Medical ultrasound imaging

Introduction

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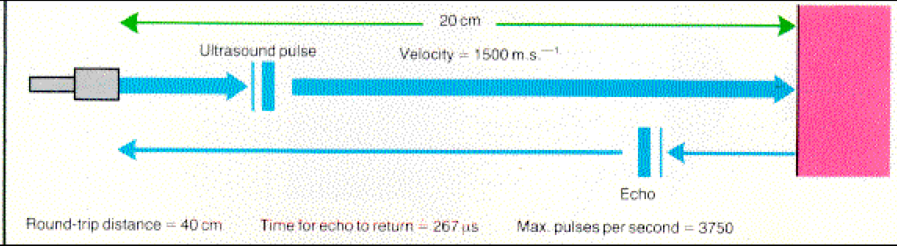
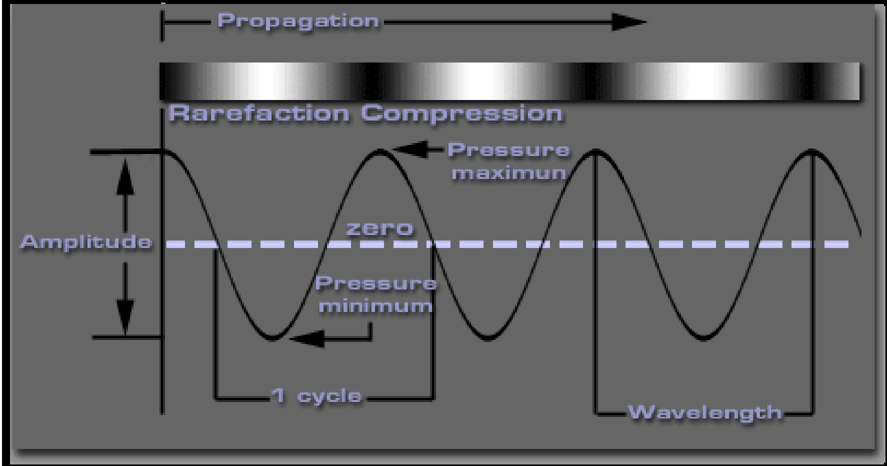
[1] Using images from J.Hozman, E.Dove, A. Stoylen

Medical ultrasound basics

- Acoustic waves, frequency $2 \sim 50\,\mathrm{MHz}$
- Measure the time and intensity of the echo
- Harmless
- Stopped by air and dense tissues (bone)

Ultrasound Principle





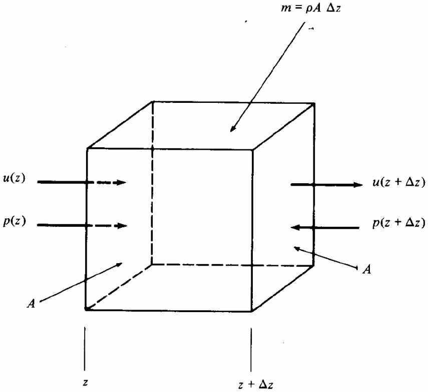
Sinusoidal pressure source

Physical quantities

Ultrasound

Property	Symbol	Unit	Usual values
Speed	c	m/s	$1350 \sim 1800\,\mathrm{m/s}$
Wavelength	λ	m	$0.1 \sim 0.8\,\mathrm{mm}$
Frequency	f	Hz	$2 \sim 20\,\mathrm{MHz}$
Density	ϱ	$\mathrm{kg/m^3}$	$\sim 1000\,\mathrm{kg/m^3}$
Intensity	I	$\mathrm{W/m^2}$	$1 \sim 10\,\mathrm{mW/cm^2}$

Elementary volume



Speed u, pressure p, density ϱ, area A, mass m.

Newton's law

Motion along z:

$$F = ma = m\frac{\mathrm{d}u}{\mathrm{d}t} = m\left(\frac{\partial u}{\partial t} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial t}\right) \approx m\frac{\partial u}{\partial t}$$

force $F = pA$:

$$\left(p(z) - p(z + \Delta z)\right) A = m\frac{\partial u}{\partial t}$$

for $\Delta z \ll z$:

$$-\frac{\partial p}{\partial z}\Delta z\, A = m\frac{\partial u}{\partial t}$$

as $m = \rho A\, \Delta z$

$$-\frac{\partial p}{\partial z} = \rho\frac{\partial u}{\partial t}$$

Conservation of mass law

Difference of entering and exiting mass, density change:

$$A\Big(u(z+\Delta z)\rho(z+\Delta z) - u(z)\rho(z)\Big) = -A\,\Delta z \frac{\partial \rho}{\partial t}$$

for $\Delta z \ll z$:

$$\frac{\partial \rho u}{\partial z} = -\frac{\partial \rho}{\partial t}$$

density $\rho = \rho_0 + \rho_1, \quad \rho_0 = \text{const},\ \rho_1 \ll \rho_0$:

$$\rho_0 \frac{\partial u}{\partial z} = -\frac{\partial \rho_1}{\partial t}$$

Compressibility *(stlačitelnost)* $\frac{\rho_1}{\rho_0} = Kp, \quad K = 1/E$:

$$\frac{\partial u}{\partial z} = -K\frac{\partial p}{\partial t}$$

1D wave equation

$$\rho\frac{\partial u}{\partial t}+\frac{\partial p}{\partial z}=0 \quad \text{derive by } z$$
$$\frac{\partial u}{\partial z}+K\frac{\partial p}{\partial t}=0 \quad \text{derive by } t$$
$$\rho\frac{\partial^2 u}{\partial t\partial z}+\frac{\partial^2 p}{\partial z^2}=0$$
$$\frac{\partial^2 u}{\partial z\partial t}+K\frac{\partial^2 p}{\partial t^2}=0$$

subtract

$$\frac{\partial^2 p}{\partial z^2}-K\rho\frac{\partial^2 p}{\partial t^2}=0$$

similarly

$$\frac{\partial^2 u}{\partial z^2}-K\rho\frac{\partial^2 u}{\partial t^2}=0$$

Wave equation solution

Harmonic wave:

$$p = p_+ \cos(\underbrace{\omega t - kz}_{\phi})$$

where k is the wave number *(vlnové číslo)* [rad/m].

Wave speed (phase velocity):

$$\phi_0 = \omega t - kz \quad \rightarrow \quad z = \frac{\omega}{k} t - \frac{\phi_0}{k}$$

$$\boxed{c = \omega / k}$$

$$\boxed{c = \lambda f} \quad \text{because} \quad \omega = 2\pi f, \quad k = \frac{2\pi}{\lambda}$$

Wave speed

$$p = p_+ \cos(\underbrace{\omega t - kz}_{\phi})$$

$$\frac{\partial^2 p}{\partial z^2} = -p_+ k^2 \cos(\omega t - kz)$$

$$\frac{\partial^2 p}{\partial t^2} = -p_+ \omega^2 \cos(\omega t - kz)$$

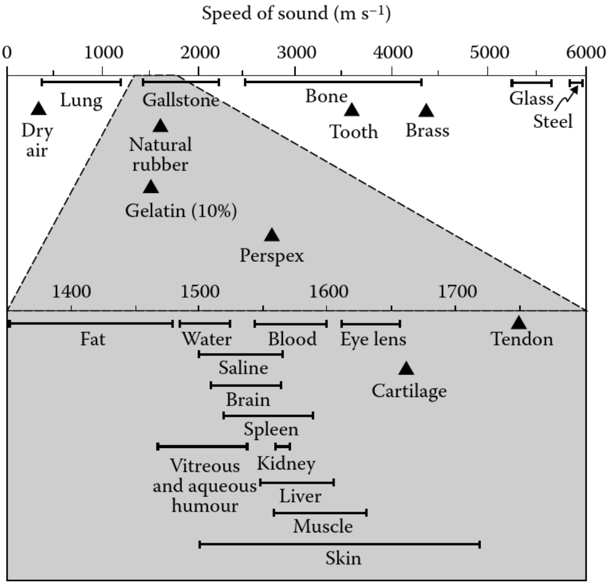
The wave equation

$$\frac{\partial^2 p}{\partial z^2} = K\rho \frac{\partial^2 p}{\partial t^2}$$

holds if

$$k^2 = \rho K \omega^2 \quad \rightarrow \quad \boxed{c = \frac{1}{\sqrt{\rho K}} = \sqrt{\frac{E}{\rho}}} \quad \text{because} \quad c = \frac{\omega}{k}$$

Speed of sound



Other wave equation solution

$$p = p_- \cos(\omega t + kz)$$

Any forward or backward wave (by linearity and harmonic decomposition).

$$\boxed{p = f_+(z + ct) + f_-(z - ct)}$$

Forward and backward wave combination:

$$p = p' \Big(\cos(\omega t - kz) + \cos(\omega t + kz) \Big)$$

Standing wave:

$$p = 2p' \cos(\omega t) \cos(kz)$$

Acoustic impedance

$$Z_a = \frac{p \text{ (pressure)}}{Q \text{ (flow)}} \; [\mathrm{Pa \cdot s/m^3}]$$

"acoustic Ohm".

For an infinite tube:

$$Z_a = \frac{\rho_0 c}{S}$$

$Z = \rho_0 c$ is a characteristic acoustic impedance.

Unit [kg/s $\cdot \mathrm{m}^2$]=1 Rayl.

Acoustic impedance (2)

Acoustic impedance

$$Z_a = \frac{p}{Q}$$

Specific acoustic impedance

$$Z_{\mathrm{sp}} = Z_{\mathrm{a}} S = \frac{p}{Q} S = \frac{p}{u} \quad \text{as flow} \quad Q = Su$$

Characteristic acoustic impedance

$$Z = \varrho_0 c = \sqrt{\frac{\rho_0}{K}}$$

For plane waves in lossless medium

$$Z = Z_{\mathrm{sp}}$$

Wave intensity

Kinetic and potential energy density (phase shifted by 90°)

$$i = \frac{1}{2}\left(Zu^2 + \frac{p^2}{Z}\right) \quad [\mathrm{W/m^2}]$$

Effective values

$$I = U^2 Z = \frac{P^2}{Z}$$

Often expressed in dB

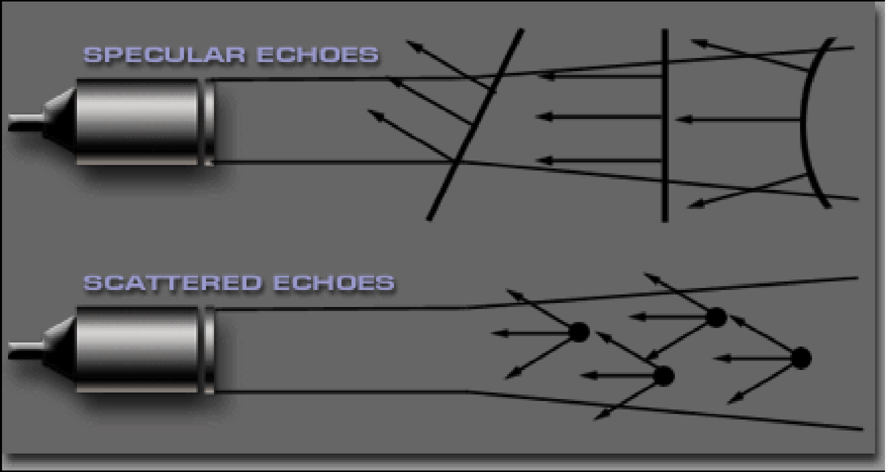
$$10\log_{10}\frac{I_1}{I_2} = 20\log_{10}\frac{P_1}{P_2} = 20\log_{10}\frac{U_1}{U_2}$$

Speed and impedance variations

Material	Density ρ (kgm^{-3})	Speed c (ms^{-1})	Characteristic impedance Z (kgm^{-2}s^{-1}) $\times 10^6$	Absorption coefficient α (dB cm^{-1}) at 1 MHz
Water	1000	1480	1.5	0.0022
Blood	1060	1570	1.62	(0.15)
Bone	1380–1810	4080	3.75-7.38	(14.2–25.2)
Brain	1030	1558	1.55–1.66	(0.75)
Fat	920	1450	1.35	(0.63)
Kidney	1040	1560	1.62	–
Liver	1060	1570	1.64–1.68	(1.2)
Lung	400	650	0.26	(40)
Muscle	1070	1584	1.65–1.74	(0.96–1.4)
Spleen	1060	1566	1.65–1.67	–

Ray/tissue interaction types

- $d \gg \lambda$
  - Geometric (specular) reflection and refraction.
  - Strong reflection.
  - Diaphragm, vessels, tissue/bone interface, tissue/lung interface, . . .
- $d \ll \lambda$
  - Scattered reflection. Stochastic non-directional scattering and interference.
  - Main tissue signal. Speckle.
  - Most soft tissues, blood.


SPECULAR ECHOES
SCATTERED ECHOES

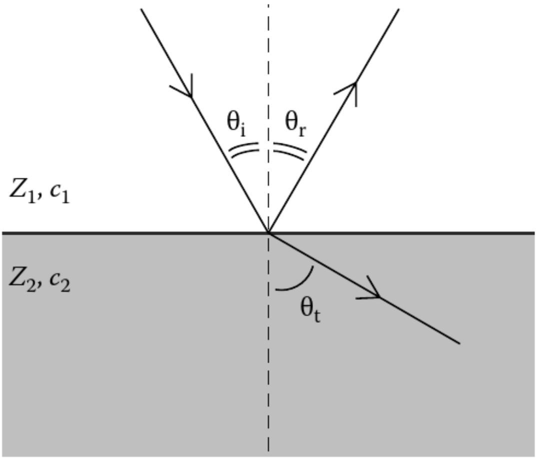
Specular Reflection

- The first, specular echoes, originate from relatively large, strongly reflective, regularly shaped objects with smooth surfaces. These reflections are angle dependent, and are described by reflectivity equation . This type of reflection is called specular reflection.

Scattered Reflection

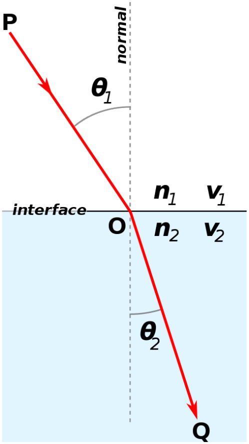
- The second type of echoes are scattered that originate from small, weakly reflective, irregularly shaped objects, and are less angle-dependent and less intense. The mathematical treatment of non-specular reflection (sometimes called “speckle”) involves the Rayleigh probability density function. This type of reflection, however, sometimes dominates medical images, as you will see in the laboratory demonstrations.

Reflection and refraction



$$\theta_i = \theta_r$$

Snell's law



$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{v_2}{v_1} = \frac{c_2}{c_1}$$

Fermat's principle of least time.

Reflectivity

Amplitude reflection coefficient for normal incidence $\theta_i = \theta_r = 0$

$$R_a = \frac{P_r}{P_i} = \frac{U_r}{U_i} = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

Reflectivity for Various Tissues

Materials at Interface	*Reflectivity*
Brain-skull bone	0.66
Fat-muscle	0.10
Fat-kidney	0.08
Muscle-blood	0.03
Soft tissue-water	0.05
Soft tissue-air	0.9995

Reflectivity (2)

Power/intensity reflection coefficient

$$R = \frac{I_r}{I_i} = R_a^2 = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

Reflectivity (2)

Power/intensity reflection coefficient

$$R = \frac{I_r}{I_i} = R_a^2 = \left(\frac{Z_2 - Z_1}{Z_2 + Z_1}\right)^2$$

Energy conservation law

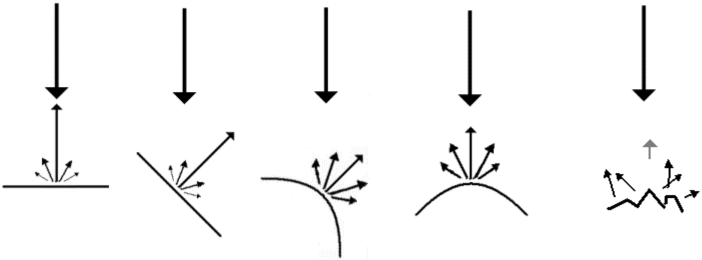
$$I_i = I_r + I_t \quad \longrightarrow \quad R = 1 - \frac{I_t}{I_i}$$

Reflectivity (3)

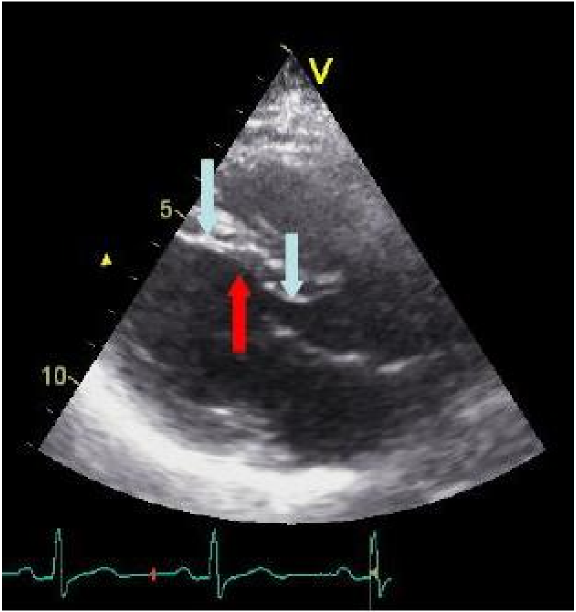
Reflection for arbitrary angle

$$R_a = \frac{Z_2 \cos\theta_i - Z_1 \cos\theta_t}{Z_2 \cos\theta_i + Z_1 \cos\theta_t}$$

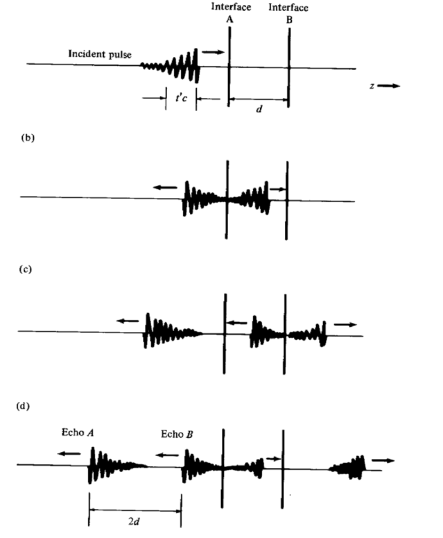
Directional dependency of reflection



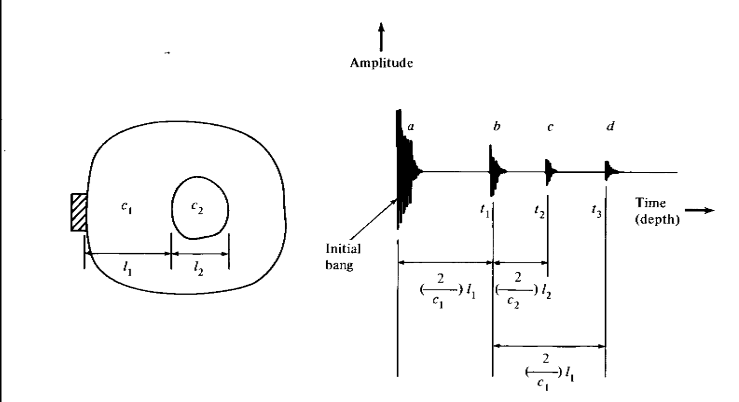
Directional dependency of reflection



Echoes from Two Interfaces



Echoes from Internal Organ



Attenuation

Signal attenuation reasons:

- Wavefront divergence
- Scattering (elastic)
- Absorption (tissue heating)

Attenuation (2)

- Amplitude attenuation

$$P(x) = P_0 \mathrm{e}^{-\mu x}$$

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- Power/intensity attenuation

$$I(x) = I_0 \mathrm{e}^{-2\mu x}$$

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- Half-value layer (HVL)

$$\frac{\log 2}{\mu}$$

Attenuation (2)

- Amplitude attenuation

$$P(x) = P_0 \mathrm{e}^{-\mu x}$$

- Power/intensity attenuation

$$I(x) = I_0 \mathrm{e}^{-2\mu x}$$

- Half-value layer (HVL)

$$\frac{\log 2}{\mu}$$

- Half-power distance (HPD)

$$\frac{\log 2}{2\mu}$$

Attenuation and frequency

Attenuation increases approximately linearly with frequency

$$\mu \propto f$$

Penetration (approximate)

frequency [MHz]	depth [cm]
3.5	$10 \sim 20$
5.0	$5 \sim 10$
7.5	$2.5 \sim 5$
10.0	$1 \sim 4$

Ultrasound Attenuation

Material	*Half–power distance (cm)*
Water	380
Blood	15
Soft tissue	5 to 1
except muscle	1 to 0.6
Bone	0.7 to 0.2
Air	0.08
Lung	0.05

Tissue attenuation variations

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Half amplitude

$$20 \log_{10} \frac{1}{2} \approx -6\,\mathrm{dB}$$

Half power

$$20 \log_{10} \frac{1}{\sqrt{2}} = 10 \log_{10} \frac{1}{2} \approx -3\,\mathrm{dB}$$

Tissue attenuation variations

Half amplitude

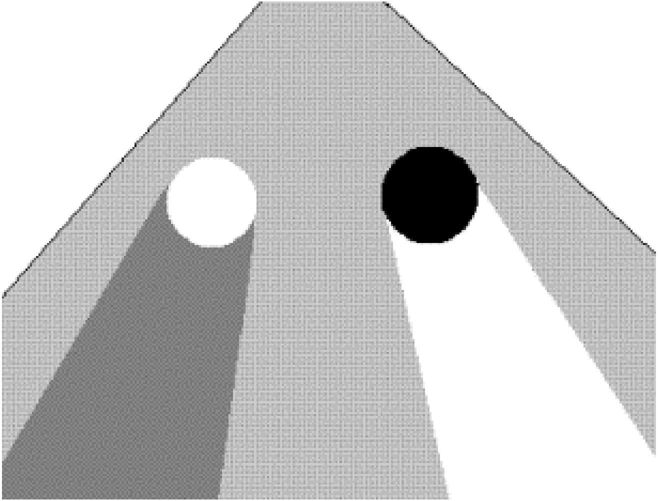
$$20 \log_{10} \frac{1}{2} \approx -6\,\mathrm{dB}$$

Half power

$$20 \log_{10} \frac{1}{\sqrt{2}} = 10 \log_{10} \frac{1}{2} \approx -3\,\mathrm{dB}$$

At $f = 3.5\,\mathrm{MHz}$, $\mu/f = 0.0022\,\mathrm{dB/cm/MHz}$ corresponds to
$\mathrm{HPD} = \frac{3\,\mathrm{dB}}{0.0022 \cdot 3.5\,\mathrm{MHz}} \approx 390\,\mathrm{cm}$

Shadows and enhancements



left: high reflexivity, right: high transmissibility

Shadows and enhancements

Medical ultrasound devices

Medical ultrasound devices

Medical ultrasound devices

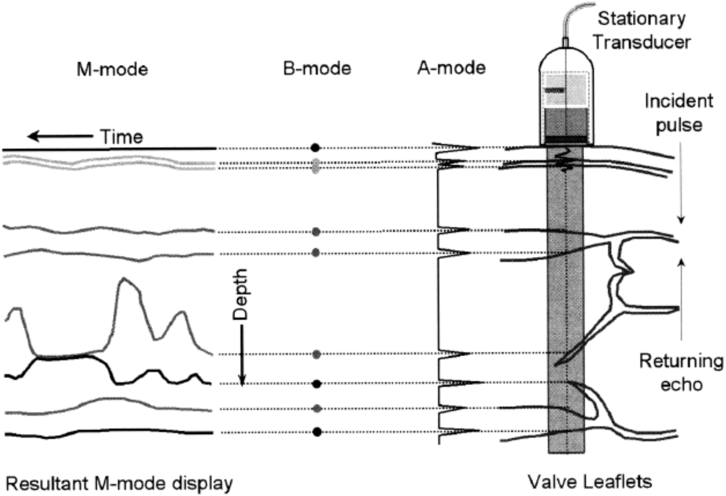
Medical applications of ultrasound imaging

- Cardiology (heart)
- Gynecology: breast, fetus (pregnancy)
- Internal organs: liver, kidney, thyroid gland
- Intravascular ultrasound
- *Therapeutic ultrasound: shock wave (kidney stone), thermal effects (rehabilitation)*

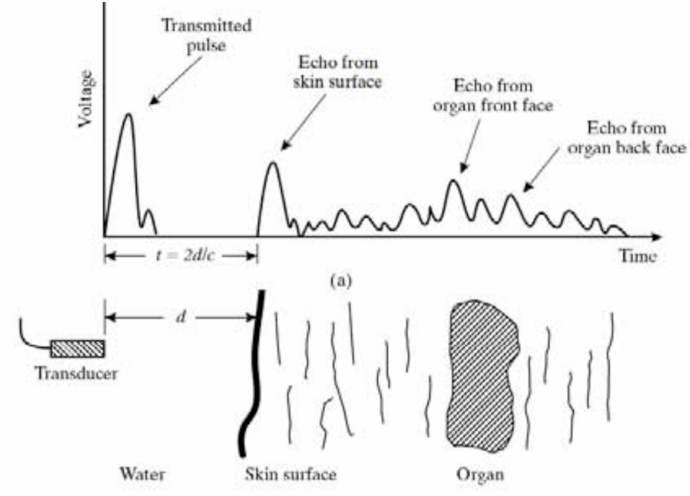
Imaging modes

- A osciloscopic, intensity/time
- B **2D in the probe plane**
- C *2D perpendicular*
- M/TM 1D+time
- Q Doppler (speed)

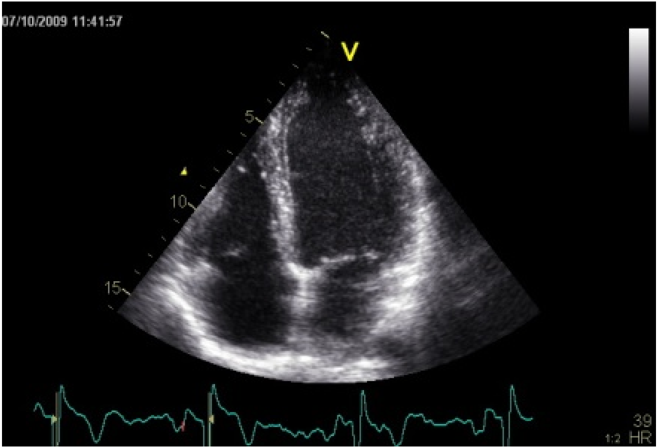
Imaging modes (2)



A-mode
(Amplitude)

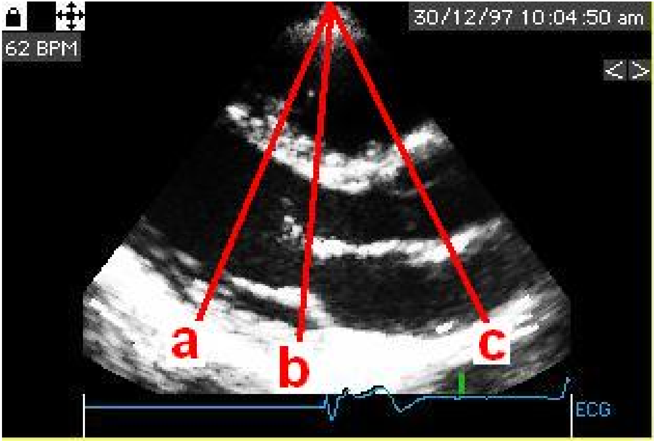


B-mode

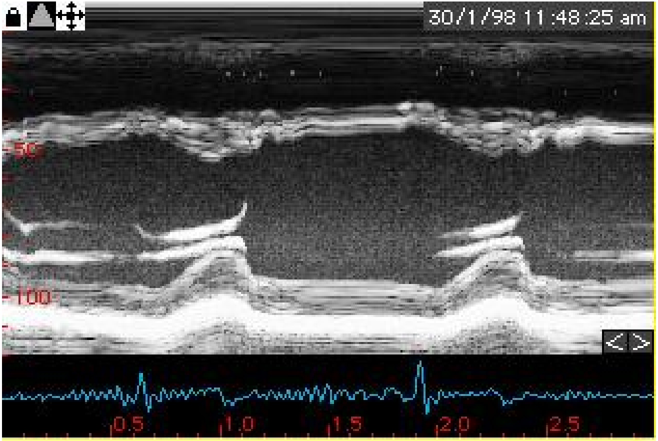


Heart

(T)M-mode

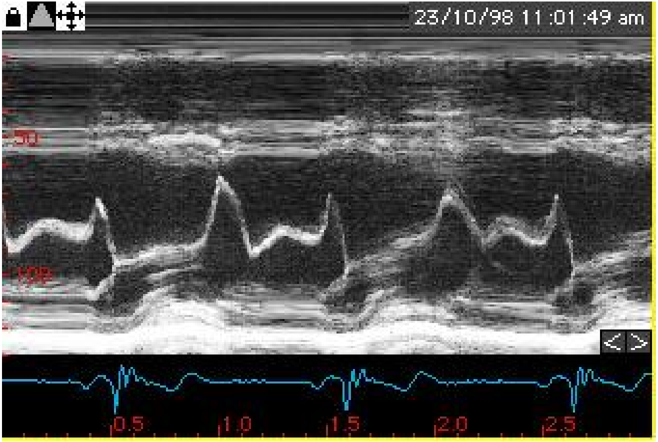


(T)M-mode



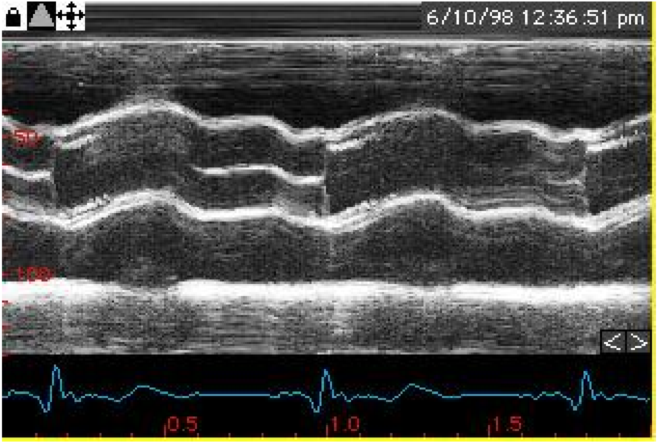
a

(T)M-mode



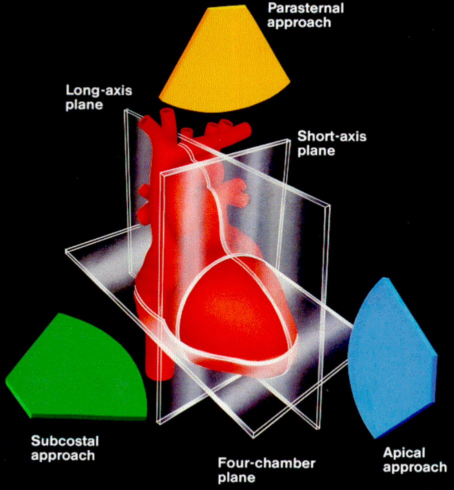
b

(T)M-mode

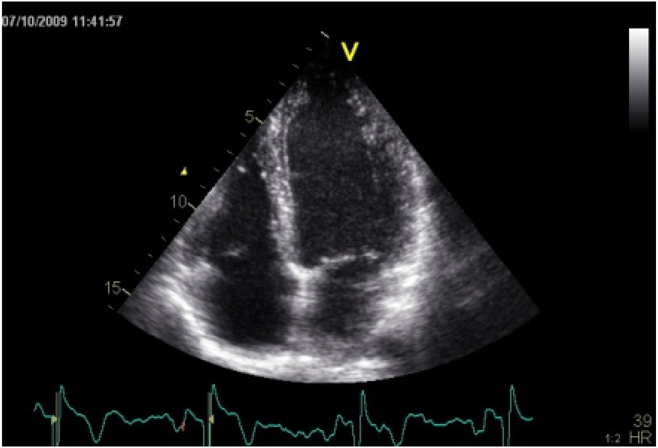


c

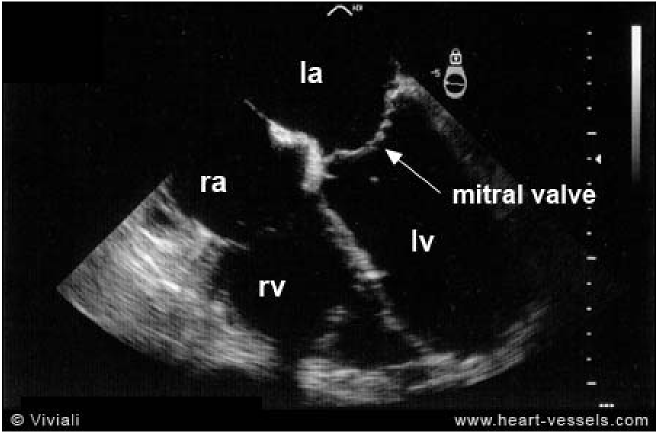
Conventional Cardiac 2D Ultrasound



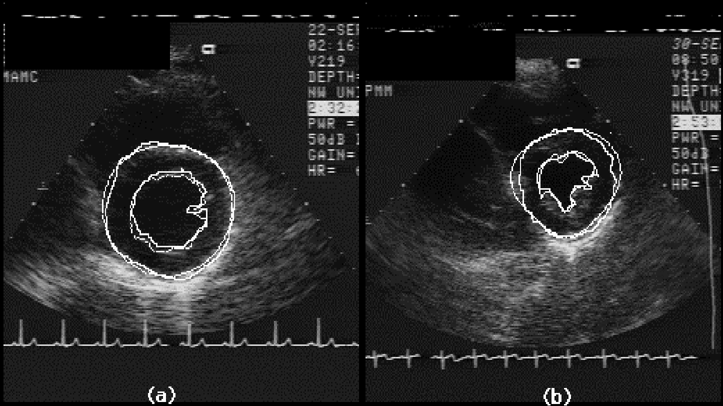
Heart



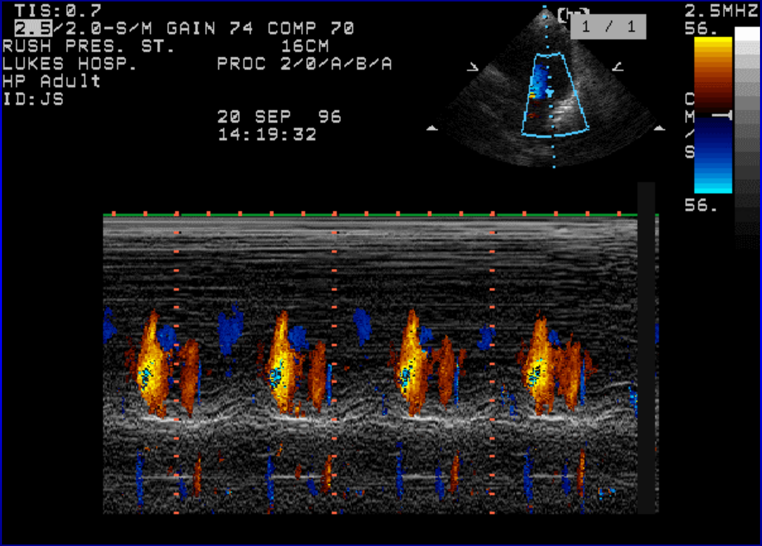
Heart



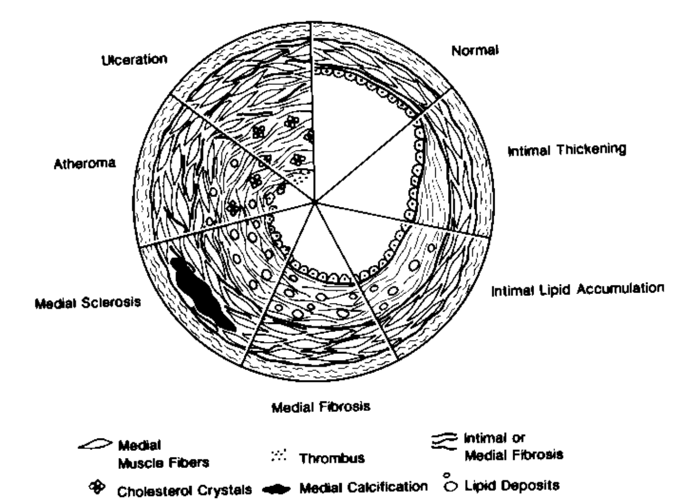
Traditional Ultrasound Images



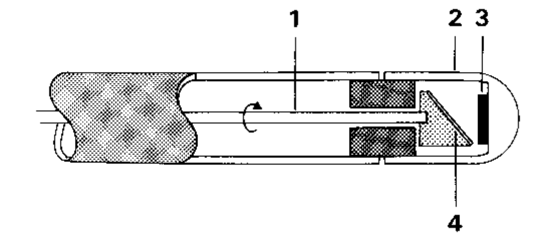
End-diastole End-systole


TIS:0.7
2.5/2.0-S/M GAIN 74 COMP 70
RUSH PRES. ST.
16CM
LUKES HOSP.
PROC 2/0/A/B/A
HP Adult
ID:JS
20 SEP 96
14:19:32
1 / 1
2.5MHZ
56.
C
M
/
S
56.

Progression of Vascular Disease

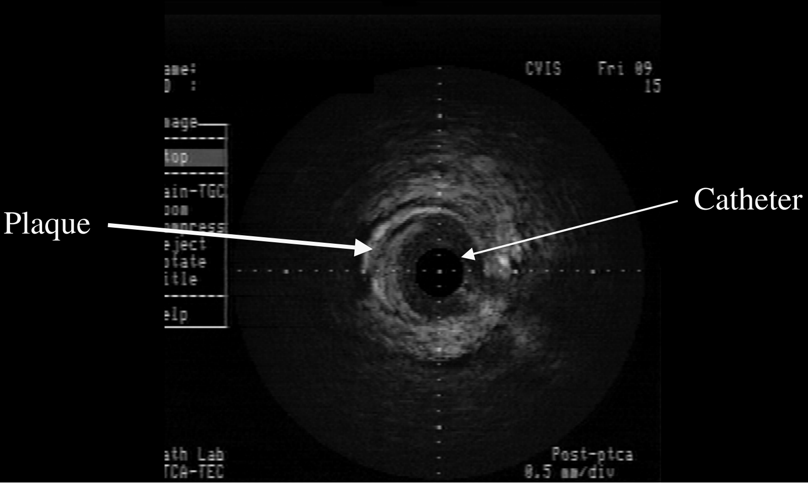


IVUS Catheter

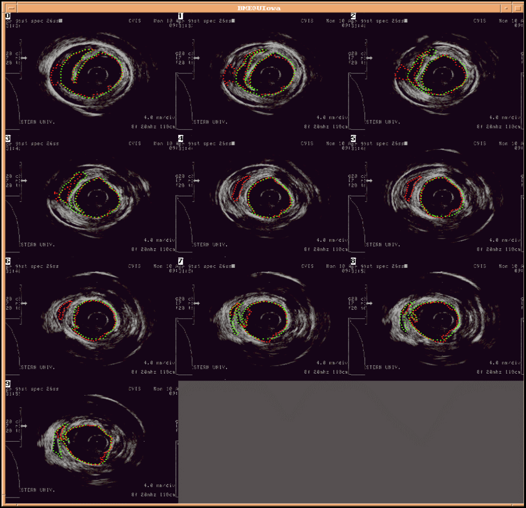


- 1 - Rotating shaft
- 2 - Acoustic window
- 3 - Ultrasound crystal
- 4 - Rotating beveled acoustic mirror

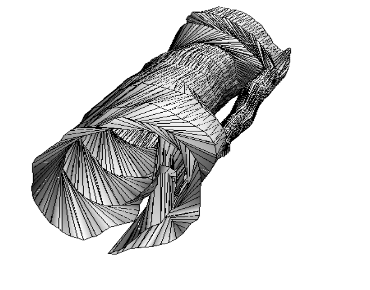
Slightly Diseased Artery in Cross-section

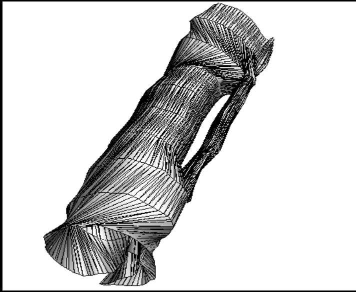


An array of Images



3D IVUS

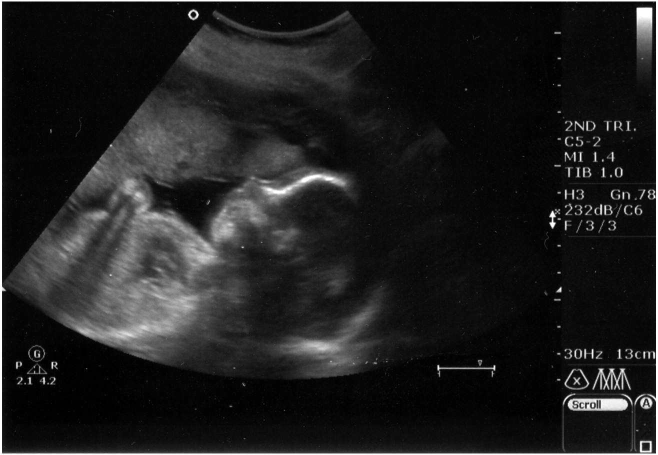




Other ultrasound examples

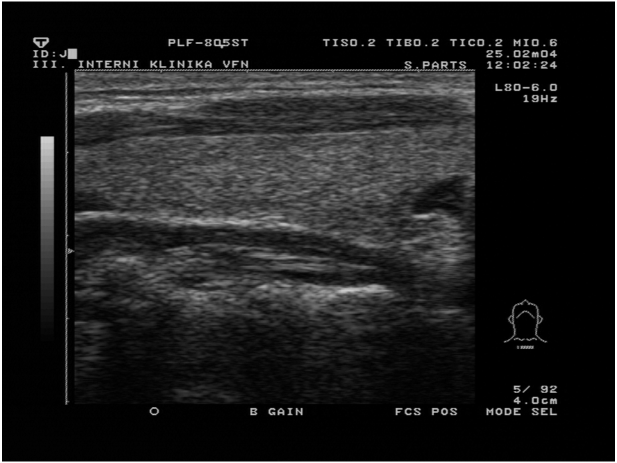
Early fetus

Other ultrasound examples



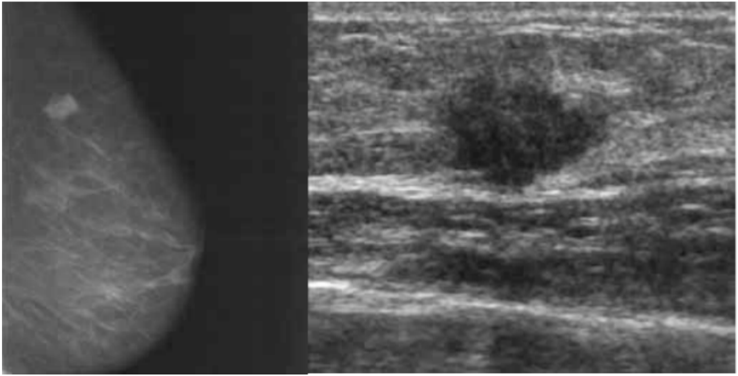
Bigger fetus

Other ultrasound examples



Thyroid gland

Other ultrasound examples

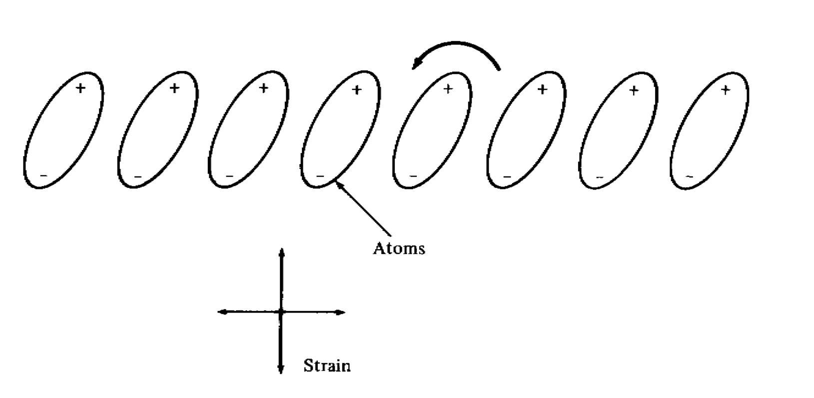


Breast

Pressure Generation

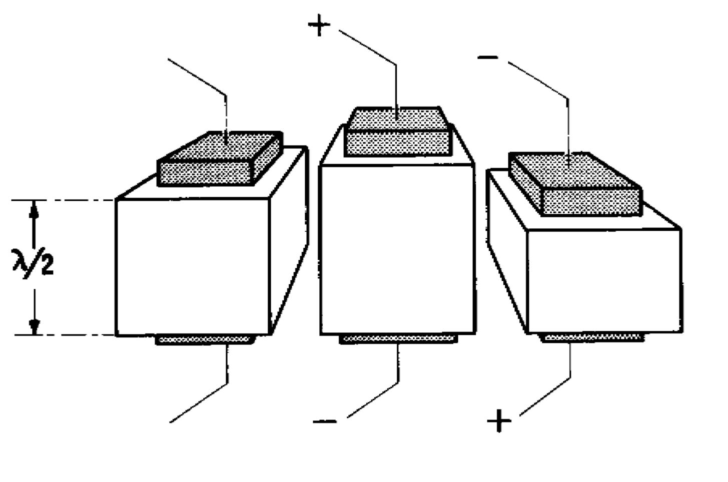
- Piezoelectric crystal
- ‘piezo’ means pressure, so piezoelectric means
  - pressure generated when electric field is applied
  - electric energy generated when pressure is applied

Charged Piezoelectric Molecules



Highly simplified effect of E field

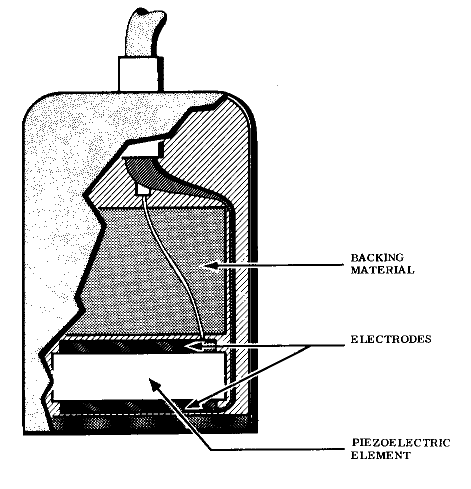
Piezoelectric Effect



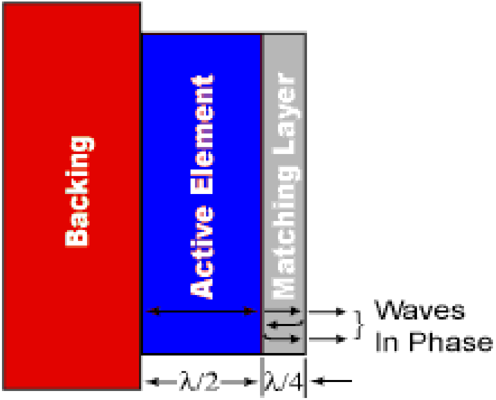
Transducer materials

- **PZT** — lead zirconate titanate, ceramic
  - High $Z \longrightarrow$ strong reflection
  - high resonance quality Q — frequency selective, high sensitivity
- **PVDF** — polyvinylidine difluoride, plastic
  - Low $Z \longrightarrow$ low reflection
  - low resonance quality Q — wider bandwith, lower sensitivity
- Composite materials
- Capacitive transducers

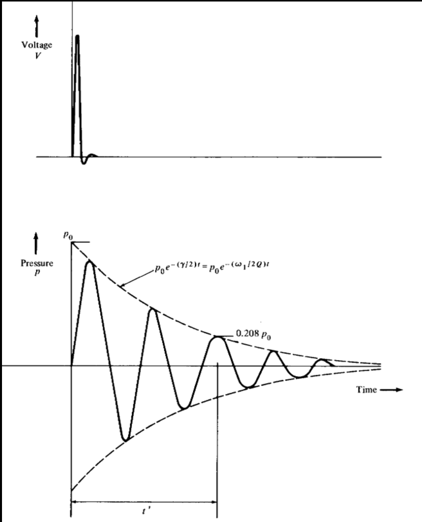
Transducer



Impedance matching layer

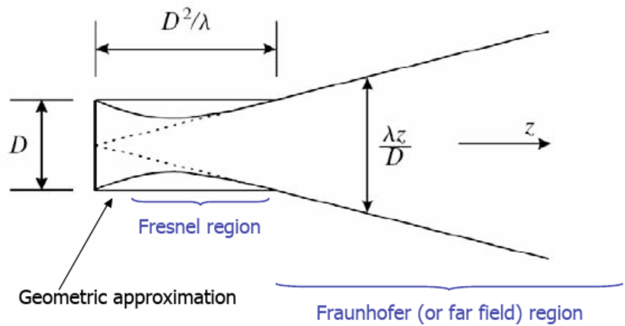


Pressure Radiated by Sharp Pulse



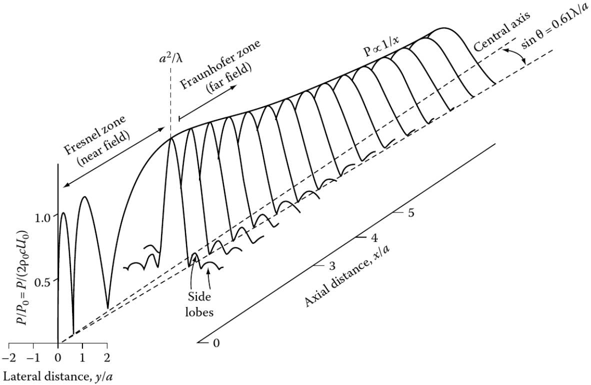
Beam pattern

Plane/unfocused source

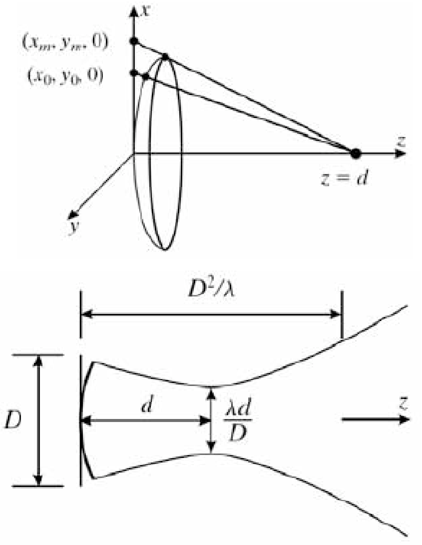


Beam pattern

Plane/unfocused source

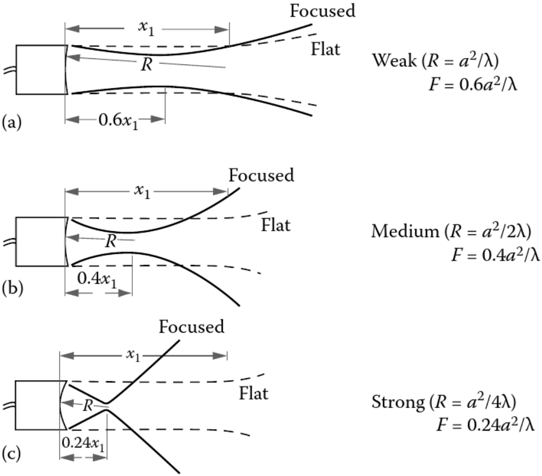


Focused beam pattern

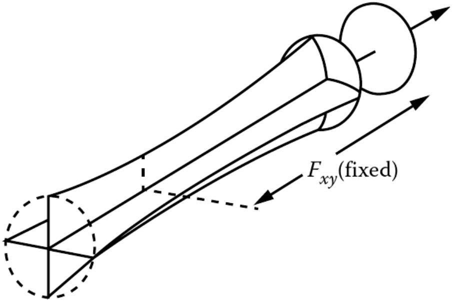


Ultrasound lens

Focused beam pattern

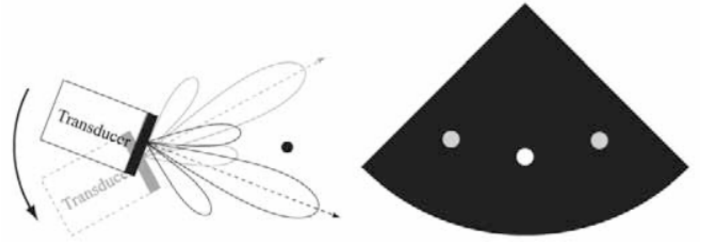


Focused beam pattern

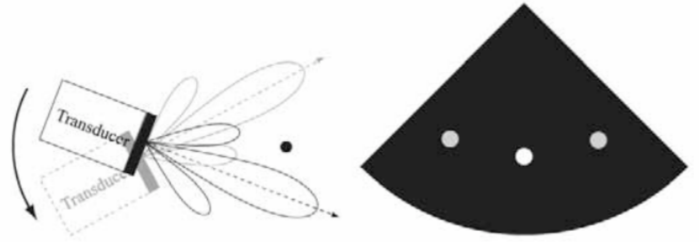


3D profile. Axial, transversal and lateral resolution

Lobes



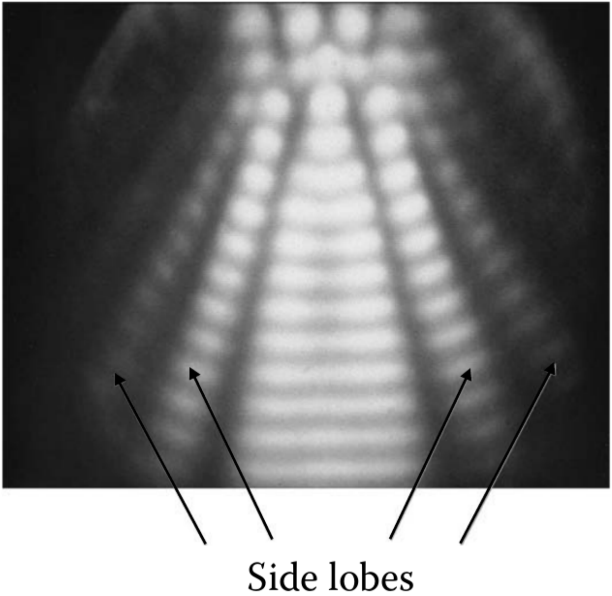
Lobes



Main lobe — contains 84 % energy, angle

$$\sin\theta \approx \frac{1.22\lambda}{D}$$

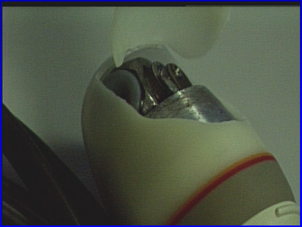
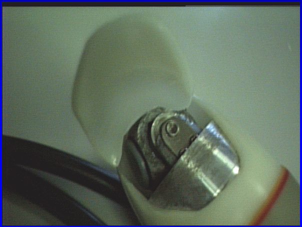
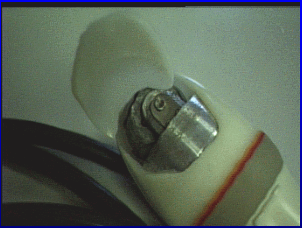
Lobes



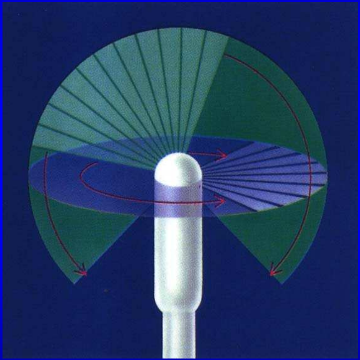
Beam steering

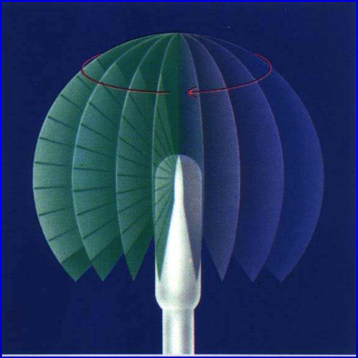
- Mechanical
- Electrical

UZV sonda s mech. rozkladem - Siemens

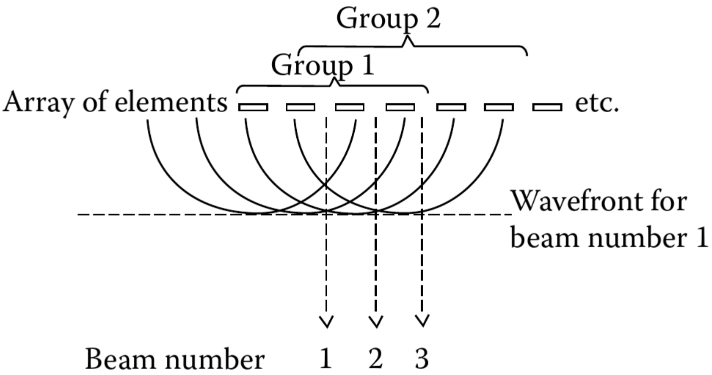





UZV sonda s mech. rozkladem - Siemens



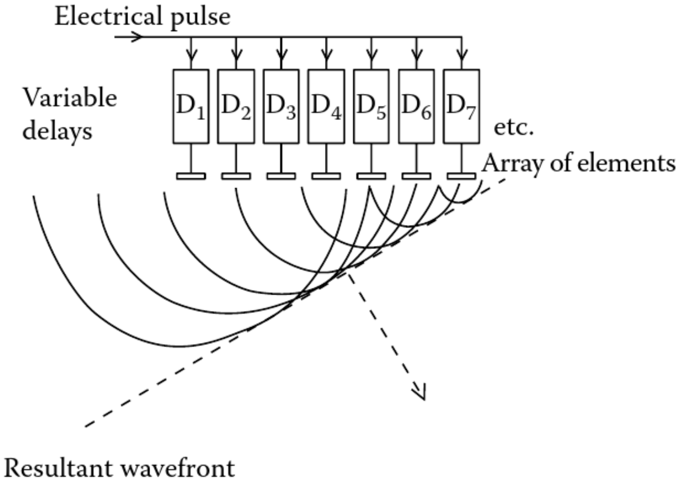


Electronic beam steering



lateral scan

Electronic beam steering

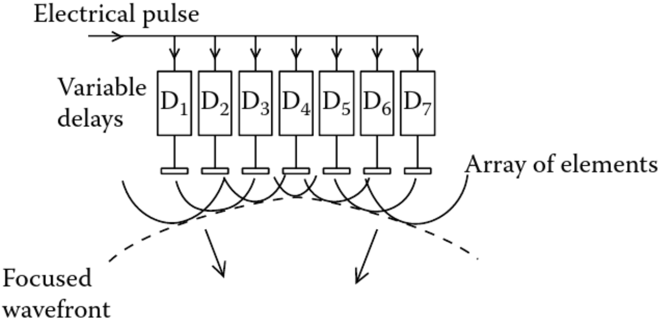


sector steering

Focusing types

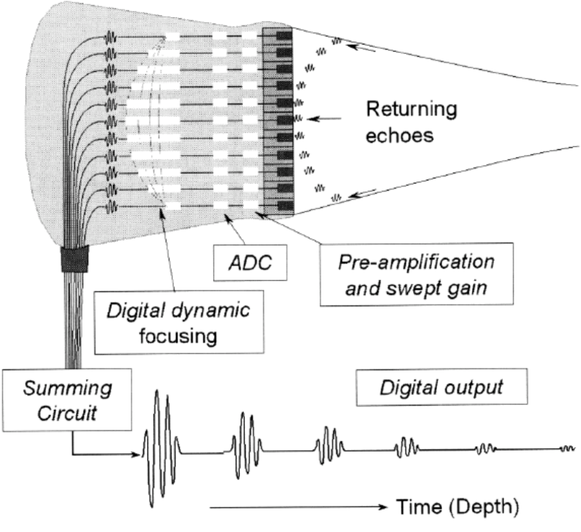
- Ultrasound lens
- Electronic

Electronic beam focusing



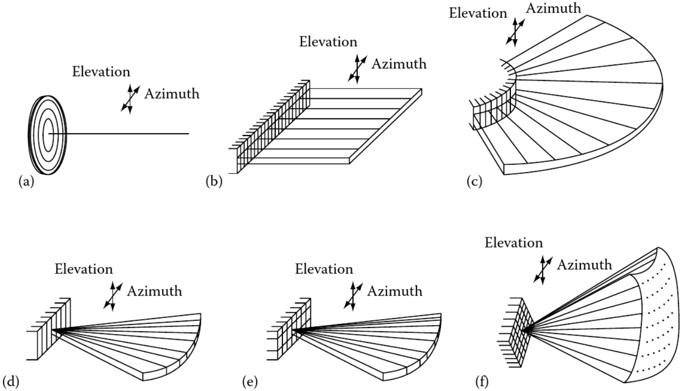
at transmit

Electronic beam focusing



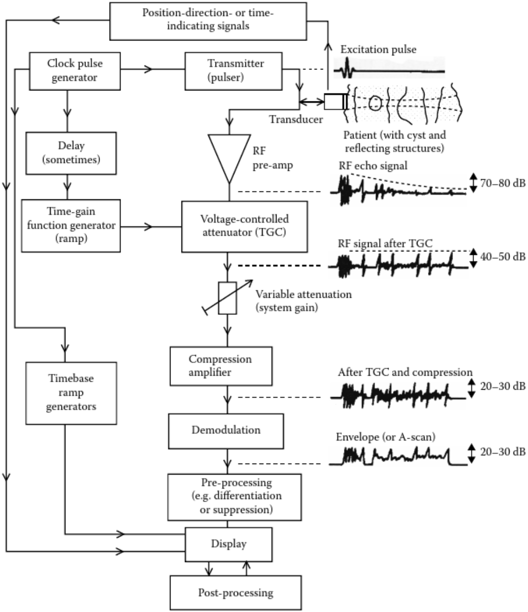
at receive

Transducer array configurations



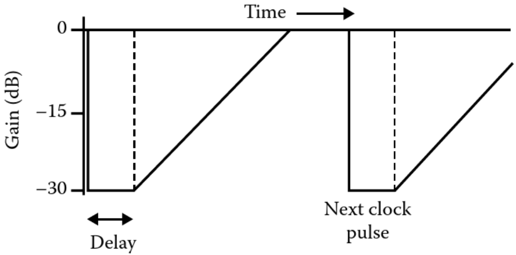
annular, linear, sector, phased-array, 1.5D phased array, 2D phased array

Scanner block diagram



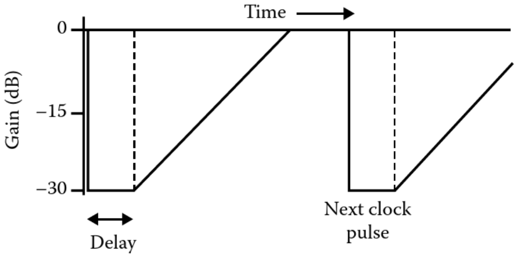
RF processing

- Time gain control



RF processing

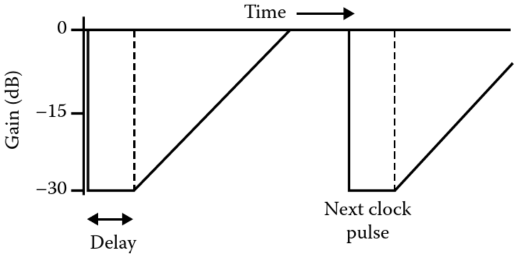
- Time gain control



- Demodulation — RF to envelope, (quadrature) detector

RF processing

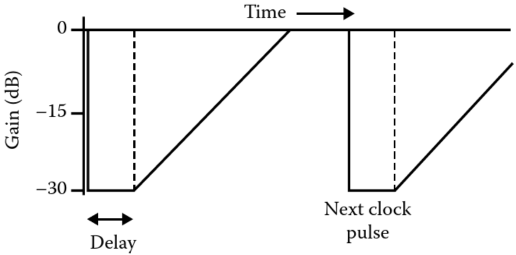
- Time gain control



- Demodulation — RF to envelope, (quadrature) detector
- Compression amplifier (50 dB range to 20 $\sim$ 30 dB range)

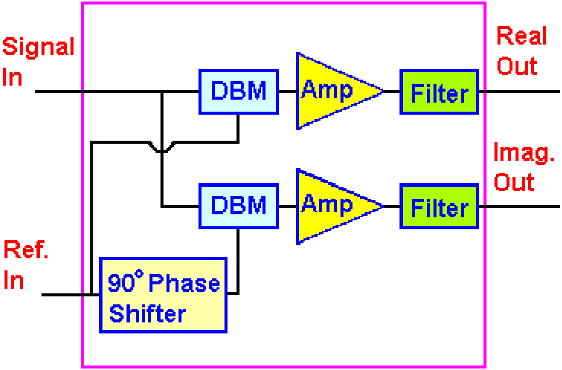
RF processing

- Time gain control



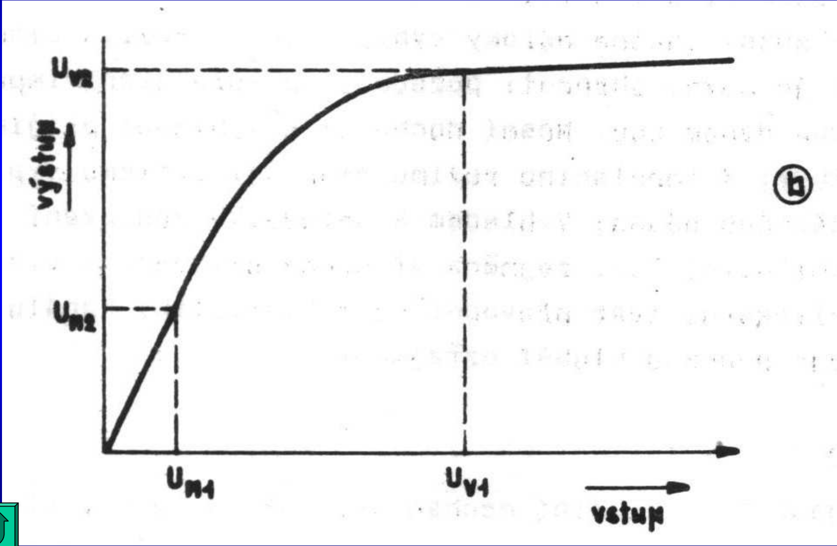
- Demodulation — RF to envelope, (quadrature) detector
- Compression amplifier (50 dB range to $20 \sim 30$ dB range)
- Geometry conversion (interpolation)

Quadrature detector

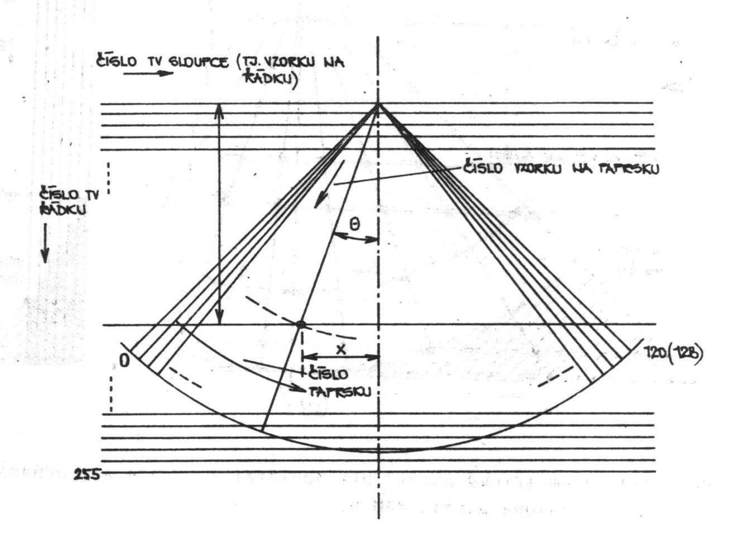


- *Input:* $g_a = \cos(at), \quad g_b = \cos(bt)$
- *Output:* $g = g_a g_b = \frac{1}{2}\cos((a+b)t) + \frac{1}{2}\cos((a-b)t)$
- Signal $\cos((a+b)t)$ can be filtered (low-pass filter)
- Difference frequence signal $s_r = \cos((a-b)t)$
- "Imaginary" signal s_i shifted by 90°: $\sin((a-b)t)$

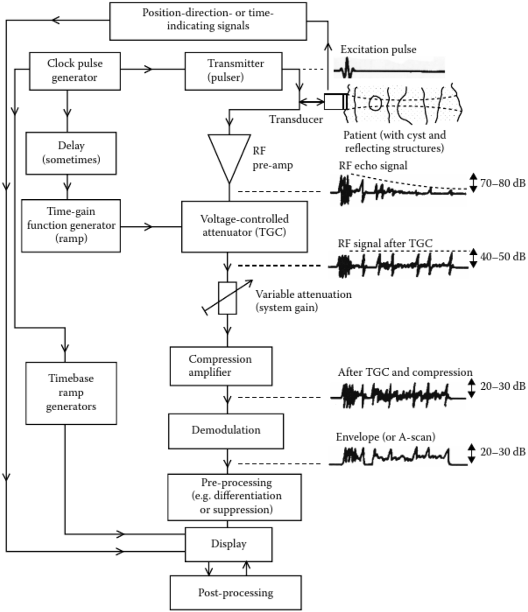
Amplitudově řízené zesilovače



Geom. vztah sekt. sním. a TV zobr. rastru



Scanner block diagram



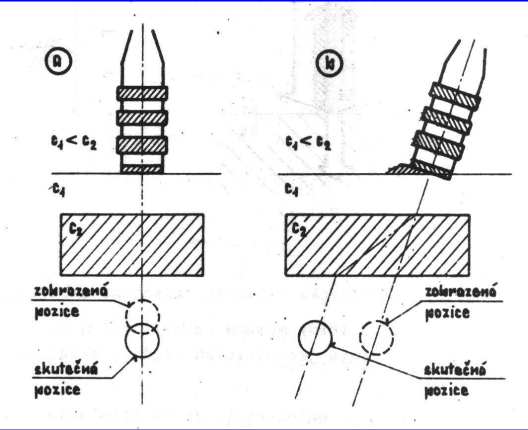
Artefacts

Due to

- Ultrasound speed variability
- Reflection
- Finite beam width
- Movement

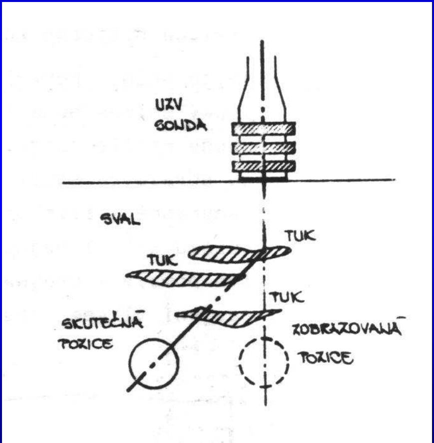
Geometrická distorze UZV zobrazení

- změnou rychlosti šíření UZV vlny,



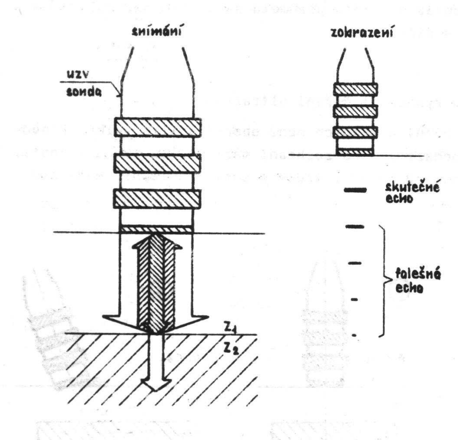
Geometrická distorze UZV zobrazení

- skladbou tkání,



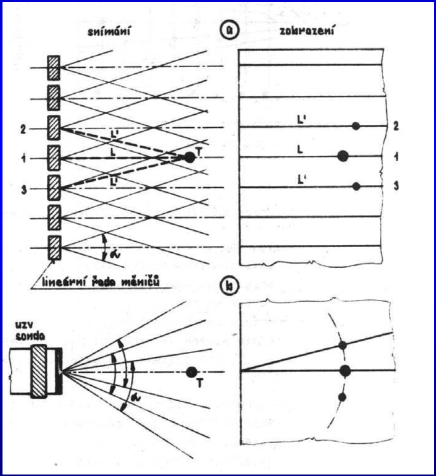
Geometrická distorze UZV zobrazení

- násobnou reflexí,



Geometrická distorze UZV zobrazení

- vlivem konečné šířky UZV svazku,



Geometrická distorze UZV zobrazení

- pohybem tkáňových struktur,