3. $\displaystyle\int \frac{1}{\cos^4 x}\,\mathrm{d}x$ (viz příklad 8.19 a))

$$R(u,v) = \frac{1}{v^4},$$

$$R(-u,v) = \frac{1}{v^4} \neq -R(u,v),$$

$$R(u,-v) = \frac{1}{(-v)^4} \neq -R(u,v),$$

$$R(-u,-v) = \frac{1}{(-v)^4} = R(u,v)$$

z jednodušších substitucí lze použít jen substituci $t = \operatorname{tg} x$

Musí platit $\cos x \neq 0$, tj. $x \in (-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi) = I_k$, $k \in \mathbb{Z}$.

$$\int \frac{1}{\cos^4 x}\,\mathrm{d}x = \int \frac{1}{\cos^2 x} \cdot \frac{1}{\cos^2 x}\,\mathrm{d}x = \left| \begin{array}{rcl} \operatorname{tg} x & = & t \\ \frac{1}{\cos^2 x}\,\mathrm{d}x & = & \mathrm{d}t \end{array} \right| =$$

$$= \int \frac{1}{\frac{1}{1+t^2}}\,\mathrm{d}t = \int (1+t^2)\,\mathrm{d}t =$$

$$= t + \frac{t^3}{3} + c = \operatorname{tg} x + \frac{\operatorname{tg}^3 x}{3} + c$$

na $(-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi)$; $k \in \mathbb{Z}$

Musí platit $\cos x \neq 0$, tj. $x \in (-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi) = I_k$, $k \in \mathbb{Z}$.

$$\int \frac{1}{\cos^4 x}\,\mathrm{d}x = \left| \begin{array}{rcl} \operatorname{tg} x & = & t \\ x & = & \operatorname{arctg} t + k\pi \quad - \text{ prostá} \\ \mathrm{d}x & = & \frac{1}{t^2+1}\,\mathrm{d}t \end{array} \right| =$$

$$= \int \frac{1}{\left(\frac{1}{1+t^2}\right)^2} \cdot \frac{1}{t^2+1}\,\mathrm{d}t = \int (1+t^2)\,\mathrm{d}t =$$

$$= t + \frac{t^3}{3} + c = \operatorname{tg} x + \frac{\operatorname{tg}^3 x}{3} + c$$

na $(-\frac{\pi}{2} + k\pi, \frac{\pi}{2} + k\pi)$; $k \in \mathbb{Z}$