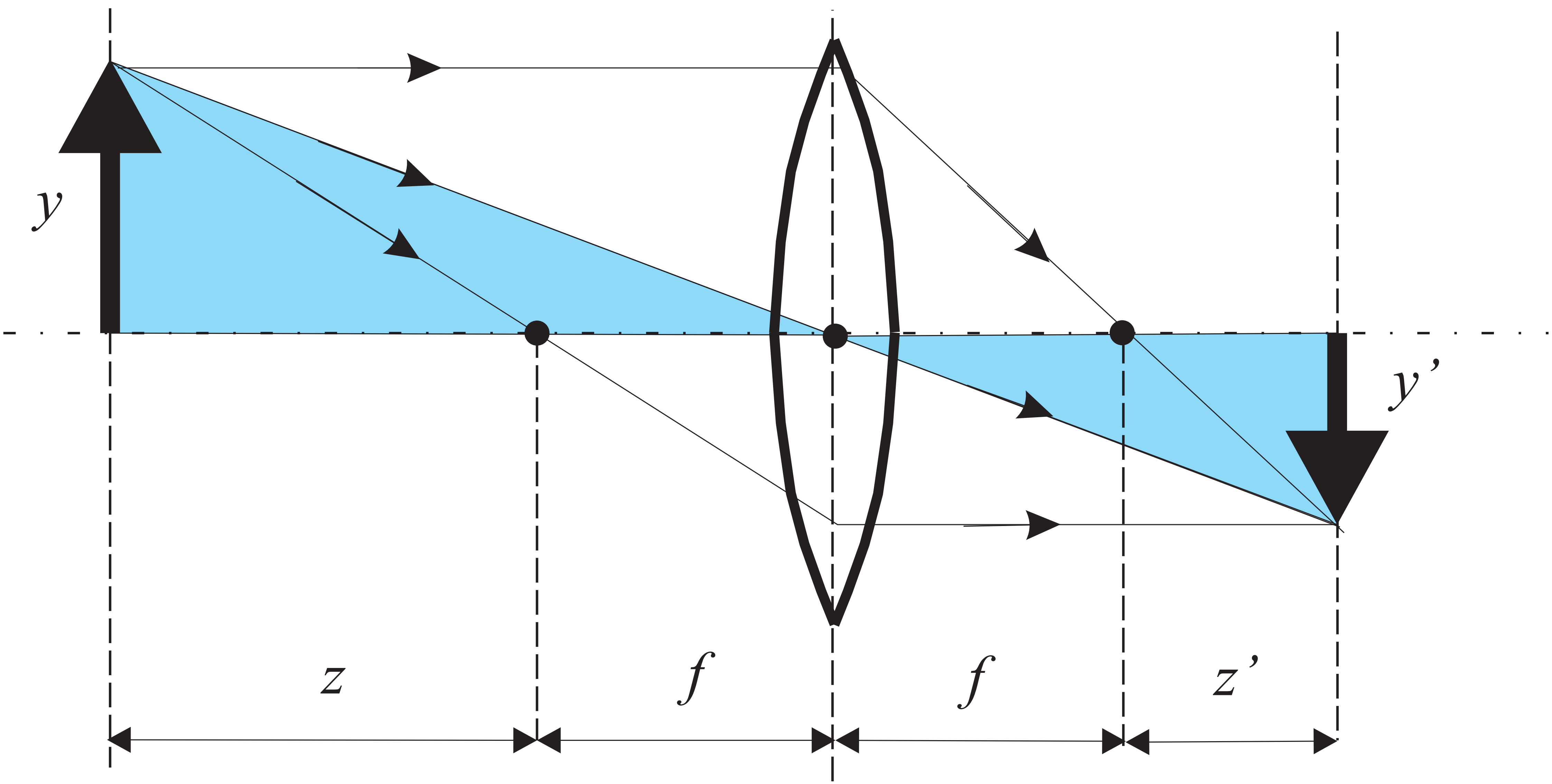





α1
n1
n2
α2



object plane
image plane
principal point
image focal
point
d
object focal
point
z
f
f
z'
principal plane


y
y'
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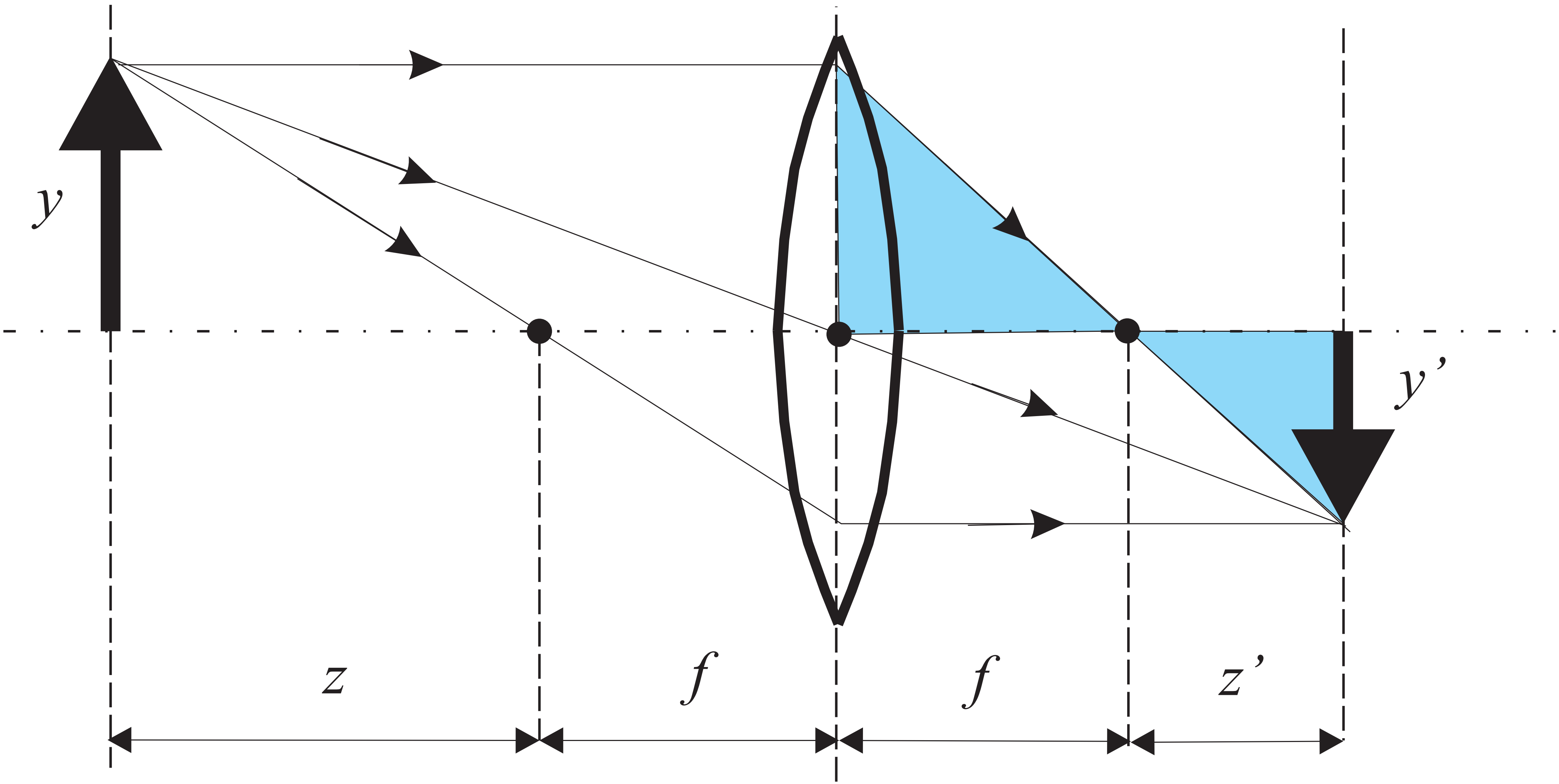
z'



y
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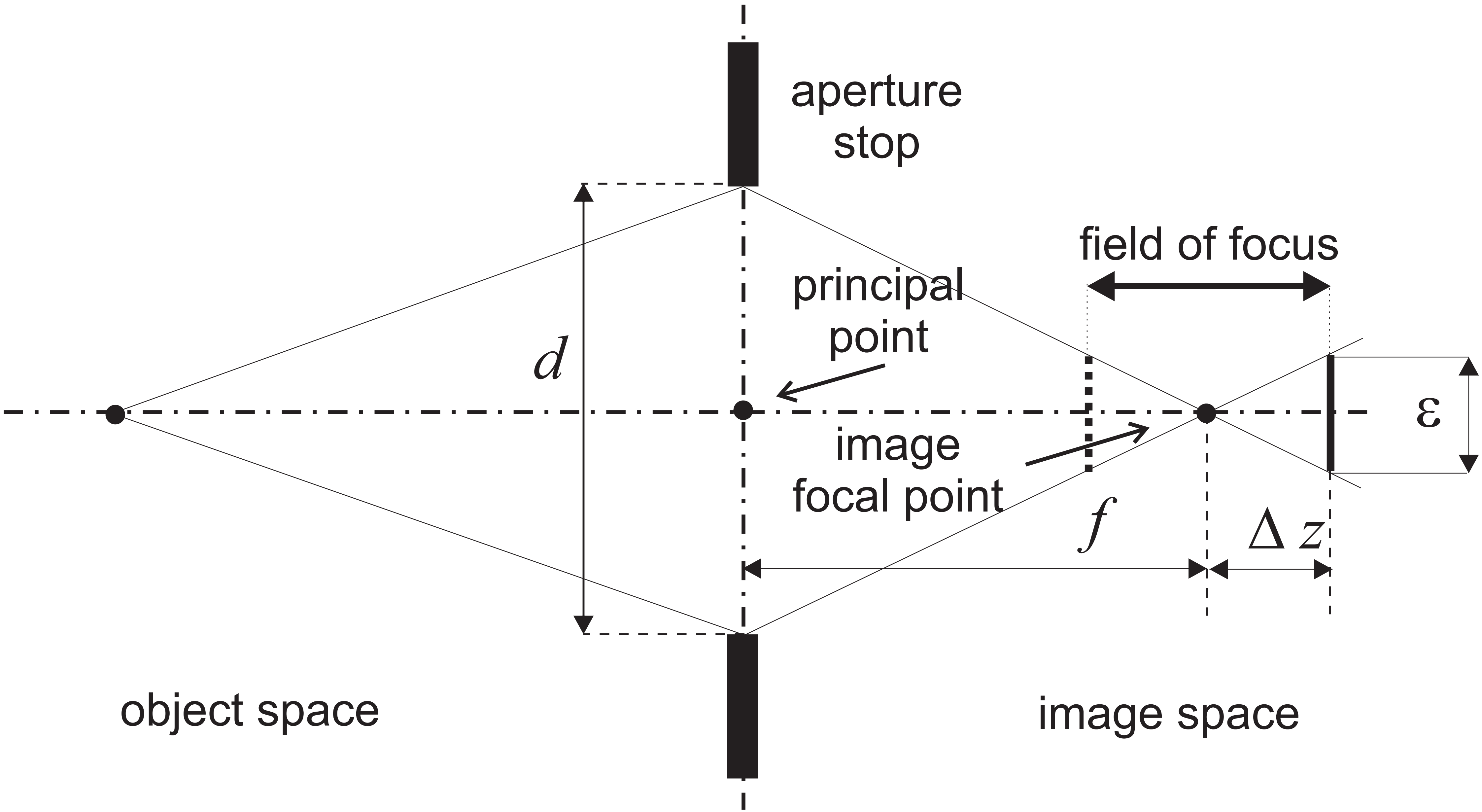
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object space
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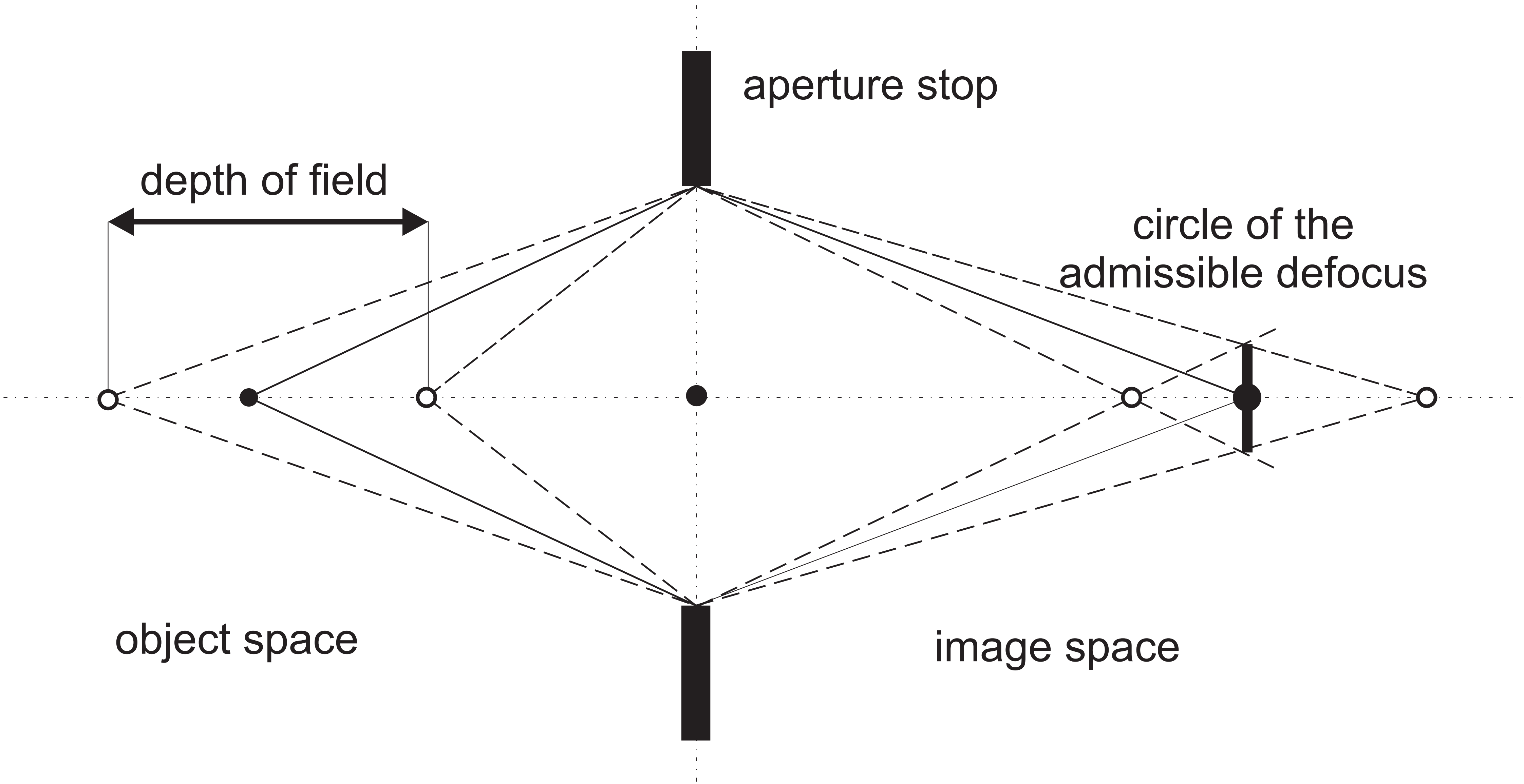

aperture stop
depth of field
circle of the
admissible defocus
object space
image space

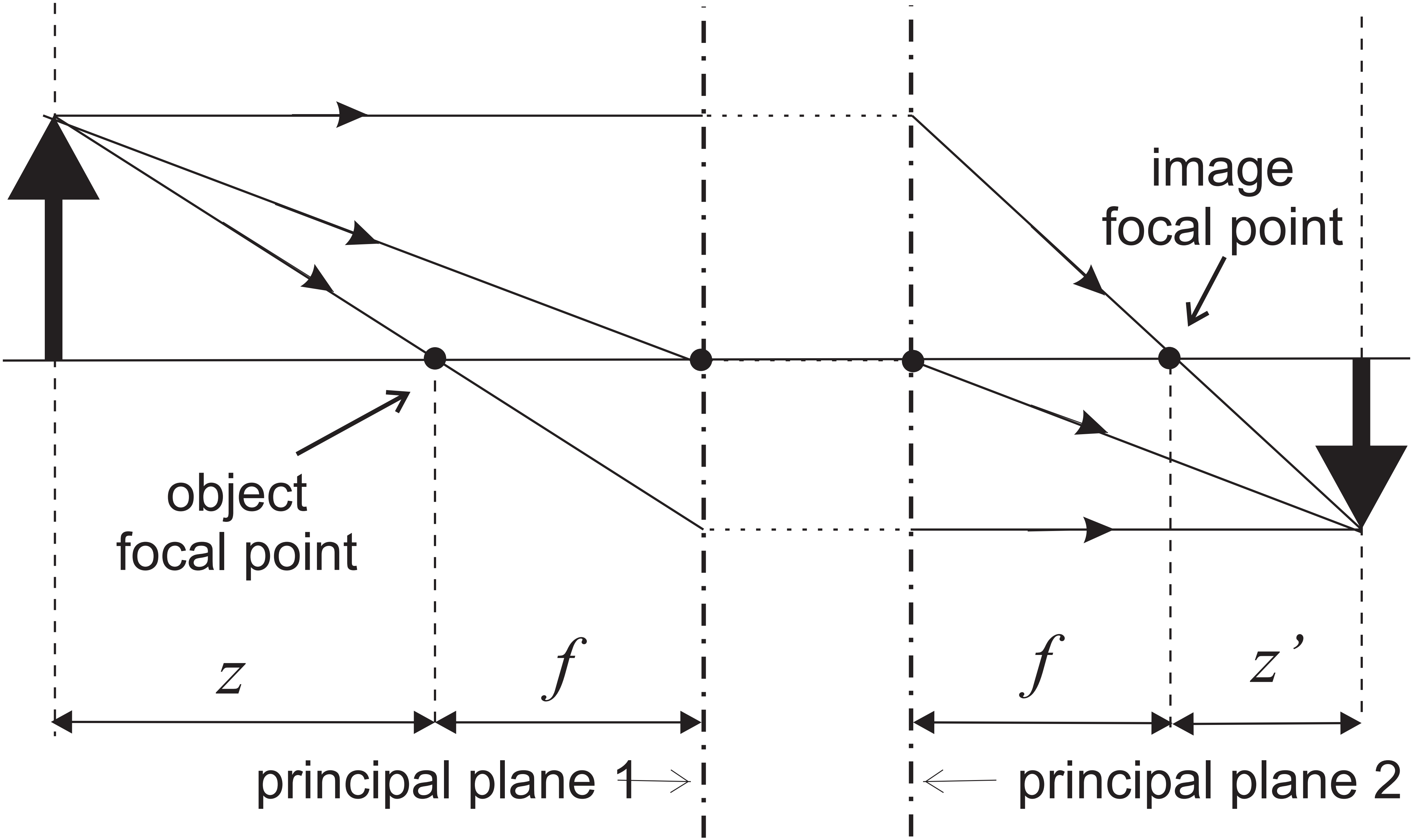

Depth of Field


DEPTH OF FIELD
DEPTH OF FIELD
DEPTH OF FIELD
DEPTH OF FIELD
DEPTH OF FIELD


F/4 50mm


F/4 85mm


F/4 180mm



image
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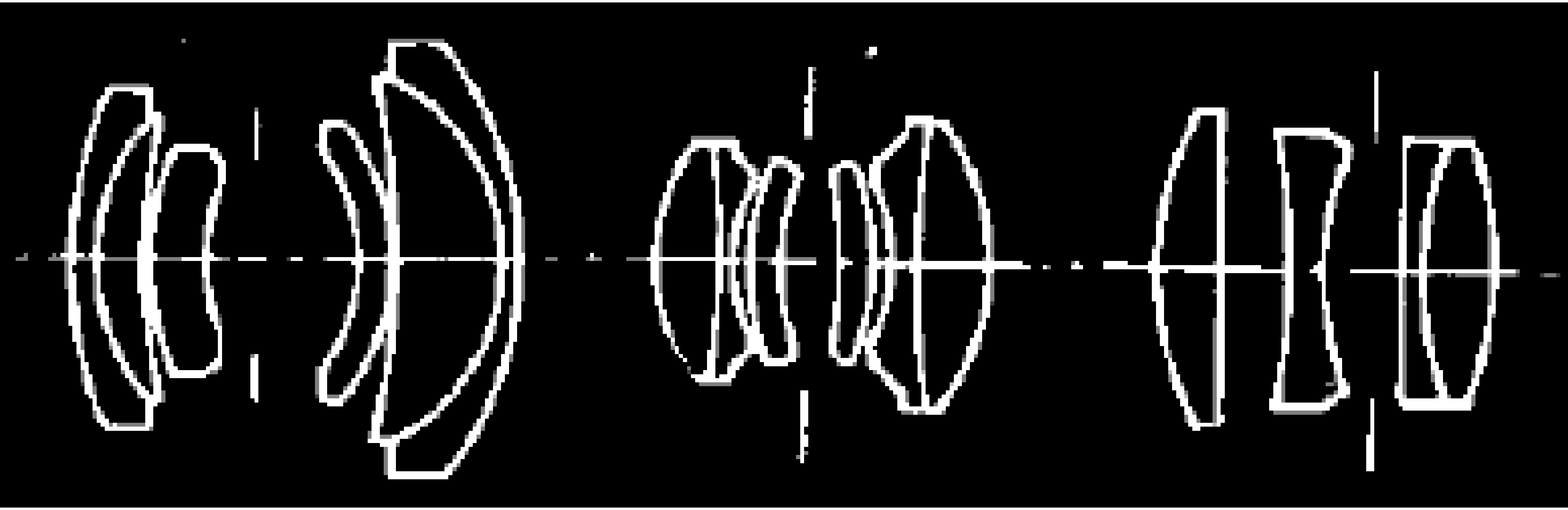
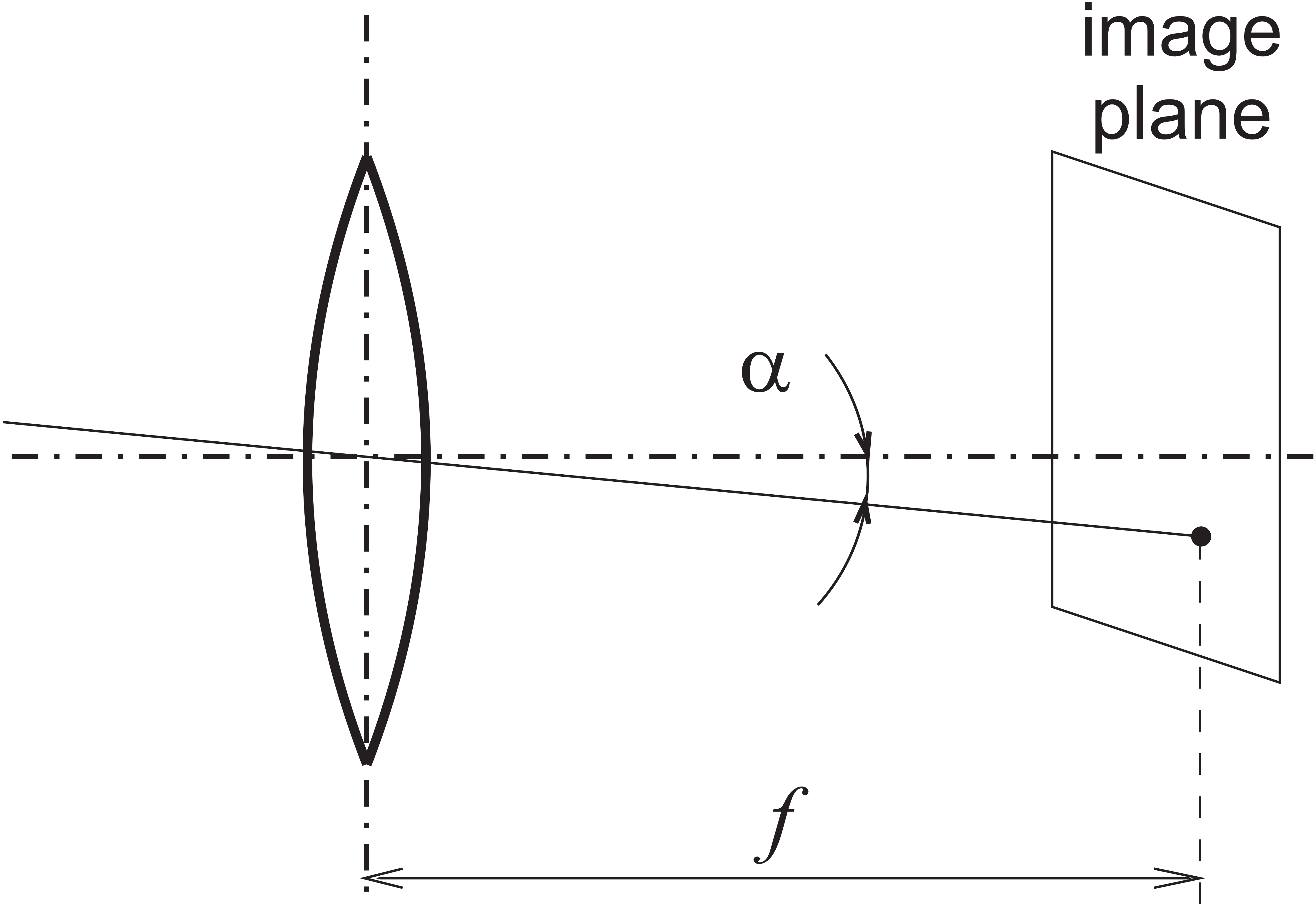
principal plane 1
principal plane 2

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Carl Zeiss
Planar 1,4/50
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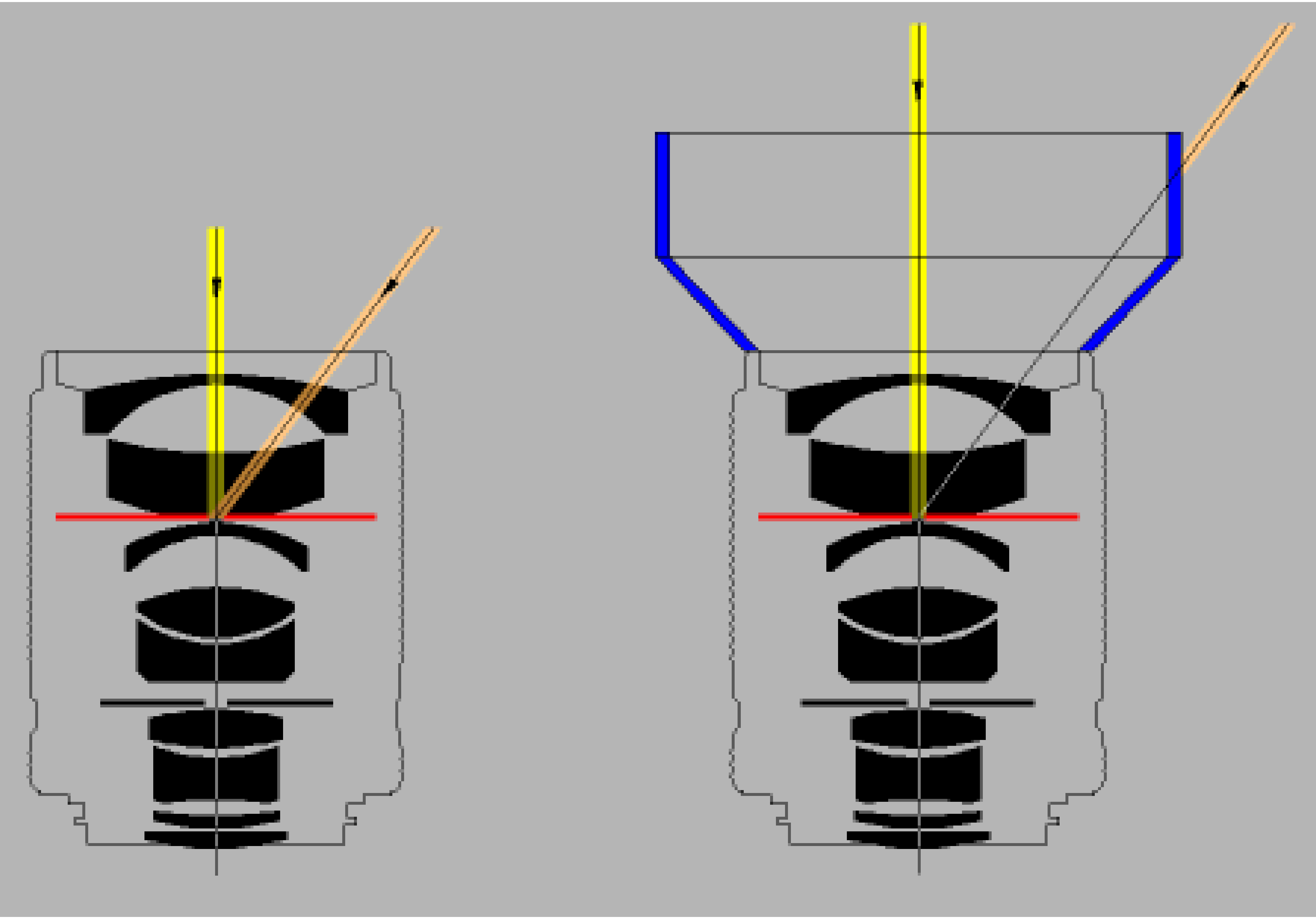


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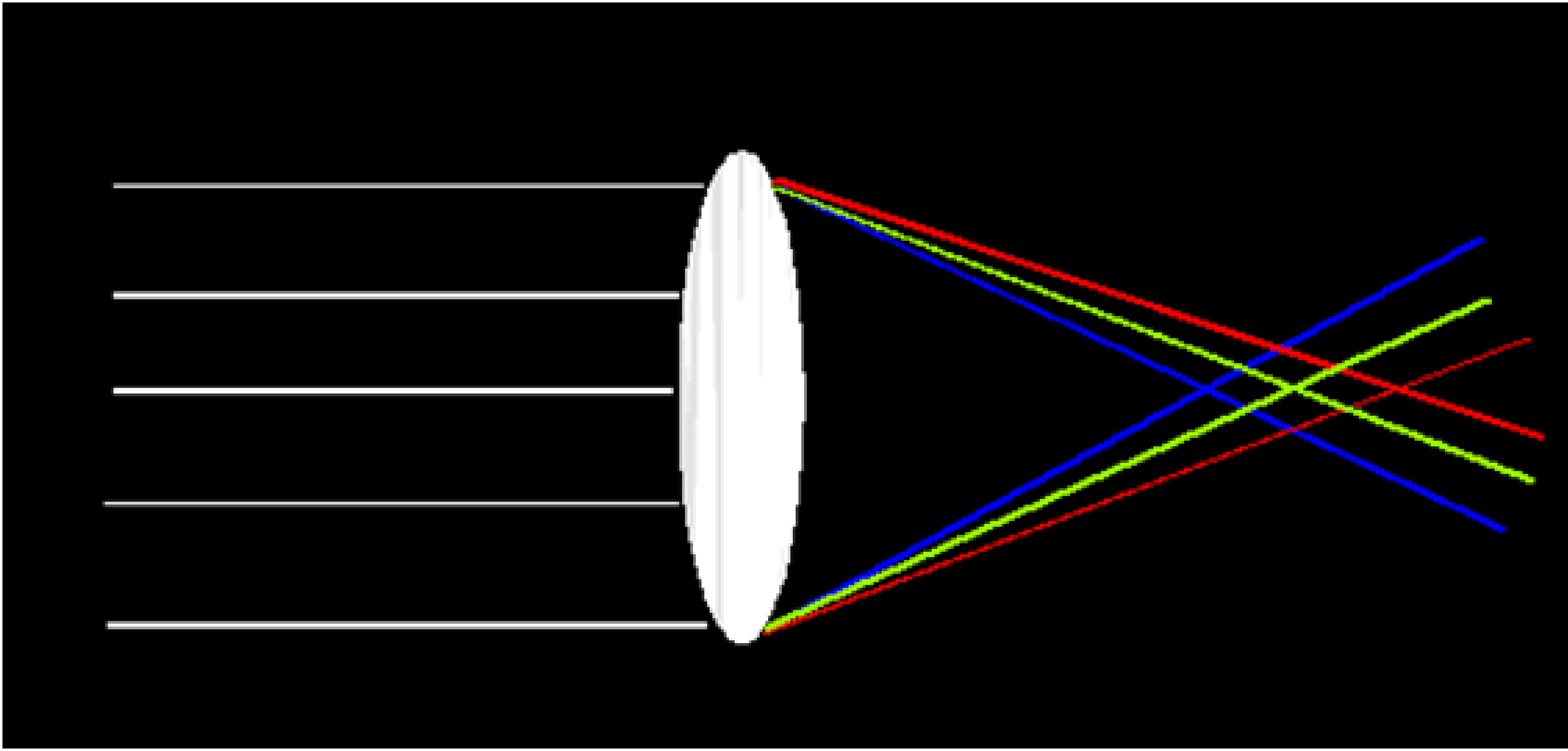


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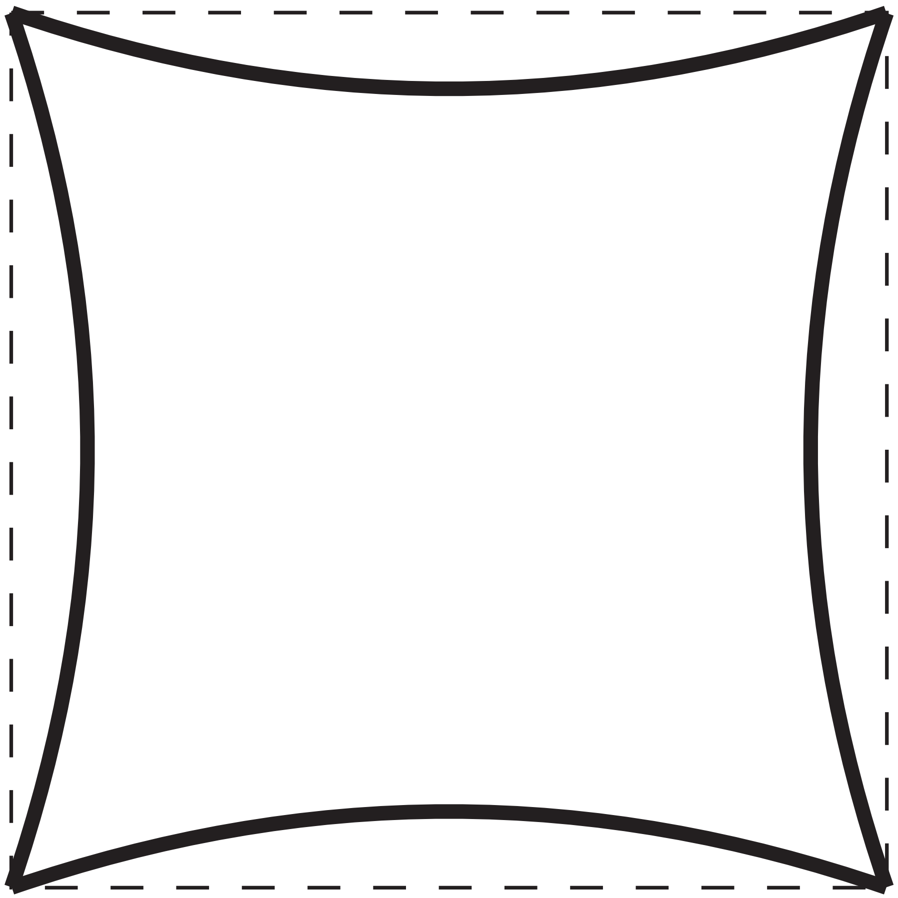
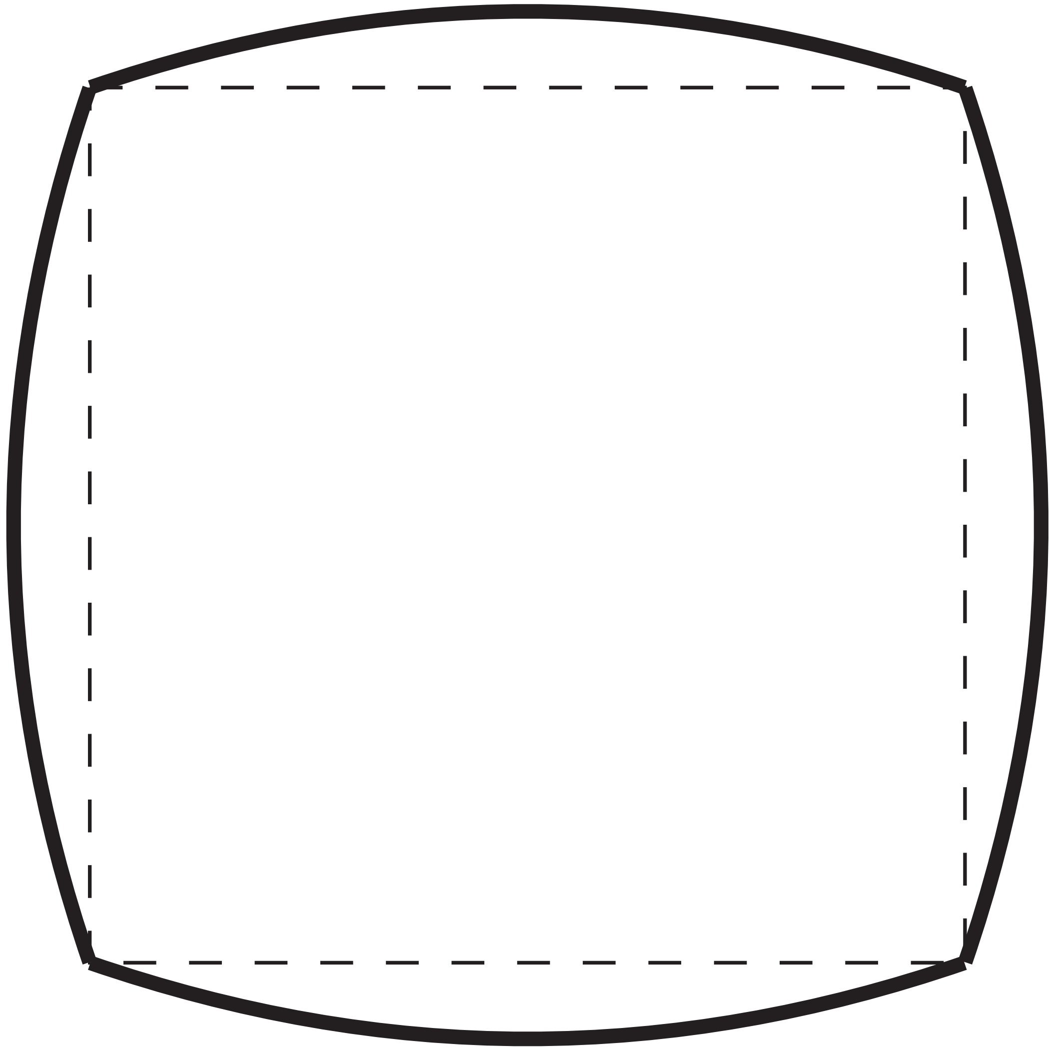
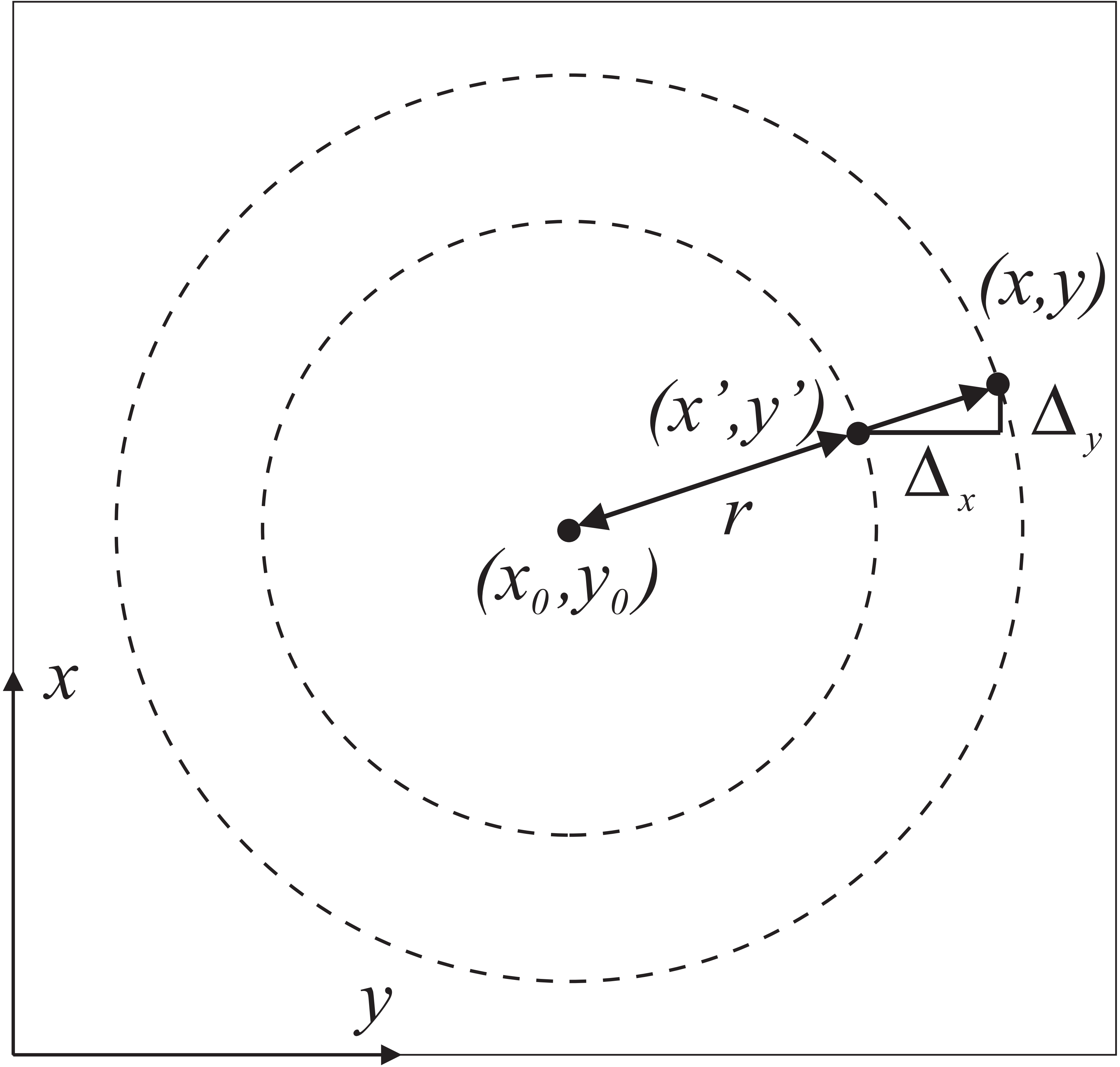


pincushion



barrel



(x,y)
(x',y')
Δy
Δx
r
(x0,y0)
x
y

PŘEHLED UŽITÉ MATEMATIKY I
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LINUX
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Pattern Classification
Swan
Practical English Usage
Oxford
Multiple View Geometry


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