

Multi-goal Planning

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Lecture 06

B4M36UIR – Artificial Intelligence in Robotics



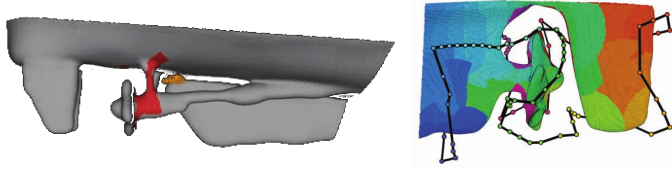
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Inspection Planning

Multi-goal Planning

Robotic Information Gathering in Inspection of Vessel's Propeller

- The planning problem is to determine a shortest inspection path for an Autonomous Underwater Vehicle (AUV) to inspect the vessel's propeller.



https://www.youtube.com/watch?v=8azP_9VnftM
Englot, B., Hover, F.S.: *Three-dimensional coverage planning for an underwater inspection robot*, International Journal of Robotics Research, 32(9-10):1048-1073, 2013.



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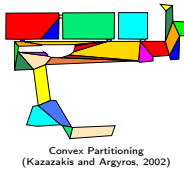
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Multi-goal Planning

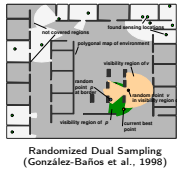
Inspection Planning – Decoupled Approach

- Determine sensing locations such that the whole environment would be inspected (seen) by visiting them. It is **Sampling design problem**.

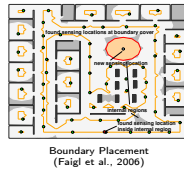
In the geometrical-based approach, a solution of the *Art Gallery Problem*.



Convex Partitioning
(Kazazakis and Argyros, 2002)



Randomized Dual Sampling
(González-Baños et al., 1998)



Boundary Placement
(Faigl et al., 2006)

The problem is related to the **sensor placement** and **sampling design**.

- Create a roadmap connecting the sensing location.

Such as **visibility graph** or **randomized sampling-based method**.

- Find the inspection path visiting all the sensing locations as a solution of the multi-goal path planning (a solution of the robotic TSP).



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Multi-goal Planning

Overview of the Lecture

- Part 1 – Multi-goal Planning
 - Inspection Planning
 - Multi-goal Planning
- Part 2 – Unsupervised Learning for Multi-goal Planning
 - Unsupervised Learning for Multi-goal Planning
 - TSPN in Multi-goal Planning with Localization Uncertainty



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Example of Inspection Planning in Search Scenario

- Periodically visit particular locations of the environment and return to the starting locations.
- Use **available floor plans** to guide the search, e.g., finding victims in search-and-rescue scenario.



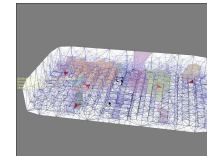
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Multi-goal Planning

Planning to Capture Areas of Interest using UAV

- Determine a cost-efficient path from which a given set of target regions is covered.
- For each target region a subspace $S \subset \mathbb{R}^3$ from which the target can be covered is determined. S represents the *neighborhood*.
- We search for the **best sequence of visits to the regions**.
Combinatorial optimization
- The PRM is utilized to construct the planning roadmap (a graph).
PRM – Probabilistic Roadmap Method – sampling-based motion planner, see lecture 8.
- The problem can be formulated as the **Traveling Salesman Problem with Neighborhoods**, as it is not necessary to visit exactly a single location to capture the area of interest.



Janoušek and Faigl, ICRA 2013



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Part I

Part 1 – Multi-goal Planning



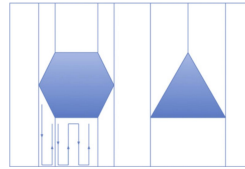
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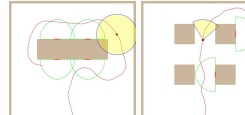
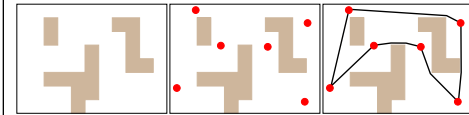
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- Inspection/coverage planning** stands to determine a plan (path) to inspect/cover the given areas or point of interest.
- We can directly find inspection/coverage plan using
 - predefined covering patterns such as *ox-plow* motion;
 - a "general" path satisfying coverage constraints.
- Decoupled approach** – Locations to be visited are determined before path planning as the **sensor placement** problem.



Trapezoidal decomposition method



Kafka, Faigl, Váňa: ICRA 2016

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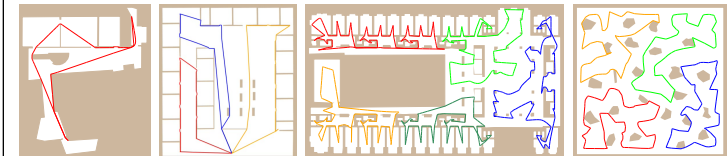
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Inspection Planning – "Continuous Sensing"

- If we do not prescribe a discrete set of sensing locations, we can formulate the problem as the **Watchman route problem**.

Given a map of the environment $\mathcal{W}$ determine the shortest, closed, and collision-free path, from which the whole environment is covered by an omnidirectional sensor with the radius ρ .



Faigl, J.: *Approximate Solution of the Multiple Watchman Routes Problem with Restricted Visibility Range*, IEEE Transactions on Neural Networks, 21(10):1668-1679, 2010.

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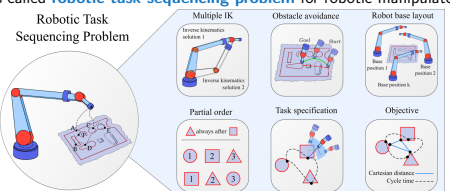
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Multi-Goal Planning

- Having a **set of locations** to be visited, determine the cost-efficient path to visit them.
 - Locations where a robotic arm or mobile robot performs some task. *The operation can be repeated-closed path.*
- The problem is called **robotic task sequencing problem** for robotic manipulators.



Alatartsev, S., Stellmacher, S., Ortmeier, F. (2015): *Robotic Task Sequencing Problem: A Survey*. Journal of Intelligent & Robotic Systems.

- The problem is also called **Multi-goal Path Planning** (MTP) problem or **Multi-goal Planning** (MGP).

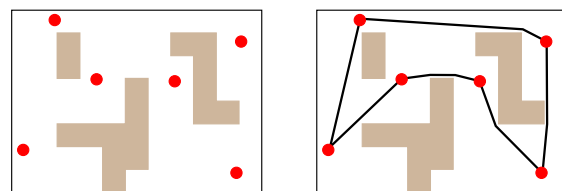
Also studied in its Multi-goal Motion Planning (MGMP) variant.

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Multi-Goal Path Planning (MTP)

- Multi-goal planning problem is a problem how to visit the given set of locations.
- It consists of **point-to-point path planning** on how to reach one location from another.
- The **challenge** is to determine the optimal sequence of the visits to the locations w.r.t. cost-efficient path to visit all the given locations.



- Determination the sequence of visits is a **combinatorial optimization problem** that can be formulated as the **Traveling Salesman Problem** (TSP).

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Traveling Salesman Problem (TSP)

Given a set of cities and the distances between each pair of cities, what is the shortest possible route that visits each city exactly once and returns to the origin city.

- The TSP can be formulated for a graph $G(V, E)$, where V denotes a set of locations (cities) and E represents edges connecting two cities with the associated travel cost c (distance), i.e., for each $v_i, v_j \in V$ there is an edge $e_{ij} \in E$, $e_{ij} = (v_i, v_j)$ with the cost c_{ij} .
- If the associated cost of the edge (v_i, v_j) is the Euclidean distance $c_{ij} = |v_i, v_j|$, the problem is called the **Euclidean TSP** (ETSP).
- It is known, the TSP is NP-hard (its decision variant) and several algorithms can be found in literature.

William J. Cook (2012) – *In Pursuit of the Traveling Salesman: Mathematics at the Limits of Computation.*

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Traveling Salesman Problem (TSP)

- Let S be a set of n sensor locations $S = \{s_1, \dots, s_n\}$, $s_i \in \mathbb{R}^2$ and $c(s_i, s_j)$ is a cost of travel from s_i to s_j .
- Traveling Salesman Problem (TSP)** is a problem to determine a closed tour visiting each $s \in S$ such that the total tour length is minimal.
 - We are searching for the optimal **sequence of visits** $\Sigma = (\sigma_1, \dots, \sigma_n)$ such that

$$\begin{aligned} \text{minimize } \Sigma \quad & L = \left(\sum_{i=1}^{n-1} c(s_{\sigma_i}, s_{\sigma_{i+1}}) \right) + c(s_{\sigma_n}, s_{\sigma_1}) \\ \text{subject to} \quad & \Sigma = (\sigma_1, \dots, \sigma_n), 1 \leq \sigma_i \leq n, \sigma_i \neq \sigma_j \text{ for } i \neq j. \end{aligned} \quad (1)$$

- The TSP can be considered on a graph $G(V, E)$ where the set of vertices V represents sensor locations S and E are edges connecting the nodes with the cost $c(s_i, s_j)$.
- For simplicity we can consider $c(s_i, s_j)$ to be Euclidean distance; otherwise, we also need to address the path/motion planning problem. **Euclidean TSP**
- If $c(s_i, s_j) \neq c(s_j, s_i)$ it is the **Asymmetric TSP**.
- The TSP is known to be NP-hard unless P=NP.

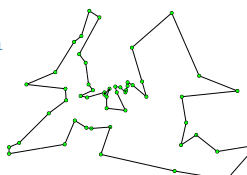
Traveling vs Travelling – <http://www.math.uwaterloo.ca/tsp/history/travelling.html>

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Existing Approaches to the TSP

- Exact solutions
 - Branch&Bound, Branch&Cut, and Integer Linear Programming (ILP).
 - Concorde-<http://www.math.uwaterloo.ca/tsp/concorde.html>
- Approximation algorithms
 - Minimum Spanning Tree (MST) heuristic with $L \leq 2L_{opt}$.
 - Christofides's algorithm with $L \leq \frac{3}{2}L_{opt}$.
- Heuristic algorithms
 - Constructive heuristic – Nearest Neighborhood (NN) algorithm;
 - 2-Opt – local search algorithm proposed by Croes 1958;
 - LKH – K. Helsgaun efficient implementation of the Lin-Kernighan heuristic (1998). <http://www.akira.ruc.dk/~keld/research/LKH/>
- Combinatorial meta-heuristics
 - Variable Neighborhood Search (VNS);
 - Greedy Randomized Adaptive Search Procedure (GRASP).
- Soft-computing techniques, evolutionary methods, and **unsupervised learning**.



Problem Berlin52 from the TSPLIB

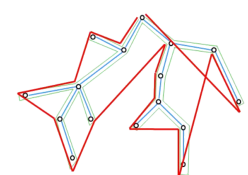
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MST-based Approximation Algorithm to the TSP

- Minimum Spanning Tree heuristic

- Compute the MST (denoted T) of the input graph G .
- Construct a graph H by doubling every edge of T .
- Shortcut repeated occurrences of a vertex in the tour.



- For the triangle inequality, the length of such a tour L is

$$L \leq 2L_{optimal},$$

where $L_{optimal}$ is the cost of the optimal solution of the TSP.

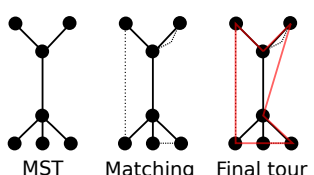
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Christofides's Algorithm to the TSP

- Christofides's algorithm

- Compute the MST of the input graph G .
- Compute the minimal matching on the odd-degree vertices.
- Shortcut a traversal of the resulting Eulerian graph.



MST Matching Final tour

- For the triangle inequality, the length of such a tour L is

$$L \leq \frac{3}{2}L_{optimal},$$

where $L_{optimal}$ is the cost of the optimal solution of the TSP.

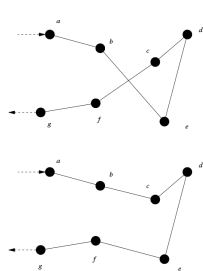
Length of the MST is $\leq L_{optimal}$
Sum of lengths of the edges in the matching $\leq \frac{1}{2}L_{optimal}$

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2-Opt Heuristic

- Use a construction heuristic to create an initial route
 - NN algorithm, cheapest insertion, farther insertion
- Repeat until no improvement is made
 - Determine swapping that can shorten the tour (i, j) for $1 \leq i \leq n$ and $i + 1 \leq j \leq n$
 - route[0] to route[i-1];
 - route[i] to route[j] in reverse order;
 - route[j] to route[end];
 - Determine length of the route;
 - Update the current route if the length is shorter than the existing solution.



Croes, G.A.: *A method for solving traveling salesman problems*, Operations Research 6:791–812, 1958.

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Overview of the Variable Neighborhood Search (VNS) for the TSP

- Variable Neighborhood Search (VNS)** is a metaheuristic for solving combinatorial optimization and global optimization problems by searching distant neighborhoods of the current **incumbent solution** using **shake** and **local search**.

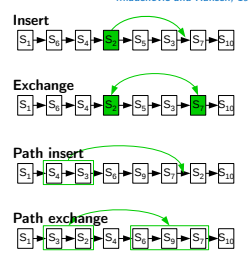
Mladenović and Hansen, 1997

- Shake** procedure explores the neighborhood of the current solution to escape from a local minima using operators:
 - Insert – moves one element;
 - Exchange – exchanges two elements.
- Local search** procedure improves the solution by
 - Path insert – moves a subsequence;
 - Path exchange – exchanges two subsequences.

Algorithm 1: VNS-based Solver to the TSP

Input: S – Set of the target locations to be visited.
Output: Σ – Found sequence of visits to locations S .

$\Sigma' \leftarrow$ Initial sequence found by cheapest insertion
while terminal condition is not met do
 $\Sigma' \leftarrow \text{shake}(\Sigma')$
 for n^{th} times do
 $\Sigma'' \leftarrow \text{localSearch}(\Sigma')$
 if Σ'' is "better" than Σ' then
 $\Sigma' \leftarrow \Sigma''$ // Select Σ'' instead of Σ'
 if Σ' is "better" than Σ then
 $\Sigma \leftarrow \Sigma'$ // Replace the incumbent sequence.
return Σ



Insert
 $S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_4 \rightarrow S_5 \rightarrow S_6 \rightarrow S_7 \rightarrow S_8$

Exchange
 $S_1 \rightarrow S_2 \rightarrow S_4 \rightarrow S_3 \rightarrow S_5 \rightarrow S_6 \rightarrow S_7 \rightarrow S_8$

Path insert
 $S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_4 \rightarrow S_5 \rightarrow S_6 \rightarrow S_7 \rightarrow S_8 \rightarrow S_9 \rightarrow S_{10}$

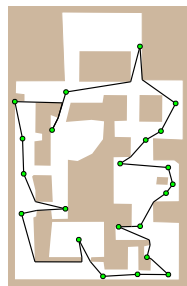
Path exchange
 $S_1 \rightarrow S_2 \rightarrow S_3 \rightarrow S_4 \rightarrow S_5 \rightarrow S_6 \rightarrow S_7 \rightarrow S_8 \rightarrow S_9 \rightarrow S_{10}$

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Multi-Goal Path Planning (MTP) Problem

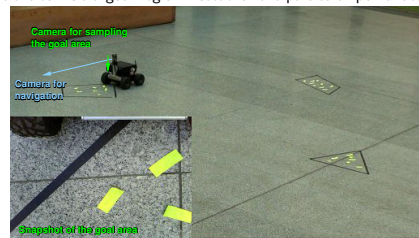
- **MTP problem** is a **robotic variant of the TSP** with the edge costs as the length of the **shortest** path connecting the locations.
- Variants of the **robotic TSP** includes additional constraints arising from limitations of real robotic systems such as
 - obstacles, curvature-constraints, sensing range, location precision.
- For n locations, we need to compute up to n^2 shortest paths.
- Having a **roadmap** (graph) representing C_{free} , the paths can be found in the graph (roadmap), from which the $G(V,E)$ for the TSP can be constructed. *Visibility graph as a roadmap for a point robot provides a straight forward solution, but such a shortest path may not be necessarily feasible for more complex robots.*
- We can determine the roadmap using randomized sampling-based motion planning techniques.

See lecture 8.



Multi-goal Path Planning with Goal Regions

- It may be sufficient to visit a goal region instead of the particular point location.



Not only a **sequence** of goals visit has to be determined, but also an **appropriate location** at each region has to be found.

The problem with goal regions can be considered as a variant of the **Traveling Salesman Problem with Neighborhoods (TSPN)**.

Traveling Salesman Problem with Neighborhoods

Given a set of n regions (neighbourhoods), what is the shortest closed path that visits each region.

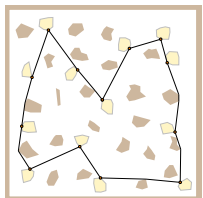
- The problem is NP-hard and APX-hard, it cannot be approximated to within factor $2 - \epsilon$, where $\epsilon > 0$. *Safra and Schwartz (2006) – Computational Complexity*
- Approximate algorithms exist for particular problem variants such as disjoint unit disk neighborhoods.
- **TSPN provides a suitable problem formulation for planning various inspection and data collection missions.**
- It enables to exploit non-zero sensing range, and thus find shortest (more cost-efficient) **data collection plans**.

Traveling Salesman Problem with Neighborhoods (TSPN)

- Instead visiting a particular location $s \in S$, $s \in \mathbb{R}^2$ as in the TSP, we request to visit a set of regions $R = \{r_1, \dots, r_n\}$, $r_i \subset \mathbb{R}^2$ to save travel cost.
- The TSP becomes the **TSP with Neighborhoods (TSPN)** where, in addition to the determination of the **sequence** Σ , we determine a suitable locations of visits $P = \{p_1, \dots, p_n\}$, $p_i \in r_i$.
- The problem is a combination of combinatorial optimization to determine Σ with **continuous optimization** to determine P .

$$\text{minimize}_{\Sigma, P} \quad L = \left(\sum_{i=1}^{n-1} c(p_{\sigma_i}, p_{\sigma_{i+1}}) \right) + c(p_{\sigma_n}, p_{\sigma_1})$$

$$\text{subject to} \quad \begin{aligned} R &= \{r_1, \dots, r_n\}, r_i \subset \mathbb{R}^2 \\ P &= \{p_1, \dots, p_n\}, p_i \in r_i \\ \Sigma &= (\sigma_1, \dots, \sigma_n), 1 \leq \sigma_i \leq n, \\ &\sigma_i \neq \sigma_j \text{ for } i \neq j \\ \text{Foreach } r_i \in R \text{ there is } p_i \in r_i. \end{aligned}$$

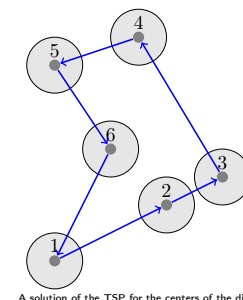


Approaches to the TSPN

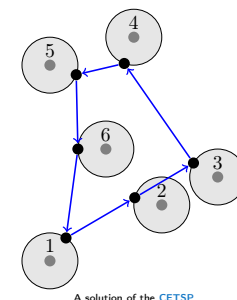
- A direct solution of the TSPN – approximation algorithms and heuristics *E.g., using evolutionary techniques or unsupervised learning*
- Euclidean TSPN with disk-shaped δ neighborhoods is called **Close Enough TSP (CETSP)**.
 - Simplified variant with regions as disks with radius δ – remote sensing with the δ communication range.
- **Decoupled approach**
 1. Determine sequence of visits Σ independently on the locations P , e.g., as a solution of the TSP using centroids of the (convex) regions R .
 2. For the sequence Σ determine the locations P to minimize the total tour length using
 - Touring polygon problem (TPP);
 - Sampling possible locations and use a forward search for finding the best locations;
 - Continuous optimization such as hill-climbing. *E.g., Local Iterative Optimization (LIO), Váňa & Faigl (IROS 2015)*
- **Sampling-based approaches**
 - For each region, sample possible locations of visits into a discrete set of locations for each region.
 - The problem can be then formulated as the **Generalized Traveling Salesman Problem (GTSP)**.

Close Enough Traveling Salesman Problem (CETSP)

- **Close Enough TSP (CETSP)** is a variant of the TSPN with disk shaped δ -neighborhoods.



A solution of the TSP for the centers of the disks

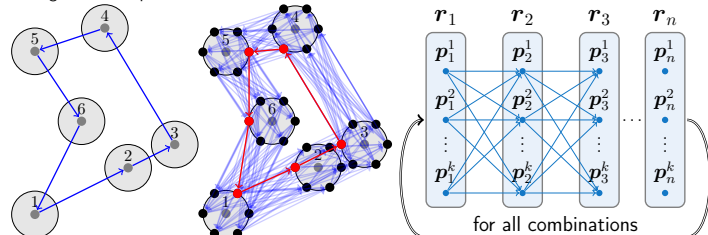


A solution of the CETSP

Decoupled Sampling-based Solution of the TSPN / CETSP

- **Decoupled** – Determine sequence of visits as a solution of the Euclidean TSP for the representatives of the regions R_i , e.g., using centroids.
- Sample each region (neighborhood) with k samples, e.g., $k = 6$.
- Construct graph and find the shortest tour in by graph search in $\mathcal{O}(nk^3)$ for n regions and nk^2 edges in the sequence.

For the closed path, we need to examine all k possible starting locations.



Iterative Refinement in the Multi-goal Planning Problem with Regions

- Let the sequence of n polygon regions be $R = (r_1, \dots, r_n)$. *Li, F., Klette, R.: Approximate algorithms for touring a sequence of polygons. 2008*
- 1. Sampling regions into a discrete set of points and determine all shortest paths between each sampled points in the sequence of visits to the regions. *E.g., using visibility graph*
- 2. **Initialization:** Construct an initial touring polygons path using a sampled point of each region. Let the path be defined by $P = (p_1, p_2, \dots, p_n)$, where $p_i \in r_i$ and $L(P)$ be the length of the shortest path induced by P .
- 3. **Refinement:** For $i = 1, 2, \dots, n$:
 - Find $p_i^* \in r_i$ minimizing the length of the path $d(p_{i-1}, p_i^*) + d(p_i^*, p_{i+1})$, where $d(p_k, p_l)$ is the path length from p_k to p_l , $p_0 = p_n$, and $p_{n+1} = p_1$.
 - If the total length of the current path over point p_i^* is shorter than over p_i , replace the point p_i by p_i^* .
- 4. Compute the path length L_{new} using the refined points.
- 5. **Termination condition:** If $L_{new} - L < \epsilon$ Stop the refinement. Otherwise $L \leftarrow L_{new}$ and go to Step 3.
- 6. **Final path construction:** Use the last points and construct the path using the shortest paths among obstacles between two consecutive points. *On-line sampling during the iterations – Local Iterative Optimization (LIO), Váňa & Faigl (IROS 2015).*



Part II

Part 2 – Unsupervised Learning for Multi-goal Planning

Unsupervised Learning based Solution of the TSP

- Iterative learning procedure where neurons (nodes) adapt to the target locations.

- Based on self-organizing map by T. Kohonen.

Somhom, S., Modares, A., Enkawa, T. (1999)

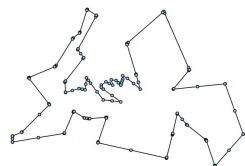
- Deployed in robotic problems such as inspection and search-and-rescue planning.

Faigl, J. et al. (2011)

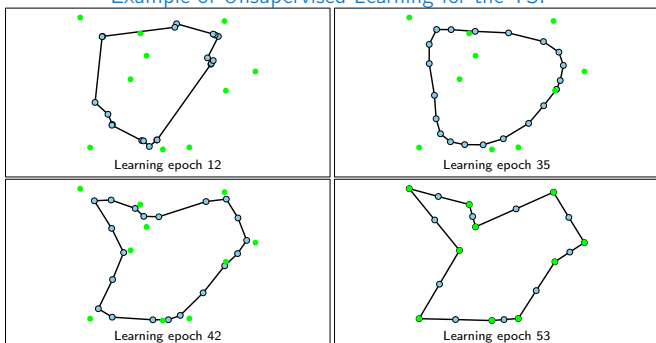
- Generalized to polygonal domain with (overlapping) regions.

- Evolved to **Growing Self-Organizing Array (GSOA)**.

A general heuristic for various routing problems with neighborhoods, including routing problems with profit aka the orienteering problem.



Example of Unsupervised Learning for the TSP



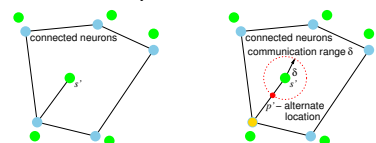
Unsupervised Learning for the TSPN

- A suitable location of the region can be sampled during the winner selection.
- We can use the centroid of the region for the shortest path computation from ν to the region r presented to the network.

- Then, an intersection point of the path with the region can be used as an alternate location.

Faigl, J. et al. (2013): *Visiting convex regions in a polygonal map*. Robotics and Autonomous Systems.

- For the Euclidean TSPN with disk-shaped δ neighborhoods, we can compute the alternate location directly from the Euclidean distance.



Unsupervised Learning based Solution of the TSP

Kohonen's type of **unsupervised** two-layered neural network (**Self-Organizing Map**)

- Neurons' **weights** represent **nodes** $\mathcal{N} = \{\nu_1, \dots, \nu_m\}$ in a **plane** (input space $\mathbb{R}^2$).

- Nodes are organized into a **ring** that evolved in the output space $\mathbb{R}^2$.

- Target locations $\mathcal{S} = \{s_1, \dots, s_n\}$ are presented to the network in a **random** order.

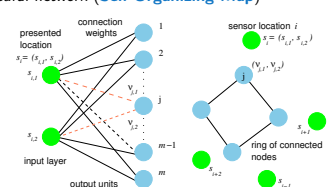
- Nodes **compete** to be winner according to their distance to the presented goal s

$$\nu^* = \operatorname{argmin}_{\nu \in \mathcal{N}} \|\mathcal{D}(\{\nu, s\})\|.$$

- The **winner** and its **neighbouring** nodes are adapted (**moved**) towards the target according to the neighbouring function

$$f(\sigma, d) = \begin{cases} e^{-\frac{d^2}{2\sigma^2}} & \text{for } d < m/n, \\ 0 & \text{otherwise,} \end{cases}$$

- Best matching unit ν to the presented prototype s is determined according to the distance function $\|\mathcal{D}(\nu, s)\|$.



- For the Euclidean TSP, $\mathcal{D}$ is the Euclidean distance
- However, for problems with obstacles, the multi-goal path planning, $\mathcal{D}$ should correspond to the length of the shortest, collision-free path.

Fort, J.C. (1988), Angéniol, B. et al. (1988), Somhom, S. et al. (1997), and further improvements.

Unsupervised Learning for the Multi-Goal Path Planning

- Unsupervised learning procedure for the **Multi-goal Path Planning (MTP)** problem a robotic variant of the Traveling Salesman Problem (TSP).

Algorithm 2: SOM-based MTP solver

$\mathcal{N} \leftarrow \text{initialization}(\nu_1, \dots, \nu_m);$

repeat

$\text{error} \leftarrow 0;$

foreach $g \in \Pi(\mathcal{S})$ **do**

$\nu^* \leftarrow \operatorname{selectWinner} \operatorname{argmin}_{\nu \in \mathcal{N}} \|S(g, \nu)\|;$

$\text{adapt}(S(g, \nu), \mu f(\sigma, \|\mathcal{D}(g, \nu)\|));$

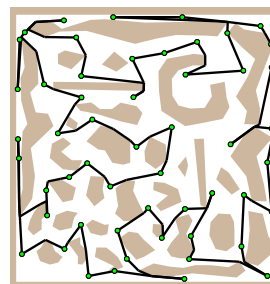
$\text{error} \leftarrow \max\{\text{error}, \|S(g, \nu^*)\|\};$

$\sigma \leftarrow (1 - \alpha)\sigma;$

until $\text{error} \leq \delta;$

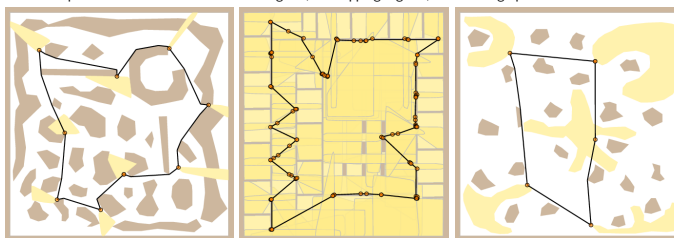
- For multi-goal path planning – the **selectWinner** and **adapt** procedures are based on the solution of the **path planning problem**.

Faigl, J., Kulich, M., Vonásek, V., Přeucil, L.: *An Application of Self-Organizing Map in the non-Euclidean Traveling Salesman Problem*, Neurocomputing, 74(5):671-679, 2011.



SOM for the Traveling Salesman Problem with Neighborhoods (TSPN)

- Unsupervised learning of the SOM for the TSP allows to generalize the adaptation procedure to the TSPN.
- It also provides solutions for non-convex regions, overlapping regions, and coverage problems.



Polygonal Goals

$n=9, T=0.32 \text{ s}$

Convex Cover Set

$n=106, T=5.1 \text{ s}$

Non-Convex Goals

$n=5, T=0.1 \text{ s}$

Faigl, J., Vonásek, V., Přeucil, L.: *Visiting Convex Regions in a Polygonal Map*, Robotics and Autonomous Systems, 61(10):1070-1083, 2013.

Unsupervised Learning based Solution of the TSP - Detail

- Target (sensor) locations $\mathcal{S} = \{s_1, \dots, s_n\}, s_i \in \mathbb{R}^2$; Neurons $\mathcal{N} = \{\nu_1, \dots, \nu_m\}, \nu_i \in \mathbb{R}^2, m = 2.5n$.
- Learning gain σ ; epoch counter i ; gain decreasing rate $\alpha = 0.1$; learning rate $\mu = 0.6$.

- $\mathcal{N} \leftarrow$ init ring of neurons as a small ring around some $s_i \in \mathcal{S}$, e.g., a circle with radius 0.5.

- $i \leftarrow 0$; $\sigma \leftarrow 12.41n + 0.06$;

- $i \leftarrow 0$

//clear inhibited neurons

- foreach** $s \in \Pi(\mathcal{S})$ (a permutation of $\mathcal{S}$)

 4.1 $\nu^* \leftarrow \operatorname{argmin}_{\nu \in \mathcal{N} \setminus I} \|\mathcal{D}(\nu, s)\|$

 4.2 **foreach** ν in d neighborhood of ν^*

$$\nu \leftarrow \nu + \mu f(\sigma, d)(s - \nu)$$

$$f(\sigma, d) = \begin{cases} e^{-\frac{d^2}{2\sigma^2}} & \text{for } d < 0.2m, \\ 0 & \text{otherwise,} \end{cases}$$

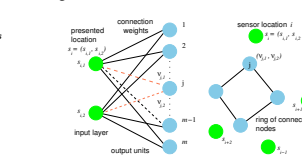
4.3 $I \leftarrow I \cup \{\nu^*\}$

// inhibit the winner

- $\sigma \leftarrow (1 - \alpha)\sigma$; $i \leftarrow i + 1$;

- If (**termination condition** is not satisfied) Goto Step 3;

Otherwise retrieve solution.



- Maximal number of learning epochs $i \leq i_{\max}$, e.g., $i_{\max} = 120$.
- Winner neurons are negligibly close to sensor locations, e.g., less than 0.001.

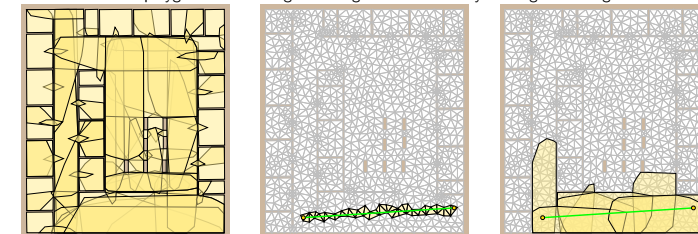
Somhom, S., Modares, A., Enkawa, T. (1999): *Competition-based neural network for the multiple travelling salesman problem with minmax objective*, Computers & Operations Research.

Faigl, J. et al. (2011): *An application of the self-organizing map in the non-Euclidean Traveling Salesman Problem*, Neurocomputing.

SOM for the TSP in the Watchman Route Problem – Inspection Planning

During the unsupervised learning, we can compute **coverage** of $\mathcal{W}$ from the current **ring** (solution represented by the neurons) and **adapt** the network **towards uncovered parts** of $\mathcal{W}$.

- Convex cover set of $\mathcal{W}$ created on top of a triangular mesh.
- Incident convex polygons with a straight line segment are found by walking in a triangular mesh.



Faigl, J.: *Approximate solution of the multiple watchman routes problem with restricted visibility range*, IEEE Transactions on Neural Networks, 21(10):1668-1679, 2010.

Growing Self-Organizing Array (GSOA)

- Growing Self-Organizing Array (GSOA)** is generalization of the unsupervised learning to routing problems motivated by data collection planning, i.e., routing with neighborhoods such as the **Close Enough TSP**.

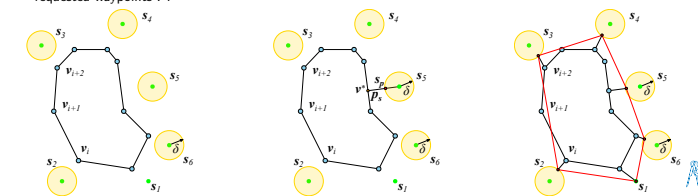
- The GSOA is an array of nodes $\mathcal{N} = \{\nu_1, \dots, \nu_M\}$ that evolves in the problem space using unsupervised learning.

- The array adapts to each $s \in \mathcal{S}$ (in a random order) and for each s a **new winner node** ν^* is determined together with the corresponding s_p , such that $\|\mathcal{D}(s_p, s)\| \leq \delta(s)$.

It adaptively adjusts the number of nodes.

- The winner and its neighborhoods are adapted (moved) towards s_p .

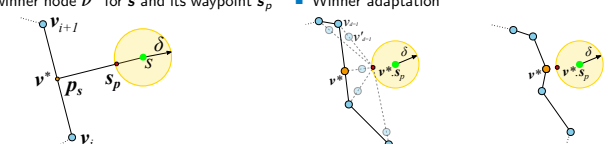
- After the adaptation to all $s \in \mathcal{S}$, each s has its ν and s_p , and the array defines the sequence Σ and the requested waypoints P .



GSOA – Winner Selection and Its Adaptation

■ Selecting winner node ν^* for s and its waypoint s_p

■ Winner adaptation

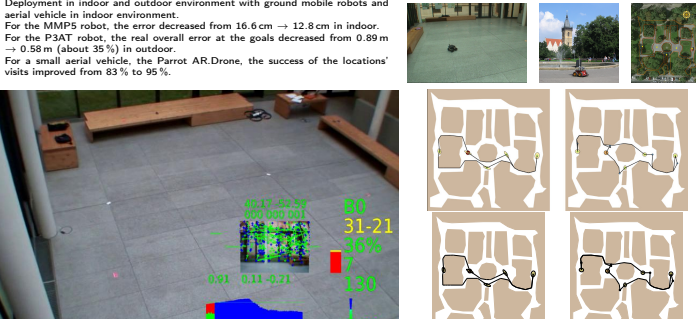


- For each $s \in S$, we create new node ν^* , and therefore, all not winning nodes are removed after processing all locations in S (one learning epoch) to balance the number of nodes in the GSOA.
- After **each learning epoch**, the **GSOA encodes a feasible solution of the CETSP**.
- The power of adaptation is decreasing using a cooling schedule after each learning epoch.
- The GSOA converges to a stable solution in tens of epochs. Number of epochs can be set.

Faigl, J. (2018): *GSOA: Growing Self-Organizing Array - Unsupervised learning for the Close-Enough Traveling Salesman Problem and other routing problems*. Neurocomputing 312: 120-134 (2018).

Example – Results on the TSPN for Planning with Localization Uncertainty

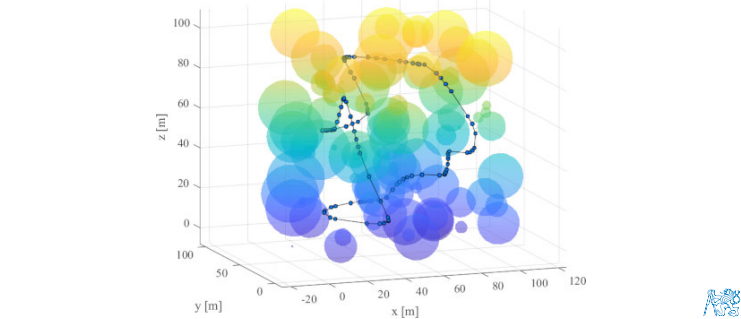
- Deployment in indoor and outdoor environment with ground mobile robots and aerial vehicle in indoor environment.
- For the MMP5 robot, the error decreased from 16.6 cm $\rightarrow$ 12.8 cm in indoor.
- For the P3AT robot, the real overall error at the goals decreased from 0.89 m $\rightarrow$ 0.58 m (about 35 %) in outdoor.
- For a small aerial vehicle, the Parrot AR.Drone, the success of the locations' visits improved from 83 % to 95 %.



Metric	TSP	TSPN
L [m]	184	202
E_{avg} [m]	0.57	0.35

TSP: L=184 m, E_{avg} =0.57 m TSPN: L=202 m, E_{avg} =0.35 m

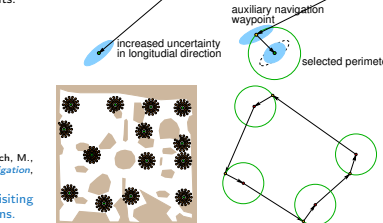
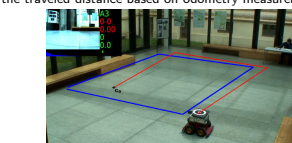
GSOA Evolution in solving the 3D CETSP



Summary of the Lecture

Example – TSPN for Planning with Localization Uncertainty

- Teach-and-repeat autonomous navigation using vision-based bearing corrections that are more precise than estimation of the traveled distance based on odometry measurements.



- The localization uncertainty can be decreased by visiting auxiliary navigation waypoints prior the target locations.
- It can be formulated as a variant of the TSPN with auxiliary navigation waypoints.
- The adaptation procedure is modified to select the auxiliary navigation waypoint to decrease the expected localization error at the target locations.

Faigl, J., Krajník, T., Vonašek, V., and Přeucil, L.: *On localization uncertainty in an autonomous inspection*, IEEE International Conference on Robotics and Automation (ICRA), 2012, pp. 1119-1124.

Topics Discussed

- Robotic information gathering in inspection missions
- Inspection planning and multi-goal path planning - coverage planning
- **Multi-goal path planning (MTP)**
 - Robotic Traveling Salesman Problem (TSP)
 - Traveling Salesman Problem with Neighborhoods (TSPN) and Close Enough Traveling Salesman Problem (CETSP)
 - Decoupled and Sampling-based approaches
 - TSP can be solved by efficient heuristics such as LKH
 - Optimal, approximation, and heuristics solutions
 - Generalized TSP (GTSP)
- Next: Data collection planning