

Wavelet texture descriptors

Šonka&Hlaváč&Boyle book, Unser 1995

Jan Kybic

2020-2025

Texture

- ▶ texture information = image information - shape & color
- ▶ perceptually shift invariant (at a given scale)
- ▶ medical imaging - stochastic (weak) texture
- ▶ random, described by statistical descriptors

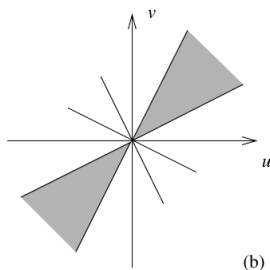
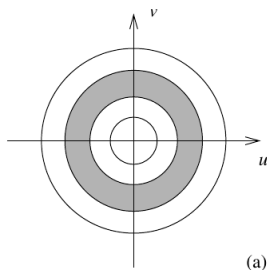
Statistical texture description

- ▶ mean, variance,

Evaluate autocorrelation coefficients for several different values of parameters p, q :

$$C_{ff}(p, q) = \frac{MN}{(M-p)(N-q)} \frac{\sum_{i=1}^{M-p} \sum_{j=1}^{N-q} f(i, j) f(i+p, j+q)}{\sum_{i=1}^M \sum_{j=1}^N f^2(i, j)}, \quad (15.1)$$

- ▶ image power spectrum $|\hat{F}|$



- ▶ *co-occurrence matrices, fractal dimensions...*

Filterbank descriptors

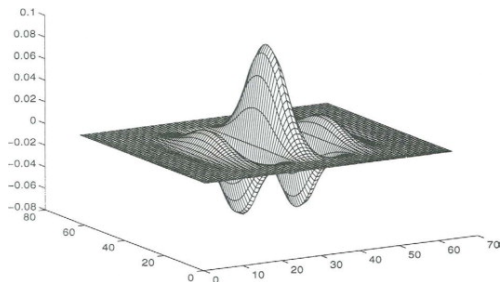
- ▶ Filterbank g_i , energy descriptors $d_i = \|f * g_i\|^2$

Filterbank descriptors

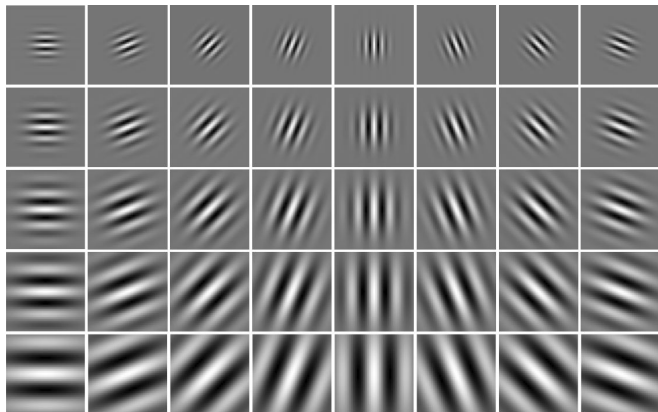
- ▶ Filterbank g_i , energy descriptors $d_i = \|f * g_i\|^2$
- ▶ Gabor filters

$$G_c[i, j] = B e^{-\frac{(i^2+j^2)}{2\sigma^2}} \cos(2\pi s(i \cos \theta + j \sin \theta))$$

$$G_s[i, j] = C e^{-\frac{(i^2+j^2)}{2\sigma^2}} \sin(2\pi s(i \cos \theta + j \sin \theta))$$

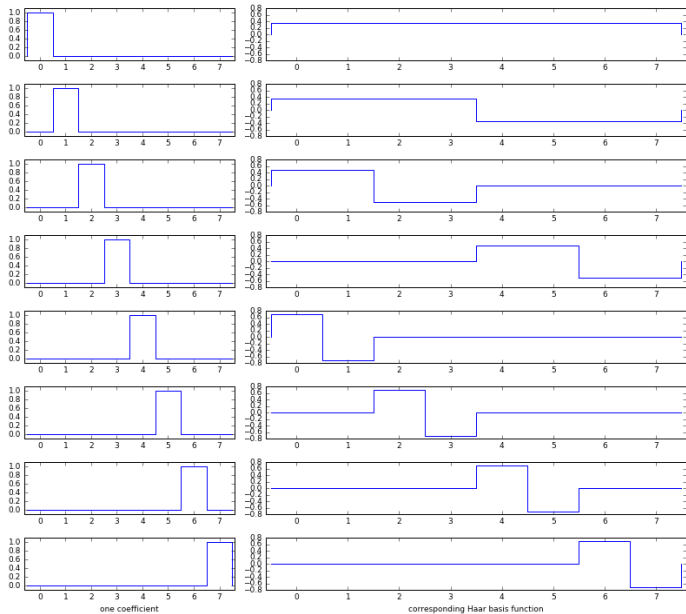


2D Gabor filters



by Mohammad Haghighat

Haar wavelets



Two-scale relationship

- ▶ Two-scale relationship

$$h_{i+1}(k) = [h]_{\uparrow 2^i} * h_i(k)$$

$$g_{i+1}(k) = [g]_{\uparrow 2^i} * h_i(k)$$

Nested subspaces

- ▶ Orthogonal basis functions

$$\varphi_{i,l}(k) = 2^{i/2} h_i(k - 2^i l),$$

$$\psi_{i,l}(k) = 2^{i/2} g_i(k - 2^i l),$$

- ▶ Nested subspaces $V_i = \text{span}\{\varphi_{i,l}\}_{l \in \mathbb{Z}}$ $V_2 \supset V_0 \supset V_1 \supset \dots \supset V_I$

- ▶ Complement space $V_{i-1} = V_i + W_i$, $V_i \perp W_i$, $W_i = \text{span}\{\psi_{i,l}\}_{l \in \mathbb{Z}}$

Wavelet representation

- ▶ Orthogonal projection to V_i

$$x_{(i)}(k) = \sum_{l \in \mathbb{Z}} s_{(i)}(l) \varphi_{i,l},$$

$$s_{(i)}(l) = \langle x(k), \varphi_{i,l}(k) \rangle_{l_2}$$

- ▶ Residual (high frequency)

$$x_{(i-1)}(k) - x_{(i)}(k) = \sum_{l \in \mathbb{Z}} d_{(i)}(l) \psi_{i,l},$$

$$d_{(i)}(l) = \langle x(k), \psi_{i,l}(k) \rangle_{l_2}$$

- ▶ Complete wavelet expansion

$$x(k) = \sum_{l \in \mathbb{Z}} s_{(I)}(l) \varphi_{I,l} + \sum_{i=1}^I \sum_{l \in \mathbb{Z}} d_{(i)}(l) \psi_{i,l}$$

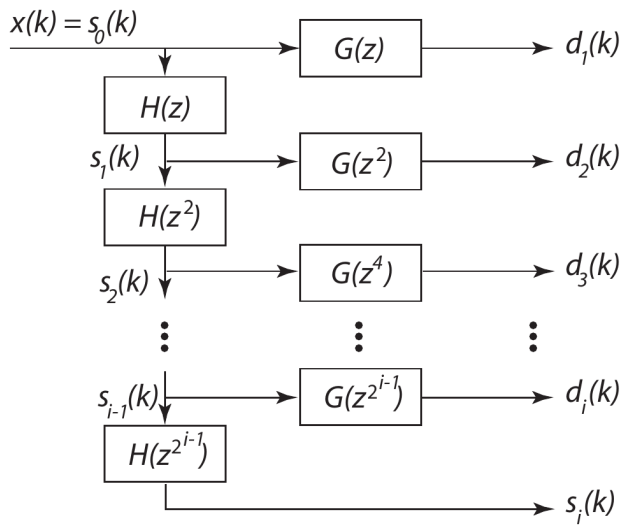
Undecimated WT

- ▶ Standard (decimated) WT is sensitive to shift
- ▶ Low pass $s_l = h_l * \tilde{x}(k)$
- ▶ High pass $d_l = g_l * \tilde{x}(k)$
- ▶ Recursive decomposition

$$s_{i+1}(k) = [h]_{\uparrow 2^i} * s_i(k)$$

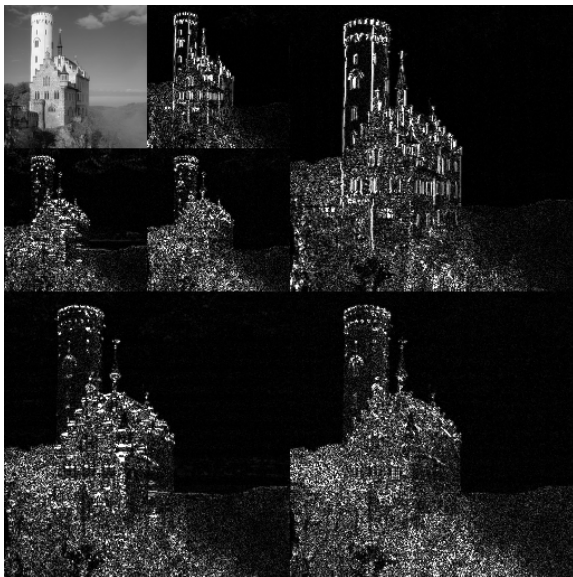
$$d_{i+1}(k) = [g]_{\uparrow 2^i} * s_i(k)$$

Recursive decomposition



2D Wavelets

Separability: $h_I(x)h_I(y)$, $h_I(x)g_I(y)$, $g_I(x)h_I(y)$, $g_I(x)g_I(y)$



Wavelet features

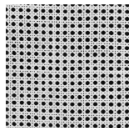
- ▶ Wavelet coefficients

$$\mathbf{y}(k, l) = (y_i(k, l))_{i=1, \dots, N} = [s_I(k, l); d_I(k, l); \dots; d_1(k, l)]^T$$

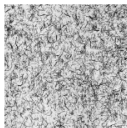
- ▶ In 2D, $3I + 1$ channels
- ▶ Texture characterized by $p(y_i)$
- ▶ Descriptors

$$v_i = \frac{1}{N_R} \sum_{(k, l) \in R} y_i^2(k, l)$$

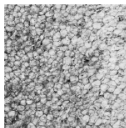
Brodatz textures



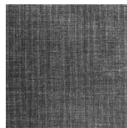
(a) 1



(b) 2



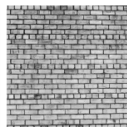
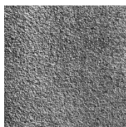
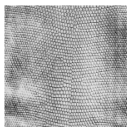
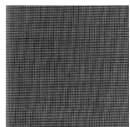
(c) 3



(d) 4



(e) 5



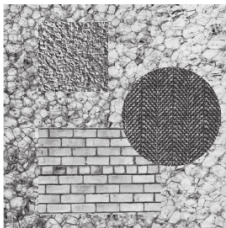
Texture classification example

	1	2	3	4	5	6	7	8	9	10
1	20	0	0	0	0	0	0	0	1	0
2	0	17	0	0	0	0	0	0	0	0
3	0	0	15	0	0	0	0	0	0	0
4	0	0	0	18	0	0	0	0	0	0
5	0	0	0	0	12	0	0	0	0	0
6	0	0	0	0	0	15	0	0	0	0
7	0	0	0	0	0	0	22	0	0	0
8	0	0	0	0	0	0	0	22	3	0
9	1	0	0	0	0	0	0	0	15	0
10	0	0	0	0	0	0	0	0	0	19

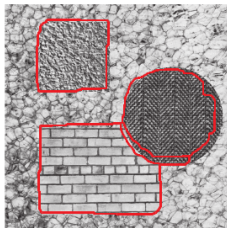
Table 15.2: Wavelet confusion matrix.

Texture segmentation

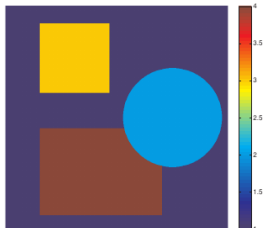
- ▶ For segmentation - average variance over a window
- ▶ $p(y_i)$ assumed normal, GMM parameters found by EM
- ▶ GraphCut regularization



(b)



(c)



(e)



(f)