Kybernetika a umělá inteligence
12. Evolutionary Algorithms: GA & GP

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Evolutionary Algorithms: GA & GP

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Contents

- **Genetic Algorithms (GAs)**
  - Simple Genetic Algorithm (SGA)
  - Areas for EA's applications
  - SGA example: Evolving strategy for an artificial ant problem
  - Schema theory – a schema, its properties, exponential growth equation and its consequences
- **Genetic Programming (GP)**
  - Tree representation, closure condition, 'strong typing'
  - Application of GP to artificial ant problem
  - Other examples

Evolutionary Algorithms: Characteristics

EA are stochastic optimization algorithms

- **Stochastic** – but not random search,
- **Use an analogy of natural evolution**
  - genetic inheritance (J.G. Mendel) – the basic principles of transference of hereditary factors from parent to offspring – genes, which present hereditary factors, are lined up on chromosomes.
  - strife for survival (Ch. Darwin) – the fundamental principle of natural selection – is the process by which individual organisms with favorable traits are more likely to survive and reproduce.
- **Not fast in some sense** – population-based algorithm,
- **Robust** – efficient in finding good solutions in difficult searches.

EA: Vocabulary

Vocabulary borrowed from natural genetics

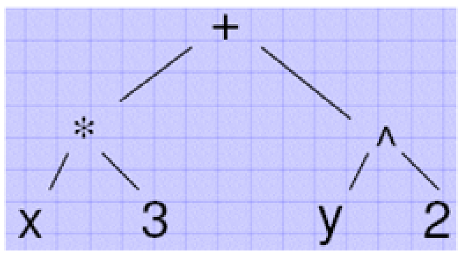
- **Individual** (chromosome + its quality measure "fitness value") – a solution to a problem.
- **Chromosome** – entire representation of the solution.
- **Fitness** – quality measure assigned to an individual, expresses how well it is adapted to the environment.
- **Gene** (also features, characters) – elementary units from which chromosomes are made.
  - each gene is located at certain place of the chromosome called locus (pl. loci),
  - a particular value for a locus is an allele.

    example: the "thickness" gene (which might be at locus 8) might be set to allele 2, meaning its second-thinnest value.
- **Genotype** – what's on the chromosome.
- **Phenotype** – what it means in the problem context (e.g., binary sequence may map to integers or reals, or order of execution, etc.).

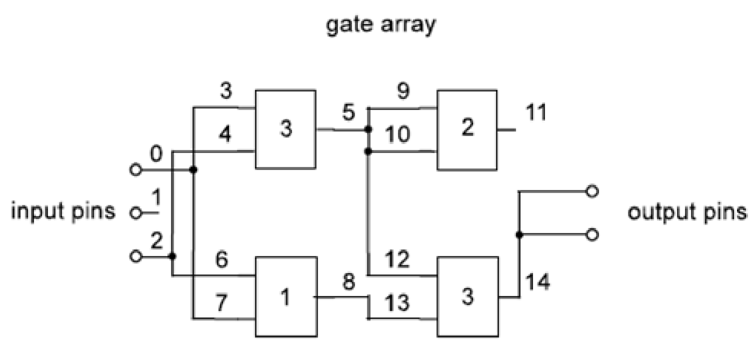
Representation

Problem can be represented as

- **binary string** – 1 0 1 1 0 1 1 0 0 1 0 1 1 0 1
- **real-valued string** – 3,24 1,78 -2,61
- **string of chars** – D→E→A→C→B
- or as a **tree**



- or as a **graph**, and others.



Evaluation Function

Objective (Fitness) function

- the only information about the sought solution the algorithm dispose of,
- must be defined for every possible chromosome.

Fitness function may be

- multimodal,
- discrete,
- multidimensional,
- nonlinear,
- noisy,
- multiobjective.

Fitness does not have to be define analytically

- simulation results,
- classification success rate.

Fitness function should not be too costly!!!

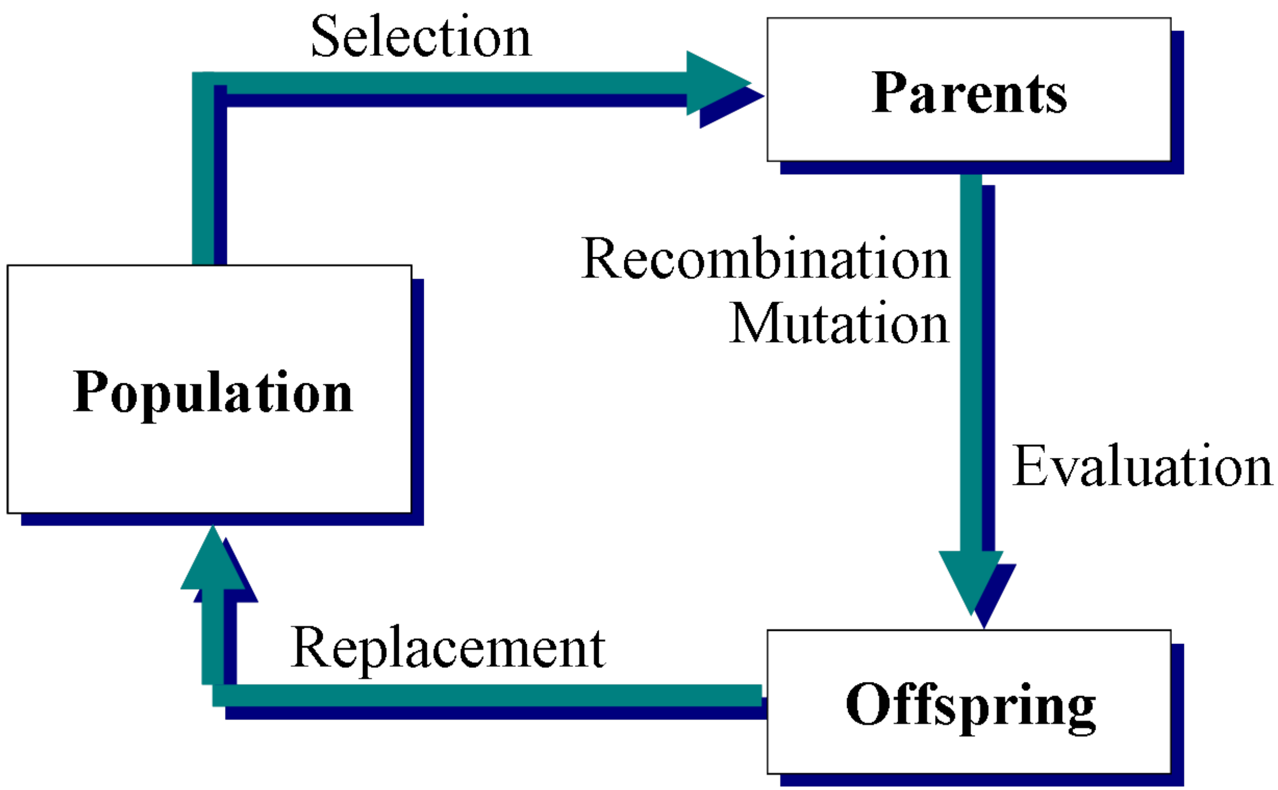
Example: Coding & Evaluation

Function optimization

- maximization of $f(x, y) = x^2 + y^2$,
- parameters x and y take on values from interval $< 0, 31 >$,
- and are code on 5 bits each.

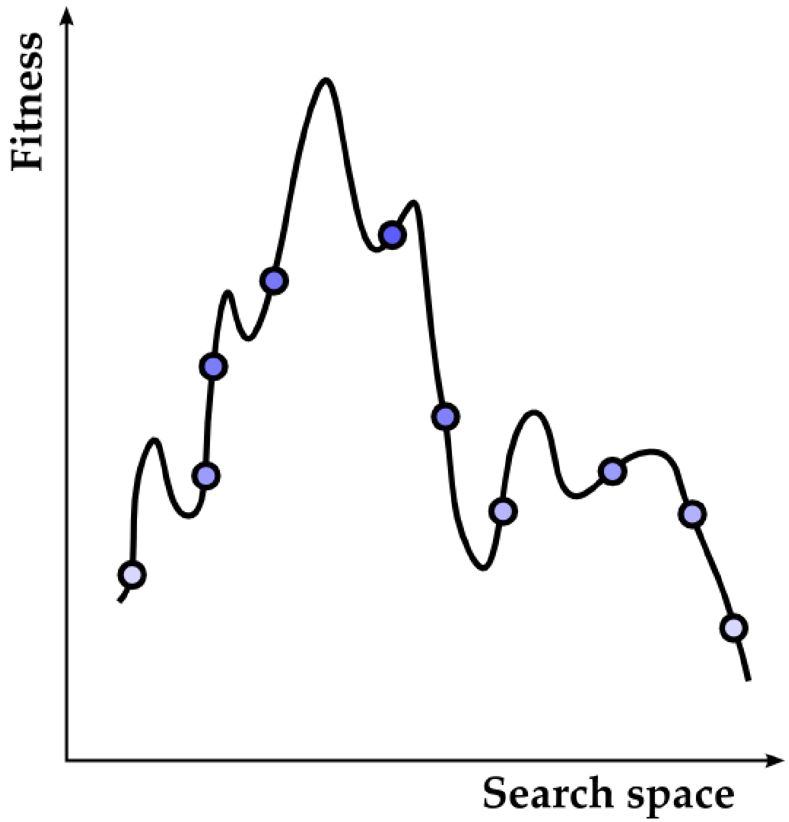
genotype	phenotype	fitness
0 0 0 0 0, 0 1 0 1 0	**0, 10**	**100**
0 0 0 0 1, 1 1 0 0 1	**1, 25**	**625 + 1 = 626**
0 1 0 1 1, 0 0 0 1 1	**11, 3**	**121 + 9 = 130**
1 1 0 1 1, 1 0 0 1 0	**27, 18**	**729 + 324 = 1053**

Evolutionary Cycle

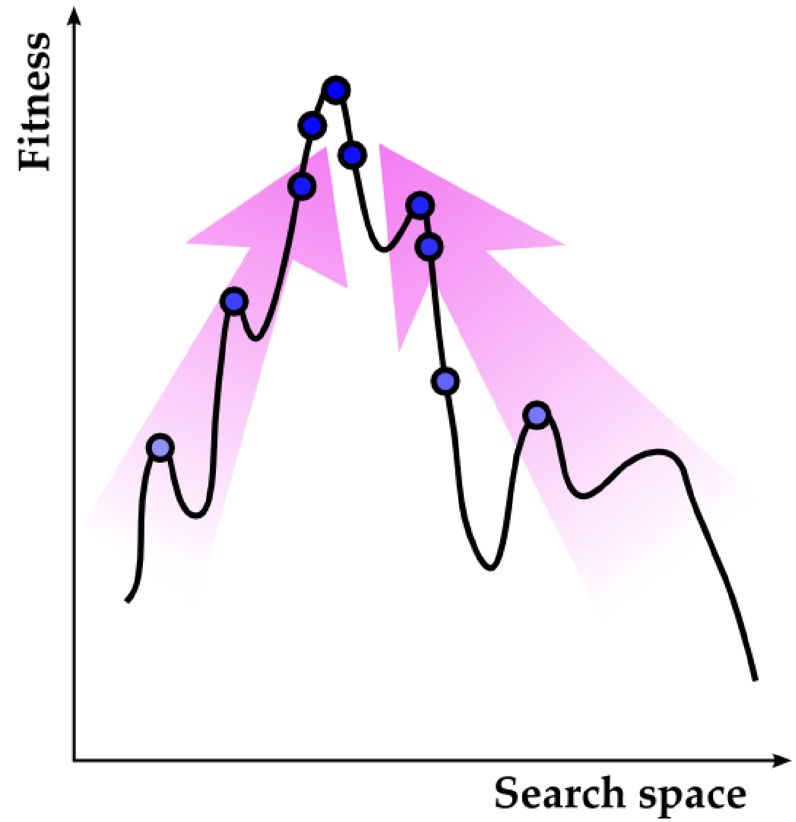


Idealized Illustration of Evolution

- Uniformly sampled population.



- Population converged to promising regions.



Initialization

Random

- randomly generated solutions,
- no prior information about the shape of the sought solution,
- relies just on "lucky" sampling of the whole search space by a finite set of samples.

Informed (pre-processing)

- (meta)heuristic routines used for seeding the initial population,
- biased random generator sampling regions of the search space that are likely to contain the sought solutions,

  + may help to find better solutions,

  + may speed up the search process,

  – may cause irreversible focusing of the search process on regions with local optima.

Reproduction

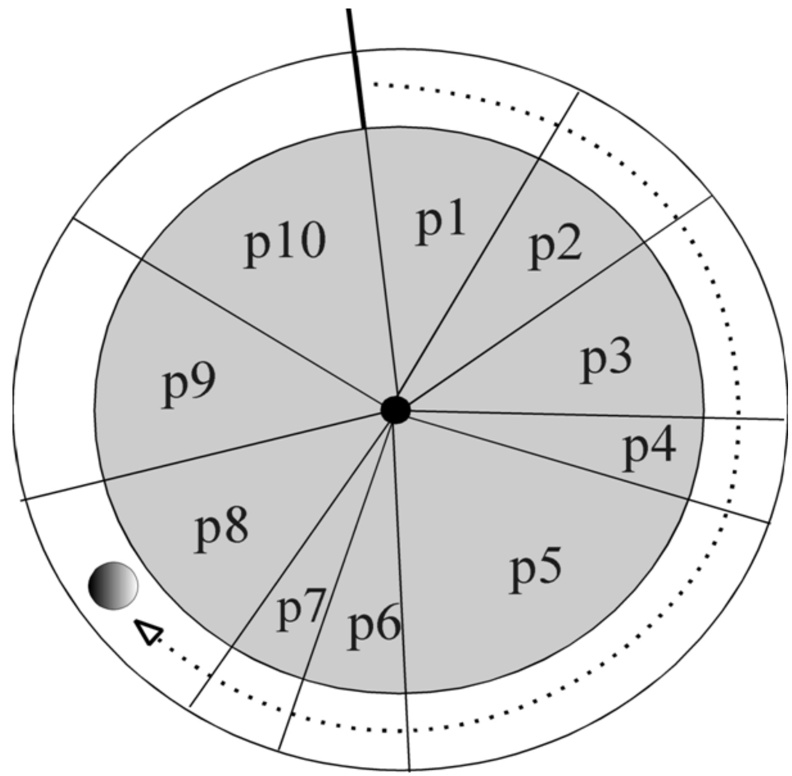
Models nature's survival-of-fittest principle

- prefers better individuals to the worse ones,
- still, every individual should have a chance to reproduce.

Roulette wheel

- probability of choosing some solution is directly proportional to its fitness value

$$P_i = \frac{f_i}{\sum_{j=1}^{PopSize} f_j}$$



Other methods

- Stochastic Universal Sampling,
- Tournament selection,
- Reminder Stochastic Sampling.

Genetic Operators: Crossover

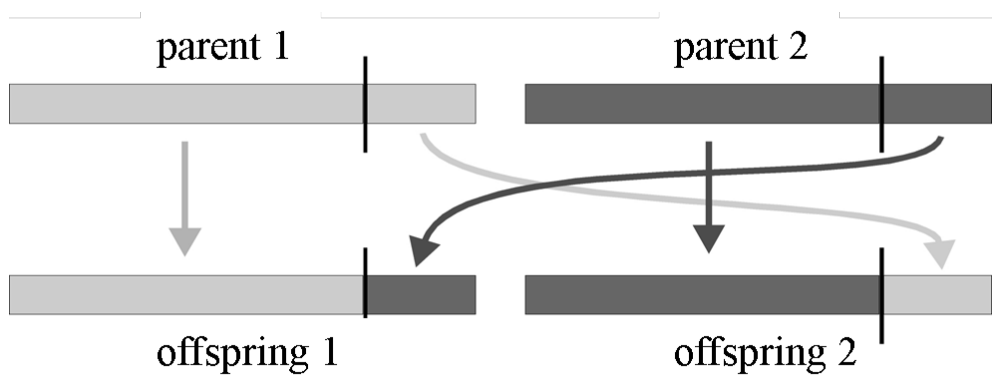
Idea

- given two well-fit solutions to the given problem, it is possible to get a new solution by properly mixing the two that is even better than both its parents.

Role of crossover

- sampling (exploration) of the search space

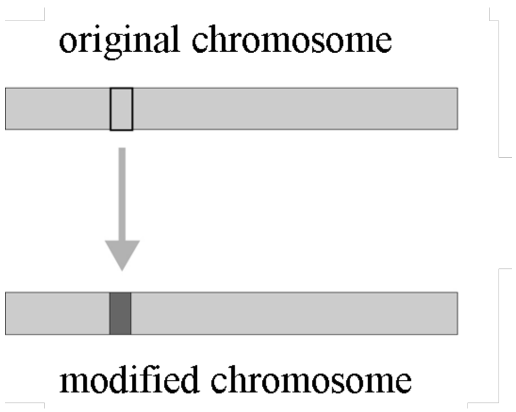
Example: 1-point crossover



Genetic Operators: Mutation

Role of mutation

- preservation of a population diversity,
- minimization of a possibility of loosing some important piece of genetic information.



Single bit-flipping mutation

Population

0	0	1	1	0	0	0	1	1	0
0	1	1	0	0	1	0	1	0	0
0	0	0	1	1	0	1	0	1	1
0	1	0	0	1	0	0	1	1	1
0	1	1	0	0	0	0	1	0	1
			.		.		.		
0	1	0	0	1	1	0	1	0	0

Example of missing genetic information

Replacement Strategy

Replacement strategy defines

- how big portion of the current generation will be replaced in each generation, and
- which solutions in the current population will be replaced by the newly generated ones.

Two extreme cases

- **Generational** – the whole old population is completely rebuild in each generation (analogy of short-lived species).
- **Steady-state** – just a few individuals are replaced in each generation (analogy of longer-lived species).

Application Areas of Evolutionary Algorithms

EAs are popular for their

- simplicity,
- effectiveness,
- robustness.

Holland: *"It's best used in areas where you don't really have a good idea what the solution might be. And it often surprises you with what you come up with."*

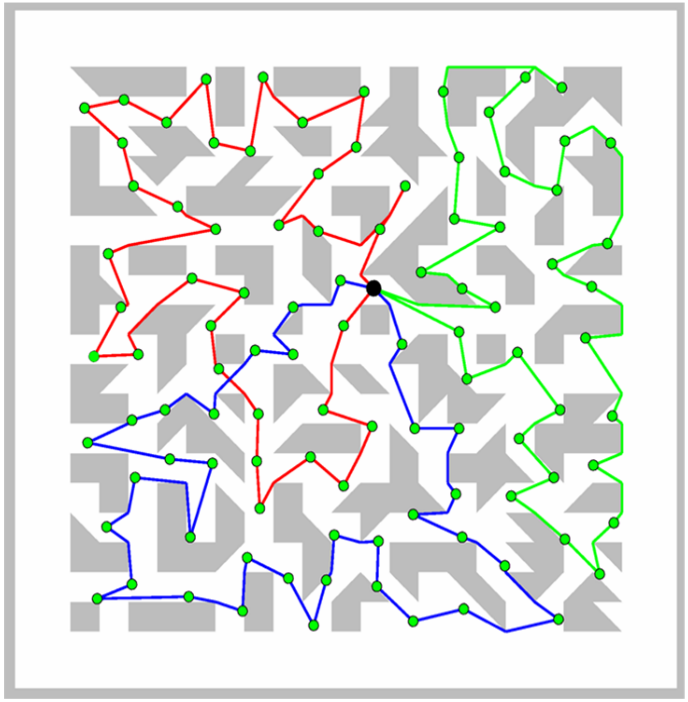
Applications

- control,
- engineering design,
- image processing,
- planning & scheduling,
- VLSI circuit design,
- network optimization & routing problems,
- optimal resource allocation,
- marketing,
- credit scoring & risk assessment,
- and many others.

Multiple Traveling Salesmen Problem

Rescue operations planning

- Given N cities and K agents, find an optimal tour for each agent so that every city is visited exactly once.
- A typical criterion to be optimized is the overall time spent by the squad (i.e., the slowest team member) during the task execution.



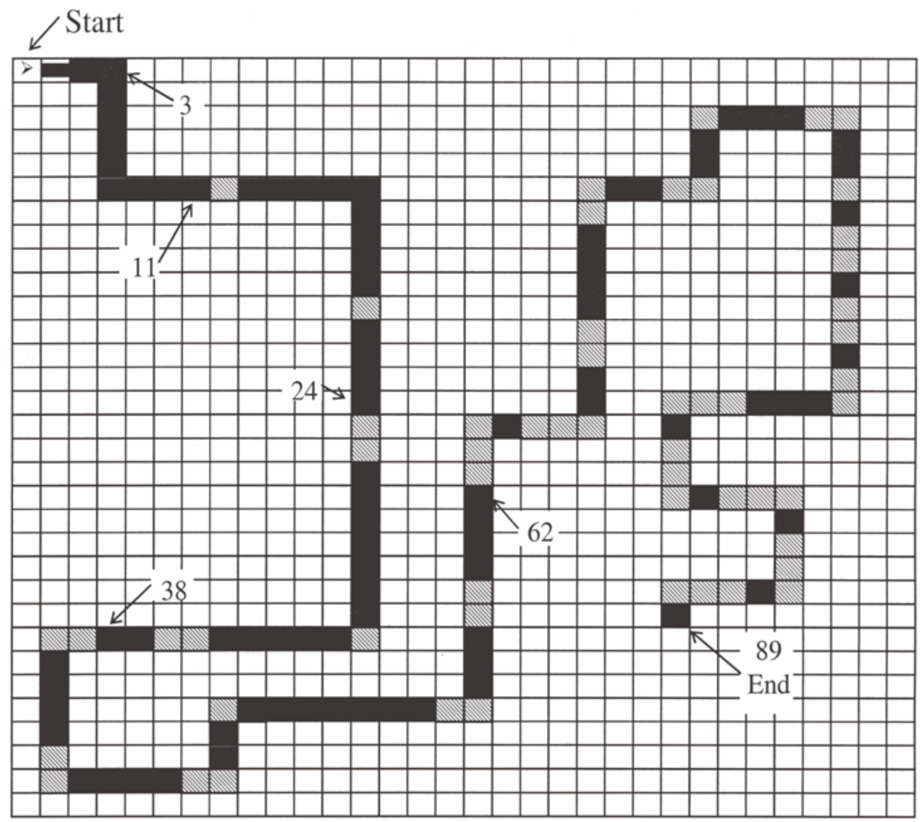
Artificial Ant Problem

Santa Fe trail

- 32×32 **grid** with 89 food pieces.
- **Obstacles**
  - $1\times, 2\times$ strait,
  - $1\times, 2\times, 3\times$ right/left.

Ant capabilities

- **detects** the food right in front of him in direction he faces.
- **actions** observable from outside
  - MOVE – makes a step and eats a food piece if there is some,
  - LEFT – turns left,
  - RIGHT – turns right,
  - NO-OP – no operation.



Goal is to find a strategy that would navigate an ant through the grid so that it finds all the food pieces in the given time (600 time steps).

Artificial Ant Problem: GA Approach

Collins a Jefferson 1991, standard GA using binary representation

Representation

- strategy represented by finite state machine,
- table of transitions coded as binary chromosomes of fixed length.

Example: 4-state FSM, 34-bit long chromosomes

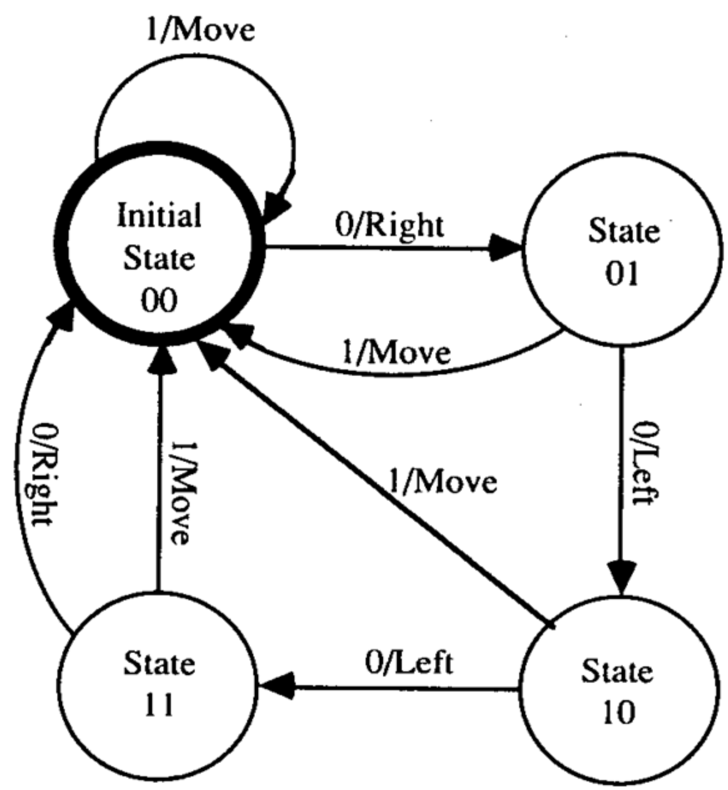
	Current state	Input	New state	Operation
1	00	0	01	10 = Right
2	00	1	00	11 = Move
3	01	0	10	01 = Left
4	01	1	00	11 = Move
5	10	0	11	01 = Left
6	10	1	00	11 = Move
7	11	0	00	10 = Right
8	11	1	00	11 = Move

00	0110	0011	1001	0011	1101	0011	0010	0011

Artificial Ant Problem: Example cont.

Ant behavior

- What happens if the ant hits an obstacle?
- What is strange with transition from state 10 to the initial state 00?
- When does the ant succeed?
- Is the number of states sufficient to solve the problem?
- Do all of the possible 32-bit chromosomes represent a feasible solution?



Artificial Ant Problem: GA result

Representation

- 32 states,
- $453 = 64 \times 7 + 5$ bits !!!

Population size: 65.536 !!!

Number of generations: 200

Total number of samples tried: 13×10^6 **!!!**

Genetic Programming (GP)

GP shares with GA the philosophy of survival and reproduction of the fittest and the analogy of naturally occurring genetic operators.

GP differs from GA in a representation, genetic operators and a scope of applications.

GP is extension of the conventional GA in which the structures undergoing adaptation are trees of dynamically varying size and shape representing hierarchical computer programs.

Applications

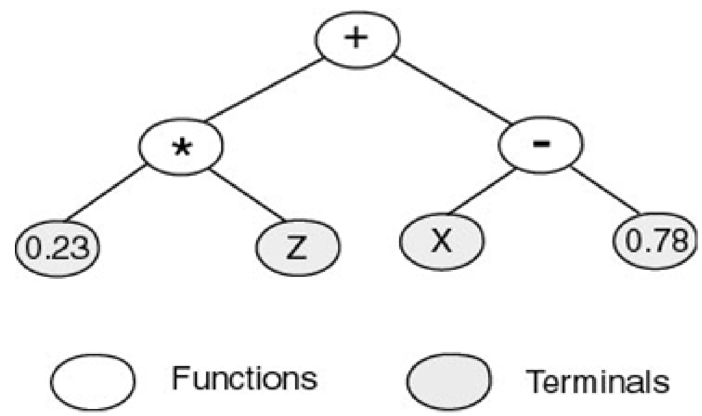
- learning programs,
- learning decision trees,
- learning rules,
- learning strategies,
- ...

GP: Representation

All possible trees are composed of **functions** (inner nodes) and **terminals** (leaf nodes) appropriate to the problem domain

- **Terminals** – inputs to the programs (independent variables), real, integer or logical constants, actions.
- **Functions**
  - arithmetic operators (+, -, *, /),
  - algebraic functions (sin, cos, exp, log),
  - logical functions (AND, OR, NOT),
  - conditional operators (If-Then-Else, cond?true:false),
  - and others.

Example: Tree representation of a LISP S-expression $0.23 * Z + X - 0.78$



Closure – each of the functions should be able to accept, as its argument, any value that may be returned by any function and any terminal.

GP Initialisation: Common Methods

GP needs a good tree-creation algorithm to create trees for the initial population and subtrees for subtree mutation.

Required characteristics:

- Light computationally complex; optimally linear in tree size.
- User control over expected tree size.
- User control over specific node appearance in trees.

`GROW` method (each branch has $depth \leq D$):

- nodes at depth $d < D_{max}$ randomly chosen from $F \cup T$,
- nodes at depth $d = D_{max}$ randomly chosen from T.

`FULL` method (each branch has $depth = D$):

- nodes at depth $d < D$ randomly chosen from function set F,
- nodes at depth $d = D$ randomly chosen from terminal set T.

`GROW`(depth d, max depth D)
Returns: a tree of depth $\leq D - d$

if ($d = D$) return a random terminal
else
  choose a random func or term f
  if (f is terminal) return f
  else
    for each argument a of f
      fill a with GROW($d + 1, D$)
    return f

GP Initialisation

Characteristics of GROW:

- does not have a size parameter – does not allow the user to create a desired size distribution,
- does not allow the user to define the expected probabilities of certain nodes appearing in trees,
- does not give the user much control over the tree structures generated.
- there is no appropriate way to create trees with either a fixed or average tree size or depth.

RAMPED HALF-AND-HALF – GROW & FULL method each deliver half of the initial population.
D is chosen between 2 to 6,

GP Initialisation

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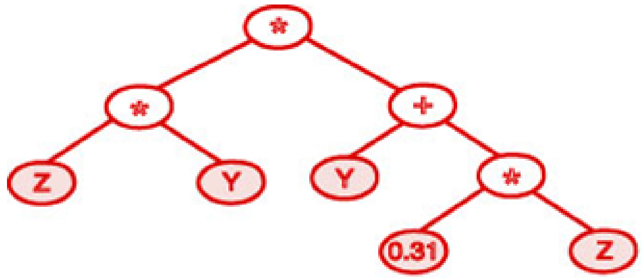
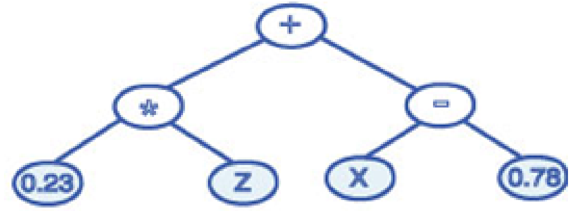
PTC1 is a modification of GROW that

- allows the user to define probabilities of appearance of functions within the tree,
- gives user a control over desired expected tree size, and guarantees that, on average, trees will be of that size.
- does not give the user any control over the variance in tree sizes,
- is fast, running in time near-linear in tree size.

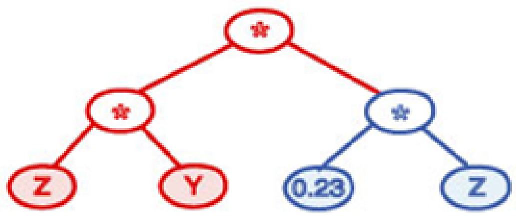
GP: Standard Crossover



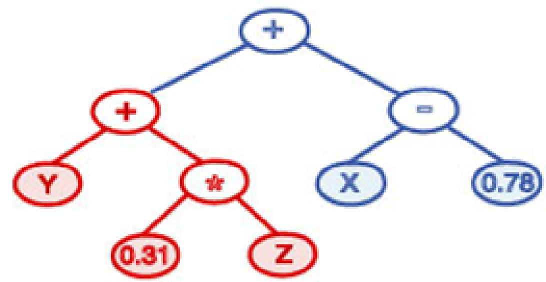
Parent 1: Z * Y * (Y + 0.31 * Z)



Parent 2: 0.23 * Z + X - 0.78



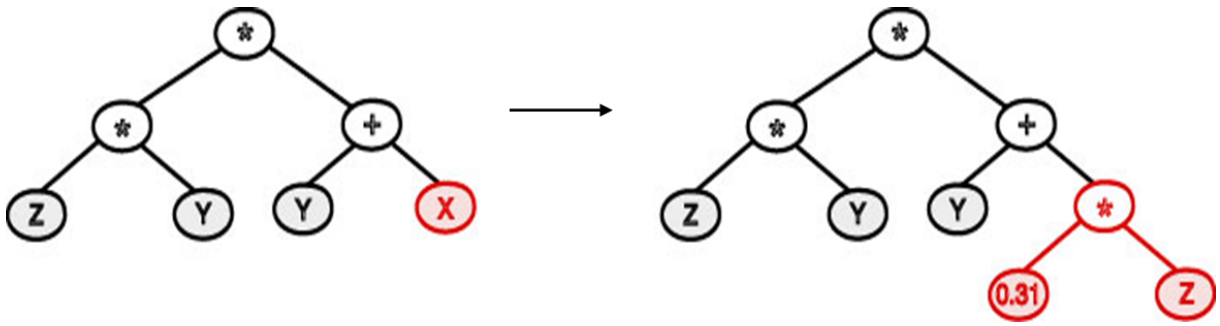
Child 1: 0.23 * Y * Z^2



Child 2: Y + 0.31 * Z + X - 0.78

GP: Subtree-Replacing Mutation

Mutation replaces selected subtree with a randomly generated new one.



GP: Selection

Commonly used are the fitness proportionate roulette wheel selection or the tournament selection.

Greedy over-selection is recommended for complex problems that require large populations (> 1000) – the motivation is to increase efficiency by increasing the chance of being selected to the fitter individuals in the population

- rank population by fitness and divide it into two groups:
  - group I: the fittest individuals that together accounting for $c = x\%$ of the sum of fitness values in the population,
  - group II: remaining less fit individuals.
- 80% of the time an individual is selected from group I in proportion to its fitness; 20% of the time, an individual is selected from group II.
- For population size = 1000, 2000, 4000, 8000, $x = 32\%$, 16%, 8%, 4%.

  %'s come from rule of thumb.

GP: Selection

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  %'s come from rule of thumb.

Example: Effect of greedy over-selection for the 6-multiplexer problem

Population size	I(M,i,z) without over-selection	I(M,i,z) with over-selection
1,000	343,000	33,000
2,000	294,000	18,000
4,000	160,000	24,000

Artificial Ant Problem

Santa Fe trail

- 32×32 **grid** with 89 food pieces.
- **Obstacles**
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Ant capabilities

- **detects** the food right in front of him in direction he faces.
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  - MOVE – makes a step and eats a food piece if there is some,
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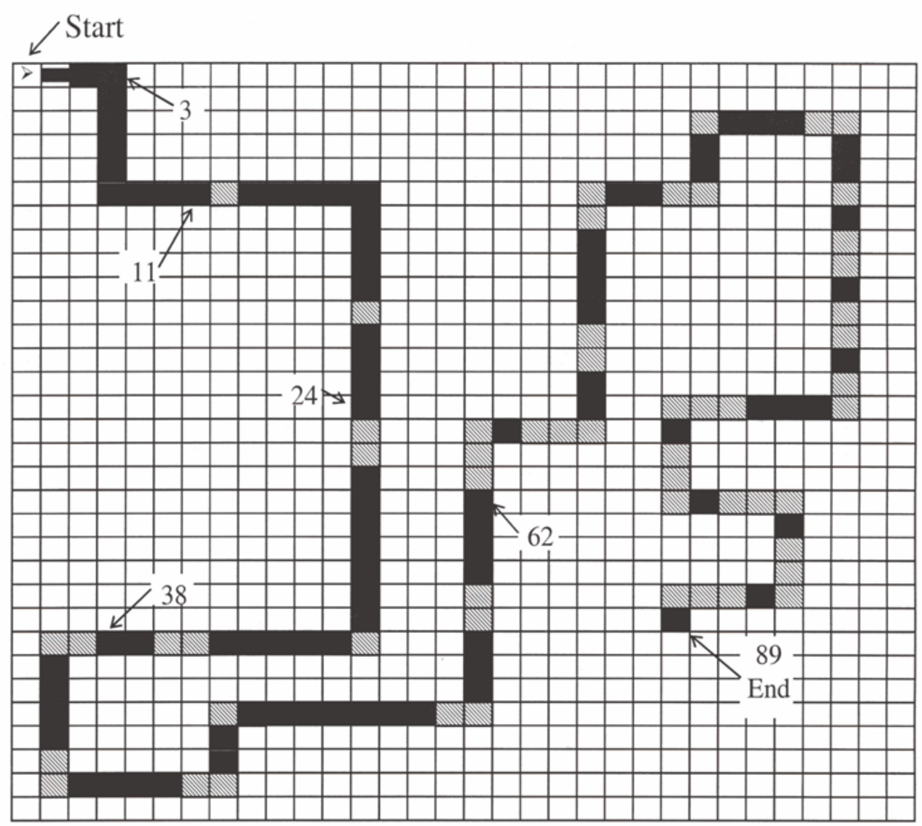


Goal is to find a strategy that would navigate an ant through the grid so that it finds all the food pieces in the given time (600 time steps).

Artificial Ant Problem: GP Approach

Terminals

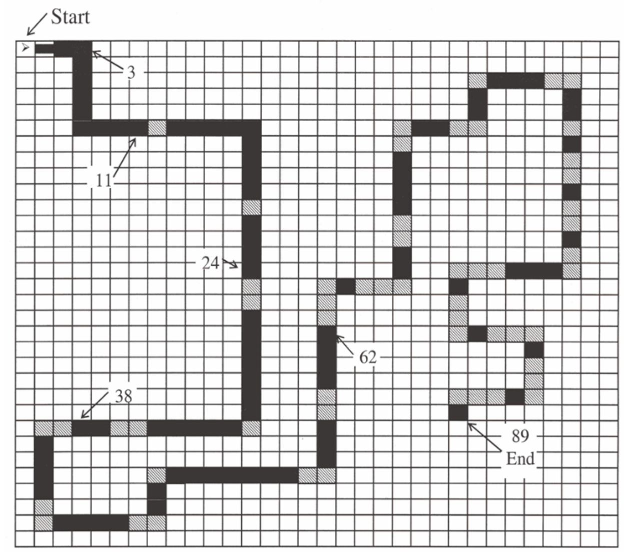
- motorial section,
- T = MOVE, LEFT, RIGHT.

Functions

- conditional IF-FOOD-AHEAD – food detection, 2 arguments (is/is_not food ahead),
- unconditional PROG2, PROG3 – sequence of 2/3 actions.

Ant repeats the program until time runs out (600 time steps) or all the food has been eaten.

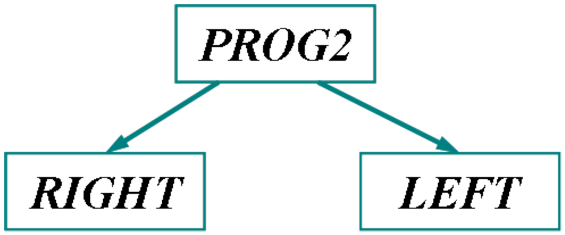
Santa Fe trail



Artificial Ant Problem: GP Approach cont.

Typical solutions in the initial population

- this solution



  completely fails in finding and eating the food,

- similarly this one

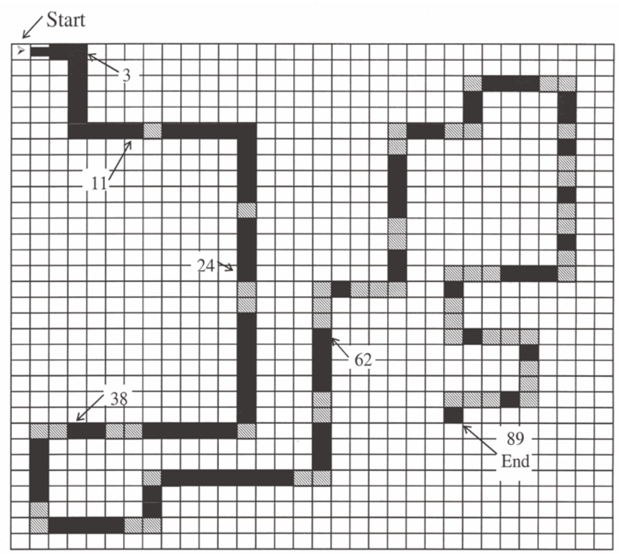
  (IF-FOOD-AHEAD (LEFT)(RIGHT)),

- this one

  (PROG2 (MOVE) (MOVE))

  just by chance finds 3 pieces of food.

Santa Fe trail



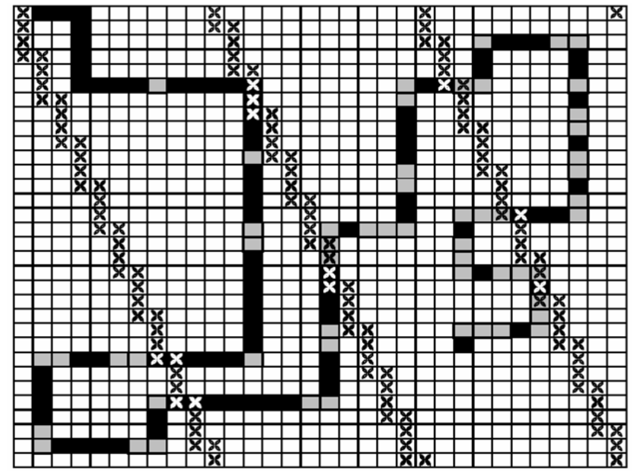
Artificial Ant Problem: GP Approach cont.

More interesting solutions

- **Quilter** – performs systematic exploration of the grid,

  (PROG3 (RIGHT)
  (PROG3 (MOVE) (MOVE) (MOVE))
  (PROG2 (LEFT) (MOVE)))

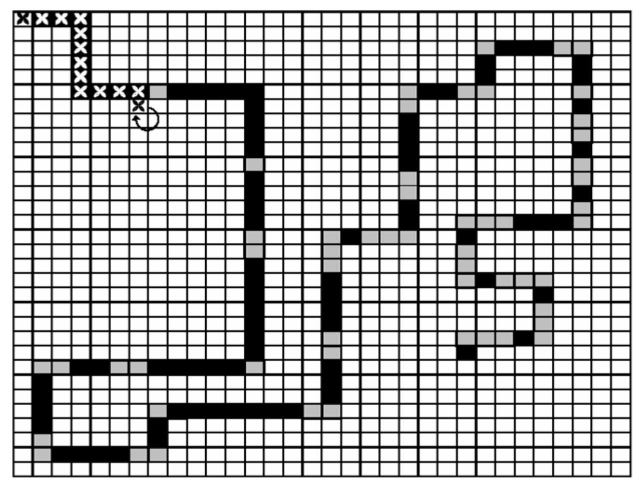
Quilter performance



- **Tracker** – perfectly tracks the food until the first obstacle occurs, then it gets trapped in an infinite loop.

  (IF-FOOD-AHEAD (MOVE) (RIGHT))

Tracker performance

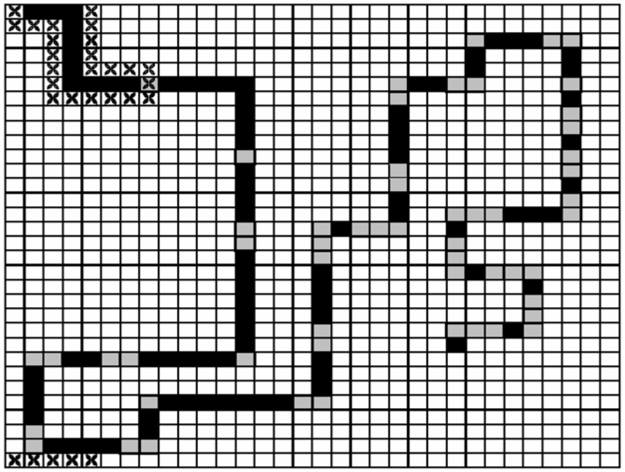


Artificial Ant Problem: GP Approach cont.

- **Avoider** – perfectly avoids food!!!

```
(I-F-A  (RIGHT)
        (I-F-A    (RIGHT)
                  (PROG2 (MOVE) (LEFT))))
```

Avoider performance



Average fitness in the initial population is 3.5

Artificial Ant Problem: GP result

In generation 21, the following solution was found that already navigates an ant so that he eats all 89 food pieces in the given time.

```
(I-F-A  (MOVE)
        (PROG3  (I-F-A    (MOVE)
                          (RIGHT)
                (PROG2    (RIGHT)
                          (PROG2    (LEFT)
                                    (RIGHT))))
                (PROG2    (I-F-A    (MOVE)
                                    (LEFT))
                          (MOVE))))
```

This program solves every trail with the obstacles of the same type as occurs in Santa Fe trail.

Compare the computational complexity with the GA approach!!!

GA approach: $65.536 \times 200 = 13 \times 10^6$ trials.

vs.

GP approach: $500 \times 21 = 10.500$ trials.

Example of GP in Action: Trigonometric Identity

Task is to find an equivalent expression to $cos(2x)$.

GP implementation:

- **Terminal set** $T = \{x, 1.0\}$.
- **Function set** $F = \{+, -, *, \%, sin\}$.
- **Training cases**: 20 pairs (x_i, y_i), where x_i are values evenly distributed in interval $(0, 2\pi)$.
- **Fitness**: Sum of absolute differences between desired y_i and the values returned by generated expressions.
- **Stopping criterion**: A solution found that gives the error less than 0.01.

Example of GP in Action: Trigonometric Identity cont.

1. run, 13^{th} **generation**

```
(- (- 1 (* (sin x) (sin x))) (* (sin x) (sinx)))
```

which equals (after editing) to $1 - 2 * sin^2 x$.

2. run, 34^{th} **generation**

```
(- 1 (* (* (sin x) (sin x)) 2))
```

which is just another way of writing the same expression.

Example of GP in Action: Trigonometric Identity cont.

1. run, 13^{th} **generation**

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(- (- 1 (* (sin x) (sin x))) (* (sin x) (sinx)))
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2. run, 34^{th} **generation**

```
(- 1 (* (* (sin x) (sin x)) 2))
```

which is just another way of writing the same expression.

3. run, 30^{th} **generation**

```
(sin (- (- 2 (* x 2))
        (sin (sin (sin (sin (sin (sin (* (sin (sin 1))
                                         (sin (sin 1))
                                         )))))))))
```

Example of GP in Action: Trigonometric Identity cont.

1. run, 13^{th} generation

```
(- (- 1 (* (sin x) (sin x))) (* (sin x) (sinx)))
```

which equals (after editing) to $1 - 2 * sin^2 x$.

2. run, 34^{th} generation

```
(- 1 (* (* (sin x) (sin x)) 2))
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which is just another way of writing the same expression.

3. run, 30^{th} generation

```
(sin (- (- 2 (* x 2))
        (sin (sin (sin (sin (sin (sin (* (sin (sin 1))
                                         (sin (sin 1))
                                         )))))))))
```

(2 minus the expression on the 2nd and 3rd rows) is almost $\pi/2$ so the discovered identity is

$$cos(2x) = sin(\pi/2 - 2x).$$

EA Materials: Reading, Demos, Software

Reading

- D. E. Goldberg: Genetic Algorithms in Search, Optimization, and Machine Learning, Addison-Wesley, 1989.
- Z. Michalewicz: Genetic Algorithms + Data Structures = Evolution Programs, Springer, 1998.
- Poli, R., Langdon, W., McPhee, N.F.: *A Field Guide to Genetic Programming*, 2008, http://www.gp-field-guide.org.uk/
- Koza, J.: Genetic Programming: On the Programming of Computers by Means of Natural Selection, MIT Press, 1992.

HUMIES: Human-Competitive Results

- http://www.genetic-programming.org/hc2011/combined.html

Demos

- M. Obitko: Introduction to genetic algorithms with java applets, http://cs.felk.cvut.cz/ xobitko/ga/

Software

- ECJ 16 – A Java-based Evolutionary Computation Research System http://cs.gmu.edu/ eclab/projects/ecj/