## **1. Revision of Prerequisites. Hybrid Algorithms. Memetic Algorithms.**

Petr Pošík

Dept. of Cybernetics CTU FEE

This lecture is based on the following works: 'A taxonomy of hybrid metaheuristics' by E. G. Talbi, Journal of Heuristics, 2002, and 'A tutorial for competent memetic algorithms' by N. Krasnogor and J. Smith, IEEE Trans. on Evolutionary Computation, 2005.

## Contents

Revision

Hybrid Optimization Algorithms

Memetic Algorithms

Revision

**Hybrid Optimization Algorithms** 

Revision

Optimization, black-box, local-search, EAs

Hybrid Optimization Algorithms

Memetic Algorithms

## Revision

Revision Optimization, black-box, local-search, EAs

Hybrid Optimization Algorithms

Memetic Algorithms

✓ What is *optimization*? Give some examples of optimization tasks.

Revision Optimization, black-box, local-search, EAs

Hybrid Optimization Algorithms

Memetic Algorithms

✓ What is *optimization*? Give some examples of optimization tasks.

✓ In what courses did you meet optimization?

Revision Optimization, black-box, local-search, EAs

Hybrid Optimization Algorithms

- ✓ What is *optimization*? Give some examples of optimization tasks.
- ✓ In what courses did you meet optimization?
- ✓ What *sorts of optimization tasks* do you know? What are their characteristics?

#### Revision Optimization, black-box, local-search, EAs

Hybrid Optimization Algorithms

- ✓ What is *optimization*? Give some examples of optimization tasks.
- ✓ In what courses did you meet optimization?
- ✓ What *sorts of optimization tasks* do you know? What are their characteristics?
- ✓ What is the difference between *exact methods* and *heuristics*?

#### Revision Optimization, black-box, local-search, EAs

Hybrid Optimization Algorithms

- ✓ What is *optimization*? Give some examples of optimization tasks.
- ✓ In what courses did you meet optimization?
- ✓ What *sorts of optimization tasks* do you know? What are their characteristics?
- ✓ What is the difference between *exact methods* and *heuristics*?
- ✓ What is the difference between *constructive* and *generative* methods?

Revision Optimization, black-box, local-search, EAs

Hybrid Optimization Algorithms

- ✓ What is *optimization*? Give some examples of optimization tasks.
- ✓ In what courses did you meet optimization?
- ✓ What *sorts of optimization tasks* do you know? What are their characteristics?
- ✓ What is the difference between *exact methods* and *heuristics*?
- ✓ What is the difference between *constructive* and *generative* methods?
- ✓ What is the *black-box optimization*? What can you do to solve such problems?

Revision Optimization, black-box, local-search, EAs

Hybrid Optimization Algorithms

- ✓ What is *optimization*? Give some examples of optimization tasks.
- ✓ In what courses did you meet optimization?
- ✓ What *sorts of optimization tasks* do you know? What are their characteristics?
- ✓ What is the difference between *exact methods* and *heuristics*?
- ✓ What is the difference between *constructive* and *generative* methods?
- ✓ What is the *black-box optimization*? What can you do to solve such problems?
- ✓ What is the difference between *local* and *global* search?

Revision Optimization, black-box, local-search, EAs

Hybrid Optimization Algorithms

- What is *optimization*? Give some examples of optimization tasks.
- ✓ In what courses did you meet optimization?
- ✓ What *sorts of optimization tasks* do you know? What are their characteristics?
- ✓ What is the difference between *exact methods* and *heuristics*?
- ✓ What is the difference between *constructive* and *generative* methods?
- ✓ What is the *black-box optimization*? What can you do to solve such problems?
- ✓ What is the difference between *local* and *global* search?
- ✓ What is *hill-climbing*? How does it work? When does it work? What is the difference between *first-improving* and *best-improving* strategy?

Revision Optimization, black-box, local-search, EAs

Hybrid Optimization Algorithms

- ' What is *optimization*? Give some examples of optimization tasks.
- ✓ In what courses did you meet optimization?
- ✓ What *sorts of optimization tasks* do you know? What are their characteristics?
- ✓ What is the difference between *exact methods* and *heuristics*?
- ✓ What is the difference between *constructive* and *generative* methods?
- ✓ What is the *black-box optimization*? What can you do to solve such problems?
- ✓ What is the difference between *local* and *global* search?
- ✓ What is *hill-climbing*? How does it work? When does it work? What is the difference between *first-improving* and *best-improving* strategy?
- ✓ What is the problem with local search methods? What are the possibilities to solve it? Name other single-state methods and describe their principles.

Revision Optimization, black-box, local-search, EAs

Hybrid Optimization Algorithms

- ' What is *optimization*? Give some examples of optimization tasks.
- ✓ In what courses did you meet optimization?
- ✓ What *sorts of optimization tasks* do you know? What are their characteristics?
- ✓ What is the difference between *exact methods* and *heuristics*?
- ✓ What is the difference between *constructive* and *generative* methods?
- ✓ What is the *black-box optimization*? What can you do to solve such problems?
- ✓ What is the difference between *local* and *global* search?
- ✓ What is *hill-climbing*? How does it work? When does it work? What is the difference between *first-improving* and *best-improving* strategy?
- What is the problem with local search methods? What are the possibilities to solve it? Name other single-state methods and describe their principles.
- ✔ What is the difference between *single-state* and *population-based* methods? What is the benefit of using a population?

Revision Optimization, black-box, local-search, EAs

Hybrid Optimization Algorithms

- ✓ What is *optimization*? Give some examples of optimization tasks.
- ✓ In what courses did you meet optimization?
- ✓ What *sorts of optimization tasks* do you know? What are their characteristics?
- ✓ What is the difference between *exact methods* and *heuristics*?
- ✓ What is the difference between *constructive* and *generative* methods?
- ✓ What is the *black-box optimization*? What can you do to solve such problems?
- ✓ What is the difference between *local* and *global* search?
- ✓ What is *hill-climbing*? How does it work? When does it work? What is the difference between *first-improving* and *best-improving* strategy?
- What is the problem with local search methods? What are the possibilities to solve it? Name other single-state methods and describe their principles.
- ✓ What is the difference between *single-state* and *population-based* methods? What is the benefit of using a population?
- ✓ Describe a simple EA. What are its main components?

Revision Optimization, black-box, local-search, EAs

Hybrid Optimization Algorithms

- ✓ What is *optimization*? Give some examples of optimization tasks.
- ✓ In what courses did you meet optimization?
- ✓ What *sorts of optimization tasks* do you know? What are their characteristics?
- ✓ What is the difference between *exact methods* and *heuristics*?
- ✓ What is the difference between *constructive* and *generative* methods?
- ✓ What is the *black-box optimization*? What can you do to solve such problems?
- ✓ What is the difference between *local* and *global* search?
- ✓ What is *hill-climbing*? How does it work? When does it work? What is the difference between *first-improving* and *best-improving* strategy?
- ✓ What is the problem with local search methods? What are the possibilities to solve it? Name other single-state methods and describe their principles.
- ✓ What is the difference between *single-state* and *population-based* methods? What is the benefit of using a population?
- ✓ Describe a simple EA. What are its main components?
- ✓ When should I use an EA? When should I use a hill-climber?

Revision Optimization, black-box, local-search, EAs

Hybrid Optimization Algorithms

- ✓ What is *optimization*? Give some examples of optimization tasks.
- ✓ In what courses did you meet optimization?
- ✓ What *sorts of optimization tasks* do you know? What are their characteristics?
- ✓ What is the difference between *exact methods* and *heuristics*?
- ✓ What is the difference between *constructive* and *generative* methods?
- ✓ What is the *black-box optimization*? What can you do to solve such problems?
- ✓ What is the difference between *local* and *global* search?
- ✓ What is *hill-climbing*? How does it work? When does it work? What is the difference between *first-improving* and *best-improving* strategy?
- ✓ What is the problem with local search methods? What are the possibilities to solve it? Name other single-state methods and describe their principles.
- ✓ What is the difference between *single-state* and *population-based* methods? What is the benefit of using a population?
- ✓ Describe a simple EA. What are its main components?
- ✓ When should I use an EA? When should I use a hill-climber?
- ✓ Do you know other *metaheuristics*?

### Revision

Hybrid Optimization Algorithms

Optimization

Algorithms

Hybridization and Classification of the approaches

The 4 Main Groups of Hybrids

Other Attributes of Hybrid Algorithms

Hybridization Design Grammar

Oraniniai

Memetic Algorithms

## **Hybrid Optimization Algorithms**

## **Optimization Algorithms**

### Revision

Hybrid Optimization Algorithms

- Optimization Algorithms
- Hybridization and Classification of the approaches
- The 4 Main Groups of Hybrids

Other Attributes of Hybrid Algorithms

Hybridization Design Grammar

Memetic Algorithms

### Single-state (single-solution) methods:

- ✓ local search (LS)
- ✓ greedy heuristic (GH)
- ✓ simulated annealing (SA)
- ✓ tabu search (TS)

**v** ...

### Population-based methods:

- ✓ genetic and evolutionary algorithms (GA, EA, ES, EP, GP, ...)
- ✓ ant colonies (ACO)
- ✓ particle swarm optimization (PSO)
- ✓ scatter search (SS)
- ...

## Hybridization and Classification of the approaches

#### Revision

Hybrid Optimization Algorithms

Optimization Algorithms

Hybridization and Classification of the approaches

The 4 Main Groups of Hybrids

Other Attributes of Hybrid Algorithms

Hybridization Design Grammar

Memetic Algorithms

Hybrid algorithm:

✓ a combination of (at least some features of) at least 2 of the above methods

## Hybridization and Classification of the approaches

#### Revision

Hybrid Optimization Algorithms

Optimization Algorithms

Hybridization and Classification of the approaches

The 4 Main Groups of Hybrids

Other Attributes of Hybrid Algorithms

Hybridization Design Grammar

Memetic Algorithms

Hybrid algorithm:

✓ a combination of (at least some features of) at least 2 of the above methods Why?

✓ The best results for many optimization problems were obtained by hybrid algorithms.

### Revision

Hybrid Optimization Algorithms

Optimization Algorithms

Hybridization and Classification of the approaches

The 4 Main Groups of Hybrids

Other Attributes of Hybrid Algorithms Hybridization Design

Grammar

Memetic Algorithms

## Hybrid algorithm:

- ✓ a combination of (at least some features of) at least 2 of the above methods Why?
- ✓ The best results for many optimization problems were obtained by hybrid algorithms.

Talbi [Tal02] classifies hybrid approaches based on several independent design issues:

- ✓ Low-level vs. high-level hybrid:
  - ✗ In low-level hybrid, a particular function of one algorithm is replaced by another optimization algorithm.
  - ✗ In high-level hybrid, the different optimization algorithms are combined without changing their inner workings.
- ✓ Relay vs. teamwork hybrid:
  - ✗ In relay hybridization, the individual algorithms are applied one after another, each using the output of the previous algorithm as its input.
  - ✗ In teamwork hybridization, each algorithm performs the search independently of the others.

[Tal02] E. G. Talbi. A taxonomy of hybrid metaheuristics. Journal of Heuristics, 8(5):541–564, 2002.

LRH (Low-level Relay Hybrid): An optimization method is embedded into a single-solution algorithm.

LRH (Low-level Relay Hybrid): An optimization method is embedded into a single-solution algorithm.

✓ LS embedded in SA. (What is the difference compared to a pure SA?)

LRH (Low-level Relay Hybrid): An optimization method is embedded into a single-solution algorithm.LS embedded in SA. (What is the difference compared to a pure SA?)

LTH (Low-level Teamwork Hybrid): An optimization method is embedded into a population-based alg.

LRH (Low-level Relay Hybrid): An optimization method is embedded into a single-solution algorithm.✓ LS embedded in SA. (What is the difference compared to a pure SA?)

LTH (Low-level Teamwork Hybrid): An optimization method is embedded into a population-based alg.

- ✓ LS embedded in EA. (Memetic algorithm, see below.)
- ✓ Greedy constructive heuristic in ACO.

LRH (Low-level Relay Hybrid): An optimization method is embedded into a single-solution algorithm.✓ LS embedded in SA. (What is the difference compared to a pure SA?)

LTH (Low-level Teamwork Hybrid): An optimization method is embedded into a population-based alg.

- ✓ LS embedded in EA. (Memetic algorithm, see below.)
- ✓ Greedy constructive heuristic in ACO.

HRH (High-level Relay Hybrid): Several optimization algorithms executed in a sequence.

LRH (Low-level Relay Hybrid): An optimization method is embedded into a single-solution algorithm.

✓ LS embedded in SA. (What is the difference compared to a pure SA?)

LTH (Low-level Teamwork Hybrid): An optimization method is embedded into a population-based alg.

- ✓ LS embedded in EA. (Memetic algorithm, see below.)
- ✓ Greedy constructive heuristic in ACO.

HRH (High-level Relay Hybrid): Several optimization algorithms executed in a sequence.

- ✓ LS builds init. population for EA. The results of EA are processed by TS.
- ✓ Iterated local search, ILS.
- ✓ Variable neighborhood search, VNS.
- ✓ Greedy randomized adaptive search procedure, GRASP.

LRH (Low-level Relay Hybrid): An optimization method is embedded into a single-solution algorithm.

✓ LS embedded in SA. (What is the difference compared to a pure SA?)

LTH (Low-level Teamwork Hybrid): An optimization method is embedded into a population-based alg.

- ✓ LS embedded in EA. (Memetic algorithm, see below.)
- ✓ Greedy constructive heuristic in ACO.

HRH (High-level Relay Hybrid): Several optimization algorithms executed in a sequence.

- ✓ LS builds init. population for EA. The results of EA are processed by TS.
- ✓ Iterated local search, ILS.
- ✓ Variable neighborhood search, VNS.
- ✓ Greedy randomized adaptive search procedure, GRASP.

**HTH (High-level Teamwork Hybrid):** Several optimization algorithms executed in parallel, cooperating to find the optimum.

LRH (Low-level Relay Hybrid): An optimization method is embedded into a single-solution algorithm.

✓ LS embedded in SA. (What is the difference compared to a pure SA?)

LTH (Low-level Teamwork Hybrid): An optimization method is embedded into a population-based alg.

- ✓ LS embedded in EA. (Memetic algorithm, see below.)
- ✓ Greedy constructive heuristic in ACO.

HRH (High-level Relay Hybrid): Several optimization algorithms executed in a sequence.

- ✓ LS builds init. population for EA. The results of EA are processed by TS.
- ✓ Iterated local search, ILS.
- ✓ Variable neighborhood search, VNS.
- ✓ Greedy randomized adaptive search procedure, GRASP.

**HTH (High-level Teamwork Hybrid):** Several optimization algorithms executed in parallel, cooperating to find the optimum.

✓ Island model of parallel GAs (one of the next lectures).

LRH (Low-level Relay Hybrid): An optimization method is embedded into a single-solution algorithm.

✓ LS embedded in SA. (What is the difference compared to a pure SA?)

LTH (Low-level Teamwork Hybrid): An optimization method is embedded into a population-based alg.

- ✓ LS embedded in EA. (Memetic algorithm, see below.)
- ✓ Greedy constructive heuristic in ACO.

HRH (High-level Relay Hybrid): Several optimization algorithms executed in a sequence.

- ✓ LS builds init. population for EA. The results of EA are processed by TS.
- ✓ Iterated local search, ILS.
- ✓ Variable neighborhood search, VNS.
- ✓ Greedy randomized adaptive search procedure, GRASP.

**HTH (High-level Teamwork Hybrid):** Several optimization algorithms executed in parallel, cooperating to find the optimum.

✓ Island model of parallel GAs (one of the next lectures).

The classification is not always unambigous, it depends on personal view.

- ✓ Is GRASP a local search with its initialization phase replaced by a constructive heuristic (LRH), or
- ✓ is it a sequence of 2 self-contained algorithms (HRH)?

## **Other Attributes of Hybrid Algorithms**

#### Revision

Hybrid Optimization Algorithms

- Optimization
- Algorithms
- Hybridization and Classification of the
- approaches

The 4 Main Groups of Hybrids

Other Attributes of Hybrid Algorithms

Hybridization Design Grammar

Memetic Algorithms

### Again, following Talbi [Tal02]:

- ✓ Homogeneous vs. heterogeneous:
  - In a homogeneous hybrid, all the combined algorithms are of the same type. (They may have different parameters.)
  - **×** In a **heterogeneous hybrid**, each algorithm may be of different type.

## **Other Attributes of Hybrid Algorithms**

#### Revision

- Hybrid Optimization Algorithms
- Optimization
- Algorithms
- Hybridization and Classification of the
- approaches
- The 4 Main Groups of Hybrids
- Other Attributes of Hybrid Algorithms
- Hybridization Design Grammar

Memetic Algorithms

## Again, following Talbi [Tal02]:

- ✓ Homogeneous vs. heterogeneous:
  - In a homogeneous hybrid, all the combined algorithms are of the same type. (They may have different parameters.)
  - ★ In a **heterogeneous hybrid**, each algorithm may be of different type.
- ✔ Global vs. partial:
  - **★** In a **global hybrid**, all algorithms search the whole space.
  - ✗ In a partial hybrid, the problem is decomposed into subproblems and each algorithm is dedicated to the search in its own subspace. (See also the lecture on coevolution.)

## **Other Attributes of Hybrid Algorithms**

### Revision

Hybrid Optimization Algorithms

- Optimization
- Algorithms

Hybridization and Classification of the

approaches

The 4 Main Groups of Hybrids

Other Attributes of Hybrid Algorithms

Hybridization Design Grammar

Memetic Algorithms

## Again, following Talbi [Tal02]:

- ✓ Homogeneous vs. heterogeneous:
  - In a homogeneous hybrid, all the combined algorithms are of the same type. (They may have different parameters.)
  - **×** In a **heterogeneous hybrid**, each algorithm may be of different type.
- ✓ Global vs. partial:
  - **×** In a **global hybrid**, all algorithms search the whole space.
  - ✗ In a partial hybrid, the problem is decomposed into subproblems and each algorithm is dedicated to the search in its own subspace. (See also the lecture on coevolution.)
- ✔ General vs. specialist:
  - ✗ In a general hybrid, all the algorithms solve the same target optimization problem.
  - ✗ In a specialist hybrid, some of the algorithms may solve a different optimization problem, helping the others to solve the original one.

[Tal02] E. G. Talbi. A taxonomy of hybrid metaheuristics. Journal of Heuristics, 8(5):541–564, 2002.

# Hybridization Design Grammar

### Revision

	<pre><hybrid-opt-algorithm> ::= <hierarchical><flat></flat></hierarchical></hybrid-opt-algorithm></pre>
Hybrid Optimization Algorithms	<pre><hierarchical> ::= <lrh>   <lth>   <hrh>   <hth></hth></hrh></lth></lrh></hierarchical></pre>
Optimization	<lrh> ::= LRH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lrh>
Algorithms	<pre><lth> ::= LTH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lth></pre>
Hybridization and Classification of the	<pre><hrh> ::= HRH(<metaheuristic> + <metaheuristic>)</metaheuristic></metaheuristic></hrh></pre>
approaches	<pre><hth> ::= HTH(<metaheuristic>)</metaheuristic></hth></pre>
The 4 Main Groups of Hybrids	HTH( <metaheuristic>, <metaheuristic>)</metaheuristic></metaheuristic>
Other Attributes of Hybrid Algorithms	<flat> ::= (<nature>, <optimization>, <function>)</function></optimization></nature></flat>
Hybridization Design	<pre><nature> ::= homogeneous   heterogeneous</nature></pre>
Grammar	<pre><optimization> ::= global   partial</optimization></pre>
Memetic Algorithms	<function> ::= general   specialist</function>
	<pre><metaheuristic> ::= <hybrid-opt-algorithm></hybrid-opt-algorithm></metaheuristic></pre>
	LS   TS   SA   GA   ES   GP   GH   ACO   SS

# Hybridization Design Grammar

### Revision

	<hybrid-opt-algorithm> ::= <hierarchical><flat></flat></hierarchical></hybrid-opt-algorithm>
Hybrid Optimization Algorithms	<hierarchical> ::= <lrh>   <lth>   <hrh>   <hth></hth></hrh></lth></lrh></hierarchical>
Optimization	<lrh> ::= LRH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lrh>
Algorithms	<lth> ::= LTH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lth>
Hybridization and Classification of the	<pre><hrh> ::= HRH(<metaheuristic> + <metaheuristic>)</metaheuristic></metaheuristic></hrh></pre>
approaches	<hr/>
The 4 Main Groups of Hybrids	HTH( <metaheuristic>, <metaheuristic>)</metaheuristic></metaheuristic>
Other Attributes of	<flat> ::= (<nature>, <optimization>, <function>)</function></optimization></nature></flat>
Hybrid Algorithms Hybridization Design	<nature> ::= homogeneous   heterogeneous</nature>
Grammar	<pre><optimization> ::= global   partial</optimization></pre>
Memetic Algorithms	<function> ::= general   specialist</function>
	<metaheuristic> ::= <hybrid-opt-algorithm></hybrid-opt-algorithm></metaheuristic>
	LS   TS   SA   GA   ES   GP   GH   ACO   SS

**Example:** HTH(HRH(GH + LTH(GA(LS))))

## Hybridization Design Grammar

### Revision

	<hybrid-opt-algorithm> ::= <hierarchical><flat></flat></hierarchical></hybrid-opt-algorithm>
Hybrid Optimization Algorithms	<hierarchical> ::= <lrh>   <lth>   <hrh>   <hth></hth></hrh></lth></lrh></hierarchical>
Optimization	<lrh> ::= LRH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lrh>
Algorithms	<lth> ::= LTH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lth>
Hybridization and Classification of the	<pre><hrh> ::= HRH(<metaheuristic> + <metaheuristic>)</metaheuristic></metaheuristic></hrh></pre>
approaches	<pre><hth> ::= HTH(<metaheuristic>)</metaheuristic></hth></pre>
The 4 Main Groups of Hybrids	HTH( <metaheuristic>, <metaheuristic>)</metaheuristic></metaheuristic>
Other Attributes of	<flat> ::= (<nature>, <optimization>, <function>)</function></optimization></nature></flat>
Hybrid Algorithms Hybridization Design	<nature> ::= homogeneous   heterogeneous</nature>
Grammar	<pre><optimization> ::= global   partial</optimization></pre>
Memetic Algorithms	<function> ::= general   specialist</function>
	<metaheuristic> ::= <hybrid-opt-algorithm></hybrid-opt-algorithm></metaheuristic>
	LS   TS   SA   GA   ES   GP   GH   ACO   SS

## **Example:** HTH(HRH(GH + LTH(GA(LS))))

Island model containing HRH(GH + LTH(GA(LS))) in each node. /

#### Revision

	<hybrid-opt-algorithm> ::= <hierarchical><flat></flat></hierarchical></hybrid-opt-algorithm>
Hybrid Optimization Algorithms	<hierarchical> ::= <lrh>   <lth>   <hrh>   <hth></hth></hrh></lth></lrh></hierarchical>
Optimization	<lrh> ::= LRH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lrh>
Algorithms	<pre><lth> ::= LTH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lth></pre>
Hybridization and Classification of the	<pre><hrh> ::= HRH(<metaheuristic> + <metaheuristic>)</metaheuristic></metaheuristic></hrh></pre>
approaches	<pre><hth> ::= HTH(<metaheuristic>)</metaheuristic></hth></pre>
The 4 Main Groups of Hybrids	HTH( <metaheuristic>, <metaheuristic>)</metaheuristic></metaheuristic>
Other Attributes of Hybrid Algorithms	<flat> ::= (<nature>, <optimization>, <function>)</function></optimization></nature></flat>
Hybridization Design	<nature> ::= homogeneous   heterogeneous</nature>
Grammar	<pre><optimization> ::= global   partial</optimization></pre>
Memetic Algorithms	<function> ::= general   specialist</function>
	<metaheuristic> ::= <hybrid-opt-algorithm></hybrid-opt-algorithm></metaheuristic>
	LS   TS   SA   GA   ES   GP   GH   ACO   SS

### **Example:** HTH(HRH(GH + LTH(GA(LS))))

- ✓ Island model containing HRH(GH + LTH(GA(LS))) in each node.
- ✓ In each node, initial population is generated using a greedy heuristic (GH)
- ✓ ... and then a genetic algorithm (GA) was run
- ✓ ... with an embedded local search (LS) improving the results each generation.

#### Revision

	<hybrid-opt-algorithm> ::= <hierarchical><flat></flat></hierarchical></hybrid-opt-algorithm>
Hybrid Optimization Algorithms	<hierarchical> ::= <lrh>   <lth>   <hrh>   <hth></hth></hrh></lth></lrh></hierarchical>
Optimization	<lrh> ::= LRH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lrh>
Algorithms	<pre><lth> ::= LTH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lth></pre>
Hybridization and Classification of the	<pre><hrh> ::= HRH(<metaheuristic> + <metaheuristic>)</metaheuristic></metaheuristic></hrh></pre>
approaches	<pre><hth> ::= HTH(<metaheuristic>)</metaheuristic></hth></pre>
The 4 Main Groups of Hybrids	HTH( <metaheuristic>, <metaheuristic>)</metaheuristic></metaheuristic>
Other Attributes of	<flat> ::= (<nature>, <optimization>, <function>)</function></optimization></nature></flat>
Hybrid Algorithms Hybridization Design	<nature> ::= homogeneous   heterogeneous</nature>
Grammar	<pre><optimization> ::= global   partial</optimization></pre>
Memetic Algorithms	<function> ::= general   specialist</function>
	<metaheuristic> ::= <hybrid-opt-algorithm></hybrid-opt-algorithm></metaheuristic>
	LS   TS   SA   GA   ES   GP   GH   ACO   SS

### **Example:** HTH(HRH(GH + LTH(GA(LS))))

- ✓ Island model containing HRH(GH + LTH(GA(LS))) in each node.
- ✓ In each node, initial population is generated using a greedy heuristic (GH)
- ✓ ... and then a genetic algorithm (GA) was run
- ✓ ... with an embedded local search (LS) improving the results each generation.

**Example:** HRH(HRH(GH + LS) + LTH(GA(HRH(GH + LS))))

#### Revision

	<pre><hybrid-opt-algorithm> ::= <hierarchical><flat></flat></hierarchical></hybrid-opt-algorithm></pre>
Hybrid Optimization Algorithms	<hierarchical> ::= <lrh>   <lth>   <hrh>   <hth></hth></hrh></lth></lrh></hierarchical>
Optimization	<lrh> ::= LRH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lrh>
Algorithms	<lth> ::= LTH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lth>
Hybridization and Classification of the	<pre><hrh> ::= HRH(<metaheuristic> + <metaheuristic>)</metaheuristic></metaheuristic></hrh></pre>
approaches	<pre><hth> ::= HTH(<metaheuristic>)</metaheuristic></hth></pre>
The 4 Main Groups of Hybrids	HTH( <metaheuristic>, <metaheuristic>)</metaheuristic></metaheuristic>
Other Attributes of	<flat> ::= (<nature>, <optimization>, <function>)</function></optimization></nature></flat>
Hybrid Algorithms Hybridization Design	<nature> ::= homogeneous   heterogeneous</nature>
Grammar	<pre><optimization> ::= global   partial</optimization></pre>
Memetic Algorithms	<function> ::= general   specialist</function>
	<metaheuristic> ::= <hybrid-opt-algorithm></hybrid-opt-algorithm></metaheuristic>
	LS   TS   SA   GA   ES   GP   GH   ACO   SS

## **Example:** HTH(HRH(GH + LTH(GA(LS))))

- ✓ Island model containing HRH(GH + LTH(GA(LS))) in each node.
- ✓ In each node, initial population is generated using a greedy heuristic (GH)
- ✓ ... and then a genetic algorithm (GA) was run
- ✓ ... with an embedded local search (LS) improving the results each generation.

### **Example:** HRH(HRH(GH + LS) + LTH(GA(HRH(GH + LS))))

✓ HRH(GH + LS) is actually the GRASP algorithm, is it clearer now?

#### Revision

	<pre><hybrid-opt-algorithm> ::= <hierarchical><flat></flat></hierarchical></hybrid-opt-algorithm></pre>
Hybrid Optimization Algorithms	<hierarchical> ::= <lrh>   <lth>   <hrh>   <hth></hth></hrh></lth></lrh></hierarchical>
Optimization	<lrh> ::= LRH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lrh>
Algorithms	<lth> ::= LTH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lth>
Hybridization and Classification of the	<pre><hrh> ::= HRH(<metaheuristic> + <metaheuristic>)</metaheuristic></metaheuristic></hrh></pre>
approaches	<pre><hth> ::= HTH(<metaheuristic>)</metaheuristic></hth></pre>
The 4 Main Groups of Hybrids	HTH( <metaheuristic>, <metaheuristic>)</metaheuristic></metaheuristic>
Other Attributes of	<flat> ::= (<nature>, <optimization>, <function>)</function></optimization></nature></flat>
Hybrid Algorithms Hybridization Design	<nature> ::= homogeneous   heterogeneous</nature>
Grammar	<pre><optimization> ::= global   partial</optimization></pre>
Memetic Algorithms	<function> ::= general   specialist</function>
	<metaheuristic> ::= <hybrid-opt-algorithm></hybrid-opt-algorithm></metaheuristic>
	LS   TS   SA   GA   ES   GP   GH   ACO   SS

### **Example:** HTH(HRH(GH + LTH(GA(LS))))

- ✓ Island model containing HRH(GH + LTH(GA(LS))) in each node.
- ✓ In each node, initial population is generated using a greedy heuristic (GH)
- ✓ ... and then a genetic algorithm (GA) was run
- ✓ ... with an embedded local search (LS) improving the results each generation.

### **Example:** HRH(HRH(GH + LS) + LTH(GA(HRH(GH + LS))))

- ✓ HRH(GH + LS) is actually the GRASP algorithm, is it clearer now?
- ✓ No? What about HRH(GRASP + LTH(GA(GRASP)))?

### Revision

	<hybrid-opt-algorithm> ::= <hierarchical><flat></flat></hierarchical></hybrid-opt-algorithm>
Hybrid Optimization Algorithms	<hierarchical> ::= <lrh>   <lth>   <hrh>   <hth></hth></hrh></lth></lrh></hierarchical>
Optimization	<lrh> ::= LRH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lrh>
Algorithms	<lth> ::= LTH(<metaheuristic>(<metaheuristic>))</metaheuristic></metaheuristic></lth>
Hybridization and Classification of the	<pre><hrh> ::= HRH(<metaheuristic> + <metaheuristic>)</metaheuristic></metaheuristic></hrh></pre>
approaches	<pre><hth> ::= HTH(<metaheuristic>)</metaheuristic></hth></pre>
The 4 Main Groups of Hybrids	HTH( <metaheuristic>, <metaheuristic>)</metaheuristic></metaheuristic>
Other Attributes of	<flat> ::= (<nature>, <optimization>, <function>)</function></optimization></nature></flat>
Hybrid Algorithms Hybridization Design	<nature> ::= homogeneous   heterogeneous</nature>
Grammar	<pre><optimization> ::= global   partial</optimization></pre>
Memetic Algorithms	<function> ::= general   specialist</function>
	<metaheuristic> ::= <hybrid-opt-algorithm></hybrid-opt-algorithm></metaheuristic>
	LS   TS   SA   GA   ES   GP   GH   ACO   SS

### **Example:** HTH(HRH(GH + LTH(GA(LS))))

- ✓ Island model containing HRH(GH + LTH(GA(LS))) in each node.
- ✓ In each node, initial population is generated using a greedy heuristic (GH)
- ✓ ... and then a genetic algorithm (GA) was run
- ✓ ... with an embedded local search (LS) improving the results each generation.

### **Example:** HRH(HRH(GH + LS) + LTH(GA(HRH(GH + LS))))

- ✓ HRH(GH + LS) is actually the GRASP algorithm, is it clearer now?
- ✓ No? What about HRH(GRASP + LTH(GA(GRASP)))?
- ✓ GRASP creates the initial population for a memetic algorithm that uses GRASP to improve solutions of GA.

#### Revision

Summary

Hybrid Optimization Algorithms

#### Memetic Algorithms

Memetic Algorithms Models of Evolution MA: Main Design Issues MAs Classification Which LS should I choose? How should I best integrate LS into EA? How should I set the global-local tradeoff?

# **Memetic Algorithms**

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms

Memetic Algorithms

Models of Evolution

MA: Main Design Issues

MAs Classification Which LS should I

choose?

How should I best integrate LS into EA?

How should I set the global-local tradeoff? Summary

## Memetic algorithm (MAs):

- ✓ hybrid of evolutionary algorithm with embedded local search, i.e.
- ✓ EA that includes one or more LS phases within its evolutionary cycle.
- ✓ MA is LTH(EA(LS)) using Talbi's taxonomy.

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms

Memetic Algorithms

Models of Evolution

MA: Main Design Issues

MAs Classification

Which LS should I

choose?

How should I best integrate LS into EA?

How should I set the global-local tradeoff? Summary

## Memetic algorithm (MAs):

- ✓ hybrid of evolutionary algorithm with embedded local search, i.e.
- ✓ EA that includes one or more LS phases within its evolutionary cycle.
- ✓ MA is **LTH(EA(LS))** using Talbi's taxonomy.

## EA vs. Local-search:

- ✓ *Local search* quickly identifies an optimum, but the optimum is often only local
- ✓ EA is more robust against getting stuck in local optima; it uses large population and searches several places of the search space in parallel. It is thus *slower*.

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms

Memetic Algorithms

Models of Evolution

MA: Main Design Issues

MAs Classification

Which LS should I

choose?

How should I best integrate LS into EA?

How should I set the global-local tradeoff? Summary

## Memetic algorithm (MAs):

- ✓ hybrid of evolutionary algorithm with embedded local search, i.e.
- ✓ EA that includes one or more LS phases within its evolutionary cycle.
- ✓ MA is LTH(EA(LS)) using Talbi's taxonomy.

## EA vs. Local-search:

- ✓ *Local search* quickly identifies an optimum, but the optimum is often only local
- ✓ EA is more robust against getting stuck in local optima; it uses large population and searches several places of the search space in parallel. It is thus *slower*.

## When implemented well,

- ✓ MAs are faster than ordinary EAs, and
- ✓ MAs are more robust against getting stuck in local optima than local search.

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms

Memetic Algorithms

Models of Evolution

MA: Main Design Issues

MAs Classification

Which LS should I

choose?

How should I best integrate LS into EA?

How should I set the global-local tradeoff? Summary

## Memetic algorithm (MAs):

- ✓ hybrid of evolutionary algorithm with embedded local search, i.e.
- ✓ EA that includes one or more LS phases within its evolutionary cycle.
- ✓ MA is LTH(EA(LS)) using Talbi's taxonomy.

## EA vs. Local-search:

- ✓ *Local search* quickly identifies an optimum, but the optimum is often only local
- ✓ EA is more robust against getting stuck in local optima; it uses large population and searches several places of the search space in parallel. It is thus *slower*.

## When implemented well,

- ✓ MAs are faster than ordinary EAs, and
- ✓ MAs are more robust against getting stuck in local optima than local search.

Talbi's taxonomy ignores many MA design issues.

# **Models of Evolution**

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms Memetic Algorithms

Models of Evolution

MA: Main Design Issues MAs Classification Which LS should I choose? How should I best integrate LS into EA? How should I set the global-local tradeoff? Summary

### **Darwinian evolution**

- ✓ the whole "plan" of an organism is inscribed in its genes
- ✓ the survival of the organism depends on its genetic constitution only
- ✓ used in ordinary EAs

# **Models of Evolution**

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms Memetic Algorithms

#### Models of Evolution

MA: Main Design Issues MAs Classification Which LS should I choose? How should I best integrate LS into EA?

How should I set the global-local tradeoff? Summary

### **Darwinian evolution**

- ✓ the whole "plan" of an organism is inscribed in its genes
- ✓ the survival of the organism depends on its genetic constitution only
- ✓ used in ordinary EAs

### Lamarckian evolution

- characteristics acquired during the organism's lifetime can be transferred to the offspring
- ✓ used in Lamarckian MAs:
  - ★ an individual in EA is improved by a local search
  - **×** the improved individual replaces the original individual

# **Models of Evolution**

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms Memetic Algorithms

#### Models of Evolution

MA: Main Design Issues MAs Classification Which LS should I choose? How should I best integrate LS into EA? How should I set the

Bow should I set the global-local tradeoff? Summary

### **Darwinian evolution**

- ✓ the whole "plan" of an organism is inscribed in its genes
- ✓ the survival of the organism depends on its genetic constitution only
- ✓ used in ordinary EAs

### Lamarckian evolution

- ✓ characteristics acquired during the organism's lifetime can be transferred to the offspring
- ✓ used in Lamarckian MAs:
  - ★ an individual in EA is improved by a local search
  - **×** the improved individual replaces the original individual

### **Baldwin effect**

- ✓ the genetic information encodes the ability to learn
- organisms able to learn have higher chance of survival, which in turn allows the "right" genes to survive and breed
- ✓ used in Baldwinian MAs:
  - ★ an individual in EA is improved by a local search
  - ✗ the original individual is preserved in the population, but its fitness is changed to the fitness of the improved individual

# MA: Main Design Issues

Revision

Hybrid Optimization Algorithms

Memetic Algorithms

Memetic Algorithms

Models of Evolution

### MA: Main Design Issues

MAs Classification Which LS should I choose? How should I best integrate LS into EA?

How should I set the global-local tradeoff? Summary What is the best trade-off between local-search and the global search provided by evolution?

# MA: Main Design Issues

#### Revision

Hybrid Optimization Algorithms

- Memetic Algorithms
- Memetic Algorithms
- Models of Evolution

### MA: Main Design Issues

MAs Classification Which LS should I choose? How should I best

integrate LS into EA?

How should I set the global-local tradeoff? Summary What is the best trade-off between local-search and the global search provided by evolution?

- ✓ Where (in which phase of the evolutionary cycle) and when (in which evolutionary cycle) should the local search be applied?
- ✓ Which individuals should be improved? (The best ones? All? Random ones?)
- ✓ How long should each local search be allowed to run? (Make a complete run? Only perform one step of LS?)
- ✓ How can the genetic operators be best combined (modified) with local search to achieve the synergy?
- ✓ Should we use the Lamarckian or Baldwinian model?

# **MAs Classification**

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms
Memetic Algorithms

Models of Evolution

MA: Main Design Issues

#### MAs Classification

Which LS should I choose? How should I best integrate LS into EA?

How should I set the global-local tradeoff? Summary Krasnogor and Smith [KS05] introduce the concept of *schedulers*:

- ✓ *Fine-grained mutation scheduler*  $fS_M$  replaces the mutation operator and organizes the mutation and LS.
- ✓ *Fine-grained crossover scheduler*  $fS_R$  replaces the crossover operator and organizes the crossover and LS.
- ✓ Coarse-grained scheduler cS replaces selection and replacement, and organizes the interplay between selection, crossover and LS. It can supply the population statistics to LS.
- ✓ *Meta scheduler mS* provides a historical information to LS inside MA.

# **MAs Classification**

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms Memetic Algorithms Models of Evolution

MA: Main Design Issues

### MAs Classification

Which LS should I choose? How should I best integrate LS into EA?

How should I set the global-local tradeoff? Summary Krasnogor and Smith [KS05] introduce the concept of *schedulers*:

- ✓ *Fine-grained mutation scheduler*  $fS_M$  replaces the mutation operator and organizes the mutation and LS.
- ✓ *Fine-grained crossover scheduler*  $fS_R$  replaces the crossover operator and organizes the crossover and LS.
- ✓ Coarse-grained scheduler cS replaces selection and replacement, and organizes the interplay between selection, crossover and LS. It can supply the population statistics to LS.
- ✓ *Meta scheduler mS* provides a historical information to LS inside MA.

A 4-bit number can then be assigned to each MA:

$$D = (b_4, b_3, b_2, b_1)$$

- ✓  $b_1 = 1 \iff$  MA contains  $fS_M$ .
- ✓  $b_2 = 1 \iff$  MA contains  $fS_R$ .
- ✓  $b_3 = 1 \iff$  MA contains *cS*.
- ✓  $b_4 = 1 \iff$  MA contains mS.

# MAs Classification

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms Memetic Algorithms Models of Evolution

MA: Main Design Issues

### MAs Classification

Which LS should I choose? How should I best integrate LS into EA?

How should I set the global-local tradeoff? Summary Krasnogor and Smith [KS05] introduce the concept of *schedulers*:

- ✓ *Fine-grained mutation scheduler*  $fS_M$  replaces the mutation operator and organizes the mutation and LS.
- ✓ *Fine-grained crossover scheduler*  $fS_R$  replaces the crossover operator and organizes the crossover and LS.
- ✓ Coarse-grained scheduler cS replaces selection and replacement, and organizes the interplay between selection, crossover and LS. It can supply the population statistics to LS.
- ✓ *Meta scheduler mS* provides a historical information to LS inside MA.

A 4-bit number can then be assigned to each MA:

$$D = (b_4, b_3, b_2, b_1)$$

- ✓  $b_1 = 1 \iff$  MA contains  $fS_M$ .
- ✓  $b_2 = 1 \iff$  MA contains  $fS_R$ .
- ✓  $b_3 = 1 \iff$  MA contains *cS*.
- ✓  $b_4 = 1 \iff$  MA contains mS.

Higher value of *D* implies more complex algorithm, but not necessarily a better one!

<sup>[</sup>KS05] N. Krasnogor and J. Smith. A tutorial for competent memetic algorithms: model, taxonomy, and design issues. *Evolutionary Computation, IEEE Transactions on,* 9(5):474–488, 2005.

## Which LS should I choose?

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms Memetic Algorithms Models of Evolution MA: Main Design Issues MAs Classification Which LS should I choose?

How should I best integrate LS into EA?

How should I set the global-local tradeoff? Summary

### It depends.

- ✓ Even within a single problem class, e.g. TSP, different LS operators are best for different instances of TSP. The choice is *instance-specific*.
- ✓ Even during one optimization run on one problem instance, different LS operators are best for different phases of evolution. The choice is time-dependent.

# Which LS should I choose?

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms Memetic Algorithms Models of Evolution MA: Main Design Issues MAs Classification Which LS should I choose?

How should I best integrate LS into EA?

How should I set the global-local tradeoff? Summary

### It depends.

- ✓ Even within a single problem class, e.g. TSP, different LS operators are best for different instances of TSP. The choice is *instance-specific*.
- ✓ Even during one optimization run on one problem instance, different LS operators are best for different phases of evolution. The choice is time-dependent.

Based on these observations, a sensible approach could be:

- ✓ do not decide a priori which LS should be used;
- ✓ incorporate several of them and consider methods to avoid spending time using the unproductive ones.
- ✓ This implies at least some way of adapting of operator probabilities, which in turn implies the presence of *cS* or *mS*.

Revision

Hybrid Optimization Algorithms

Memetic Algorithms Memetic Algorithms Models of Evolution MA: Main Design Issues MAs Classification Which LS should I choose? How should I best

integrate LS into EA?

How should I set the global-local tradeoff? Summary It depends. We can list some suggestions:

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms Memetic Algorithms Models of Evolution MA: Main Design Issues MAs Classification Which LS should I choose?

How should I best integrate LS into EA?

How should I set the global-local tradeoff? Summary It depends. We can list some suggestions:

Lamarckian vs. Baldwinian:

- ✓ Lamarckian learning can happen before or after the application of other operators.
- ✔ There is a little point in applying Baldwinian search after parent selection, but before recombination or mutation.

Revision

Hybrid Optimization Algorithms

Memetic Algorithms Memetic Algorithms Models of Evolution MA: Main Design Issues MAs Classification Which LS should I choose?

How should I best integrate LS into EA?

How should I set the global-local tradeoff? Summary It depends. We can list some suggestions:

Lamarckian vs. Baldwinian:

- ✓ Lamarckian learning can happen before or after the application of other operators.
- ✓ There is a little point in applying Baldwinian search after parent selection, but before recombination or mutation.

If Lamarckian search is continued to optimality:

- ✓ recombination and mutation will likely produce offspring worse than parents.
- ✓ We hope that the offspring will be in the basin of attraction of high-quality LO.
- ✓ To prevent selection discarding these new worse offspring, perform LS on them prior to selection.

Revision

Hybrid Optimization Algorithms

Memetic Algorithms Memetic Algorithms Models of Evolution MA: Main Design Issues MAs Classification Which LS should I choose?

How should I best integrate LS into EA?

How should I set the global-local tradeoff? Summary It depends. We can list some suggestions:

Lamarckian vs. Baldwinian:

- ✓ Lamarckian learning can happen before or after the application of other operators.
- ✓ There is a little point in applying Baldwinian search after parent selection, but before recombination or mutation.

If Lamarckian search is continued to optimality:

- ✓ recombination and mutation will likely produce offspring worse than parents.
- ✓ We hope that the offspring will be in the basin of attraction of high-quality LO.
- ✓ To prevent selection discarding these new worse offspring, perform LS on them prior to selection.

Lamarckian approach is used most often, the LS is continued to optimality.

- ✓ This leads to the loss of diversity very quickly, MA gets stuck in LO, or stagnates on a plateau.
- ✓ A *cS* can monitor the population statistics and re-introduce diversity.

# How should I set the global-local tradeoff?

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms Memetic Algorithms Models of Evolution MA: Main Design Issues MAs Classification Which LS should I choose?

How should I best integrate LS into EA?

How should I set the global-local tradeoff?

Summary

The majority of MAs apply LS to every individual in every generation of the EA. But a *cS* can be used to

- ✓ choose individuals to be optimized by LS,
- ✓ set the intensity of LS,
- ✓ set the probability of applying LS.

# How should I set the global-local tradeoff?

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms Memetic Algorithms Models of Evolution MA: Main Design Issues MAs Classification Which LS should I choose? How should I best

integrate LS into EA?

How should I set the global-local tradeoff?

Summary

The majority of MAs apply LS to every individual in every generation of the EA. But a *cS* can be used to

- ✔ choose individuals to be optimized by LS,
- ✓ set the intensity of LS,
- ✓ set the probability of applying LS.

## Example of an approach to select points for LS:

- ✓ Single-linkage clustering is used to prevent running LS too often in the same basin of attraction.
- ✓ Each basin is represented by a set of points used for LS.
- ✓ If a point is used in LS, it is assigned to the respective cluster.
- ✓ The number of clusters grows, the number of points in the cluster grows as well (if not limited somehow).
- ✓ A candidate for LS is not used, if it lies within a critical distance from an already clustered point.

## Summary

#### Revision

Hybrid Optimization Algorithms

Memetic Algorithms Memetic Algorithms Models of Evolution MA: Main Design Issues MAs Classification Which LS should I choose? How should I best integrate LS into EA? How should I set the

global-local tradeoff?

Summary

- ✓ Hybrid optimization algorithm is virtually any combination of 2 or more other algorithms.
  - ★ Many existing algorithms can be described as hybrids.
  - ✗ Using the systematic taxonomy, many of non-existing hybrids can be easily created. (This does not mean they will be better than the existing ones.)
- ✓ MAs are a broad class of hybrid optimization algorithms combining
  - ★ the robustness of evolutionary algorithms and
  - **×** the speed of local search methods.
- ✔ Their design is
  - ★ simple from a practitioner's point of view (very often they just work), but
  - ★ becomes harder when we want to exploit the most of it (extensive testing and comparisons needed), and is
  - ✗ incredibly complex from the theoretical point of view when the exact interplay between particular algorithm features should be undestood or modeled.