CORRECTION TO "A FORMAL BASIS FOR THE HEURISTIC DETERMINATION OF MINIMUM COST PATHS"

by

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Our paper on the use of heuristic information in graph searching defined a path-finding algorithm, A*, and proved that it had two important properties. In the notation of the paper, we proved that if the heuristic function $\hat{h}(n)$ is a lower bound on the true minimal cost from node n to a goal node, then A* is admissible; i.e., it would find a minimal cost path if any path to a goal node existed. Further, we proved that if the heuristic function also satisfied something called the consistency assumption, then A* was optimal; i.e., it expanded no more nodes than any other admissible algorithm A no more informed than A*. These results were summarized in a book† by one of us.

We are grateful to Professor Ron Colman of the California State University at Fullerton who wrote us to point out that our proof of optimality did not use the consistency assumption at all! Thus our result is true under more general conditions than we thought. Professor Colman correctly observed that in our proof of optimality given in Nilsson (Theorem 3-2, page 65, near the end of the proof), we claim to use the result $\hat{g}(n) = g(n)$ which does depend on the consistency assumption, whereas all we needed was the fact that $\hat{g}(n) \geq g(n)$ which does not depend on the consistency assumption at all but follows directly from the definition of $\hat{g}$. We leave it to the reader to verify for himself the result that we overlooked, namely, that the inequalities needed in the proof are still maintained using only $\hat{g} \geq g$.

We can make the corresponding strengthening corrections to Theorems 2 and 3 of our paper. In the proof of Theorem 2 (page 105) we also claim to use $\hat{g}(n) = g(n)$ when all we needed was the fact that $\hat{g}(n) \geq g(n)$. The proof of Theorem 2 also claimed to use Lemma 3 of the paper (page 105) which does depend on the consistency assumption. However, we needed only the corollary to Lemma 3, which can be proved without the consistency assumption.

The needed corollary and its proof are as follows:

Lemma 4 Suppose $\hat{h}(n) \leq h(n)$ for all n and suppose A* has not terminated. Then if node n is closed, $\hat{f}(n) \leq f(s)$.

†Nils J. Nilsson, Problem-Solving Methods in Artificial Intelligence, McGraw-Hill Book Co., New York, New York, 1971.

Proof   Node n is not a goal node, since by hypothesis A* has not terminated. By the corollary to Lemma 1 of the paper (page 103), at the time node n was closed there existed an open node n' on an optimal path P from s to a preferred goal node of s with $\hat{f}(n') \leq f(s)$. Since node n was closed in preference to n' it must have been the case that

$$\hat{f}(n) \leq \hat{f}(n') \leq f(s)$$

completing the proof.

The consistency assumption is still needed to prove Lemma 2 and 3 of the paper but our main theorems depended only on a weaker corollary. Lemma 2 (namely that $\hat{g}(n) = g(n)$ for closed nodes) is of independent interest since it allows a simpler implementation of algorithm A*. With $\hat{g}(n) = g(n)$ when node n is closed, there is no need to consider re-opening closed nodes. Thus, satisfying the consistency assumption may have practical consequences.

We thank Professor Colman for his observation.

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