Homework No. 03

Imagine a source-free volume filled with a general linear material with constitutive relations

$$\begin{aligned}
\boldsymbol{D}(\omega) &= \boldsymbol{\varepsilon}(\omega) \cdot \boldsymbol{E}(\omega) + \frac{\boldsymbol{\sigma}(\omega)}{\mathrm{j}\omega} \cdot \boldsymbol{E}(\omega) + \boldsymbol{\chi}^{\mathrm{em}}(\omega) \cdot \boldsymbol{H}(\omega)/c_0 \\
\boldsymbol{B}(\omega) &= \boldsymbol{\mu}(\omega) \cdot \boldsymbol{H}(\omega) + \boldsymbol{\chi}^{\mathrm{me}}(\omega) \cdot \boldsymbol{E}(\omega)/c_0
\end{aligned} \tag{1}$$

Poynting's theorem for time-harmonic fields can, in such cases, be written as

$$\oint_{\partial V} (\boldsymbol{E} \times \boldsymbol{H}^*) \cdot \mathrm{d}\boldsymbol{S} = -\mathrm{j}\omega \int_V (\boldsymbol{H}^* \cdot \boldsymbol{B} - \boldsymbol{E} \cdot \boldsymbol{D}^*)\, \mathrm{d}V \tag{2}$$

where

$$P_{\mathrm{out}} = \frac{1}{2} \mathrm{Re} \left\{ \oint_{\partial V} (\boldsymbol{E} \times \boldsymbol{H}^*) \cdot \mathrm{d}\boldsymbol{S} \right\} \tag{3}$$

is interpreted as a cycle-mean power flowing out of the surface ∂V. The medium inside the volume V can therefore be considered as passive if $P_{\mathrm{out}} < 0$, as active if $P_{\mathrm{out}} > 0$ and as lossless if $P_{\mathrm{out}} = 0$. Since the volume can be arbitrary, the relation must also hold pointwise.

Task No. 1: Based on the relation (1) and (2) argue that loss-less medium is described by material tensors satisfying

$$\begin{aligned}
\boldsymbol{\mu} &= \boldsymbol{\mu}^{\mathrm{H}} \\
\boldsymbol{\varepsilon} &= \boldsymbol{\varepsilon}^{\mathrm{H}} \\
\boldsymbol{\sigma} &= -\boldsymbol{\sigma}^{\mathrm{H}} \\
\boldsymbol{\chi}^{\mathrm{me}} &= \left(\boldsymbol{\chi}^{\mathrm{em}}\right)^{\mathrm{H}}
\end{aligned} \tag{4}$$