

Homework No. 01

The field-averaging process used to transform microscopic field into their macroscopic counterparts is about to be trained in this homework. To this end assume that N point charges are uniformly distributed along the x -axis. The distance between the charges is Δ . The microscopic charge density is therefore described as

$$\rho^{\text{micro}} = \sum_{n=0..N-1} q_n \delta(x - n\Delta). \quad (1)$$

Assume that the first half of charges has charge q_0 , while the second half has charge $2q_0$.

Task No. 1: Evaluate macroscopic charge density $\rho^{\text{macro}} = \langle \rho^{\text{micro}} \rangle$ by using averaging function $G(x)$ in a form of rectangular low frequency filter that cuts out all spectral components higher than k_{max} . For numerical results assume $N = 200$. Formulate the problem analytically before going to numerical evaluation.

Task No. 2: Discuss cases when $k_{\text{max}}\Delta/(2\pi) \gg 1$, $k_{\text{max}}\Delta/(2\pi) = 1/5$ and $k_{\text{max}}\Delta/(2\pi) < 2/N$. Intuitively we would expect to have $\rho^{\text{macro}}\Delta = q_0$ for $x/\Delta \in N(0, 1/2)$, $\rho^{\text{macro}}\Delta = 2q_0$ for $x/\Delta \in N(1/2, 1)$ and zero charge density elsewhere.